

The Mathematical Intelligencer encourages comments about the material in this issue. Letters to the editor should be sent to the editor-in-chief, Chandler Davis.

Some History of the Hauptvermutung

We offer a comment on the article by Peter Hilton and Ioan James on "The Whitehead Heritage" in the Winter 1997 issue of *The Mathematical Intelligencer*. The authors remarked that Whitehead was attracted to the "Hauptvermutung," but they only stated the conjecture for manifolds. In view of Whitehead's deeper involvement in complexes, it should be observed that the original conjecture was in this wider context, asserting that homeomorphic simplicial complexes are combinatorially equivalent. In this generality the first counterexamples were obtained by John Milnor in 1961, using Whitehead's own simple homotopy theory. The first positive results on the more difficult manifold case were obtained by Andrew Casson and Dennis Sullivan in 1967. Their articles were not published at the time, but may now be found in *The Hauptvermutung Book* (K-Monographs in Mathematics 1, Kluwer, 1996, A. Ranicki ed.). A counterexample to the full manifold statement was found by Larry Siebenmann in 1969. Whitehead's work was not a direct primary ingredient of the manifold work. However, by then the "handlebody" theory introduced by Smale a decade earlier had become the principal tool used in manifolds. Handlebody theory is a translation of Whitehead's cell complexes to the manifold context.

FRANK QUINN
Virginia Tech
Blacksburg, VA 24061-0123
USA
e-mail: quinn@math.vt.edu

ANDREW RANICKI
University of Edinburgh
Edinburgh
Scotland, UK
e-mail: aar@maths.ed.ac.uk

Intuitionism and Celestial Mechanics

In "The solution to the n -body problem" (*Intelligencer*, vol. 18 (1996), no. 3, 66–70), Florin Diacu refers to Brouwer's intuitionism in connection with a 1907 paper by Sundman. As he was trying to undo misleading folk-mathematics, it is unfortunate that Prof. Diacu relied on R. L. Goodstein's *Essays in the Philosophy of Mathematics* (p. 5) as a source of information on the *Annalen* affair. Thus he relayed the folk-tale that Brouwer had "rejected all submitted papers that used *reductio ad absurdum* as a method of proof," as well as the equally erroneous story of the resignation and self-reelection (minus Brouwer) of the board of the *Annalen*. He could have got the story right by referring to the well-documented article by D. van Dalen (*Intelligencer*, vol. 12 (1990), no. 4, 17–31).

None of the editors of the *Annalen* had accused Brouwer of imposing his philosophical position on authors. In the circular letter sent to his fellow-editors, Blumenthal stated, "Brouwer has been a very conscientious and capable editor" (W. P. van Stigt, *Brouwer's Intuitionism*, North-Holland, Amsterdam, 1990, p. 102). Both Carathéodory and Einstein refused to sign the dismissal notice. As to the re-election of the editorial board (minus Brouwer), here were the editors of 1928:

1. under "unter Mitwirkung von": L. Bieberbach, H. Bohr, L.E.J. Brouwer, R. Courant, W. v. Dyck, O. Hölder, Th. v. Kármán, A. Sommerfeld;

2. under "gegenwärtig herausgegeben von": D. Hilbert, A. Einstein, O. Blumenthal, C. Carathéodory; and here are the editors of 1929:

1. under "unter Mitwirkung von": O. Blumenthal, E. Hecke;

2. under "gegenwärtig herausgegeben von": D. Hilbert.

Prof. Diacu also conjectures that Brouwer might not have developed his intuitionism “had he known and understood Sundman’s work.” This seems far-fetched to me, considering Brouwer’s own pronouncements on what led him in that direction (as for example in L.E.J. Brouwer, *Collected Works*, North-Holland, Amsterdam, 1975, 472–476), as well as his general opposition to the applications of mathematics. Of many passages expressing this opposition, I quote this comment on the “games” of mathematics and logic:

De par leur nature, ils ne devraient pas s’immiscer dans la vie sociale. Celle-ci les ayant néanmoins réclamés, ils subissent l’influence des sciences pragmatiques tout en coopérant, contre leur nature, aux transformations de la vie sociale qu’on appelle le progrès. Heureusement, leurs plus beaux développements n’auront probablement jamais aucun rapport avec les questions techniques, économiques ou politiques.” (L.E.J. Brouwer,

Collected Works, Vol. 1, North-Holland, Amsterdam, 1975, p. 503).¹

There may be constructivists who reject non-constructive proof because they think it “useless from a practical point of view.” Brouwer was not one of them. His motivation for intuitionism lies elsewhere, and can be discerned only by a careful reading of his own words.

VICTOR PAMBUCCIAN
Department of Integrative Studies
Arizona State University West
P.O. Box 37100
Phoenix, AZ 85069-7100
USA
e-mail: pamb@math.west.asu.edu

Florin Diacu replies:

The excellent *Intelligencer* article by van Dalen came to my attention some time after I completed my article. Prof. Pambuccian is quite right. This illustrates that misleading folk-tales can be transmitted in print (the Goodstein book) as well as orally.

Prof. Pambuccian goes on to advance pertinent arguments against my specu-

lation on Brouwer’s thinking. Let me state my reasons to leave it standing.’

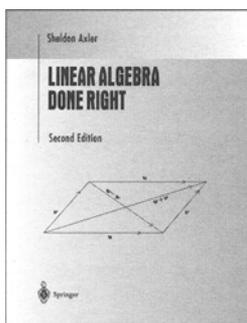
Constructivism is a nonnegligible part of Brouwer’s intuitionism. The first chapter of *On the Foundations of Mathematics*, entitled “The Construction of Mathematics,” makes clear that Brouwer has founded his theory on constructivism, independently of any other reasons he might have had. But as Gödel’s result shows the feebleness of formalism, Sundman’s proof reveals the weakness of intuitionism, which was well hidden up to now. I doubt that Brouwer would have remained insensitive to this fact, had he known it.

Finally, I would like to thank all those who have sent me comments on my article. The list of names is too long to append.

FLORIN DIACU
Department of Mathematics and Statistics
University of Victoria
Victoria, British Columbia V8W 3P4
Canada
diacu@math.uvic.ca

¹By their nature, they ought not to be involved in human affairs. However, human affairs suck them in, and they come under the influence of the practical sciences, cooperating contrary to their nature in the transformations known as progress. Fortunately, their most beautiful developments will probably never have anything to do with technical, economic, or political questions.

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