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Vol. 50, No. 1, January 1978
Printed in Belgium

On the Algebraic L-Theory of Semisimple Rings

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Communicated by A. Fröhlich

Received February 24, 1977

Let $U_n(A)$, $V_n(A)$ be the algebraic L-groups of a ring with involution A defined for $n(\mathrm{mod}\ 4)$ [3]. Thus $\begin{smallmatrix} U_{2i}(A) \\ U_{2i+1}(A) \end{smallmatrix}$ (resp. $\begin{smallmatrix} V_{2i}(A) \\ V_{2i+1}(A) \end{smallmatrix}$) is the Witt group of nonsingular $(-)^i$quadratic $\begin{smallmatrix}\text{forms}\\ \text{formations}\end{smallmatrix}$ over A involving f.g. projective (resp. f.g. free) A-modules. Here, a formation is a nonsingular $(-)^i$quadratic form together with an ordered pair of maximally self-orthogonal direct summands. $V_{2i+1}(A)$ can be interpreted as a group of stable automorphisms of nonsingular $(-)^i$quadratic forms over A.

The main result [2, Theorem 3.1] concerns $V_{2i+1}(A)$ for semisimple A. It may be proved and generalized as follows.

PROPOSITION. *If A is a semisimple ring with involution then*

$$U_{2i+1}(A) = 0,$$
$$V_{2i+1}(A) = \mathrm{coker}(U_{2i+2}(A) \to \hat{H}^0(\mathbb{Z}_2; \tilde{K}_0(A))).$$

Proof. A semisimple ring A is characterized by the property that every short exact sequence of A-modules splits [1, Theorem III.1.5]. In particular, every submodule of an A-module is a direct summand, and it is now immediate from [3, Theorem 2.3] that $U_{2i+1}(A) = 0$. The expression for $V_{2i+1}(A)$ may be deduced from the relevant part of the exact sequence of [3, Theorem 4.3]

$$U_{2i+2}(A) \to \hat{H}^0(\mathbb{Z}_2; \tilde{K}_0(A)) \to V_{2i+1}(A) \to U_{2i+1}(A).$$

The reduced Tate cohomology group is defined by

$$\hat{H}^0(\mathbb{Z}_2; \tilde{K}_0(A)) = \{[P] \in \tilde{K}_0(A) \mid [P^*] = [P]\}/\{[Q] + [Q^*] \mid [Q] \in \tilde{K}_0(A)\},$$

with $\tilde{K}_0(A)$ the reduced projective class group and $P \mapsto P^* = \mathrm{Hom}_A(P, A)$ the duality operation on f.g. projective A-modules P.

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0021-8693/78/0501-0242$02.00/0

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Printed by the St Catherine Press Ltd., Tempelhof 37, Bruges, Belgium.