MIGSAA course: Singular stochastic PDEs

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In this course, we study stochastic partial differential equations (SPDEs) from the analytical point of view, both in the *parabolic* and *dispersive* cases. In particular, by combining harmonic analysis and stochastic analysis, we study well-posedness (existence, uniqueness, and stability under perturbation) of some singular SPDEs with very rough and random forcing.

Over the last ten years, there has been a significant development in the theoretical understanding of singular stochastic PDEs in the parabolic setting, most notably the theory of regularity structures by Hairer (for which he was awarded the Fields medal in 2014) and the theory of paracontrolled distributions by Gubinelli. These two different but somewhat connected approaches turned out to be very effective in studying singular stochastic *parabolic* PDEs.

In the field of nonlinear *dispersive* PDEs (such as the nonlinear Schrödinger equations and nonlinear wave equations), multilinear harmonic analysis has played a fundamental role in the theoretical development of the subject over the last thirty years. In more recent years (since mid 2000's), a remarkable combination of PDE techniques and probability theory allowed us to obtain *probabilistic* well-posedness results (with random initial data) that are out of reach in the deterministic setting. This development further led to the study of singular stochastic *dispersive* PDEs over the last five years.

In this course, we start by going over basic concepts in stochastic analysis and PDEs to study stochastic PDEs in both parabolic and dispersive settings. Topics in the parabolic settings include:

- parabolic Φ_2-model: local well-posedness, invariant measures, etc.
- paracontrolled approach to parabolic Φ^4_3-model.

By combining the Fourier analytic approach with stochastic analysis, we also study singular stochastic dispersive PDEs such as the stochastic nonlinear wave equations and the stochastic nonlinear Schrödinger equations. Topics may include:

- 2-d stochastic wave equations with space-time white noise forcing (= canonical stochastic quantization equation),
- 3-d stochastic wave equations,
- stochastic nonlinear Schrödinger equations with rough forcing.

Prerequisites:

- real analysis and basic PDE theory.

Other particulars:

- A preferred format of this course is to hold one 2 hour lectures/week.

- This course covers the materials at the current research topics. As such, assignments and exams are not so effective as those for lower level courses. Thus, we would count the attendance and typing lecture notes (filling in details) as course assessment.
- We would like to teach this course in Spring 2021.