MIGSAA course: **Rough path theory and pathwise well-posedness of stochastic PDEs**

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The first goal of this course is to provide a brief introduction of *rough path theory*, originally introduced by Terry Lyons (1998), and further developed by Gubinelli (2004). The rough path theory provides a generalization of Lebesgue-Stieljes integrals and Young integrals (when integrands do not have sufficient regularity) by *a priori* providing higher order reference distributions, based on which rough integrals can be defined. We use it to provide a pathwise meaning of stochastic differential equations by viewing it as a rough differential equation.

We then move onto studying stochastic partial differential equations (SPDEs). One option here is to study a pathwise meaning of SPDEs with multiplicative noise by introducing a rough path structure in time. Another option here is to discuss a recent development in singular SPDEs. This is done by breaking the classically ill-posed solution map into two steps: (i) construct an enhanced data set from given random data and (ii) solve the resulting equation by constructing a deterministic continuous map (Ito-Lyons map) from the enhanced data set to a solution. We may consider equations of parabolic type (such as heat, reaction-diffusion) or dispersive type (such as Schrödinger and wave), depending on the interest of students.

Some references:

(1) P. Friz, M. Hairer, *A course on rough paths. With an introduction to regularity structures*, Universitext. Springer, Cham, 2014. xiv+251 pp.
(2) Friz, Peter K.; Victoir, Nicolas B. Multidimensional stochastic processes as rough paths. Theory and applications. Cambridge Studies in Advanced Mathematics, 120. Cambridge University Press, Cambridge, 2010. xiv+65
(3) Fabrice Baudoin's lectures: https://fabricebaudoin.wordpress.com/category/rough-paths-theory/page/3/
(4) T. Lyons, Differential equations driven by rough signals, Rev. Mat. Iberoamericana 14 (1998), no. 2, 215–310.
(5) M. Gubinelli, Controlling rough paths, J. Funct. Anal. 216 (2004), no. 1, 86–140.
(6) The SPDE part will be based on my lecture note (to be prepared).

Other particulars:

- This course covers the materials at the research materials. As such, assignments and exams are not so effective as those for lower level courses. While I may give small assignments (in particular on the rough path part), I will have students to type lecture notes (filling in details, which may include some exercises such as proving some lemmas) and count it as a course assessment. Depending on the number of students in the class, I may ask some students to present course materials and count it as a course work.