MIGSAA course: Two-dimensional statistical hydrodynamics

Tadahiro Oh (University of Edinburgh)

In this course, we study evolution equations describing the motion of fluids, in particular from the statistical point of view. We primarily study the randomly forced Navier-Stokes equation (NSE). We also study the Euler equation as the limiting equation in the fluctuation-dissipation limit of the stochastic NSE.

We start the course by going over the basic deterministic well-posedness theory of NSE (namely, existence, uniqueness, and stability under perturbation of solutions). We then move onto studying the stochastic NSE with additive stochastic forcing, first kick force in time and then white in time (with a smooth covariance function in space). By viewing the stochastic NSE as a dynamical system, we construct an invariant measure (= stationary measure) for the dynamics. This is achieved by the Bogolyubov-Krylov argument. Under some assumption, we then use the idea of coupling and prove uniqueness and hence ergodicity of the invariant measure. This allows us to show that starting with any initial data, the corresponding solution approaches to this invariant state. By taking the fluctuation-dissipation limit of the stochastic NSE, we also construct an invariant measure for the Euler equation.

Some references:

(1) S. Kuksin, *Randomly forced nonlinear PDEs and statistical hydrodynamics in 2 space dimensions,* Zürich Lectures in Advanced Mathematics. European Mathematical Society (EMS), Zürich, 2006. x+93 pp.

(2) S. Kuksin, A. Shirikyan, *Mathematics of two-dimensional turbulence,* Cambridge Tracts in Mathematics, 194. Cambridge University Press, Cambridge, 2012. xvi+320 pp.

(3) S. Albeverio, A. Cruzeiro, *Global flows with invariant (Gibbs) measures for Euler and Navier-Stokes two dimensional fluids,* Comm. Math. Phys. 129 (1990) 431–444.

Other particulars:

- A preferred format of this course is to hold two-hour lectures once a week.
- This course covers the materials at the research materials. As such, assignments and exams are not so effective as those for lower level courses. While I may give small assignments, I will have students to type lecture notes (filling in details, which may include some exercises such as proving some lemmas) and count it as a course assessment.