First-Order Methods in Nonlinear Model Predictive Control

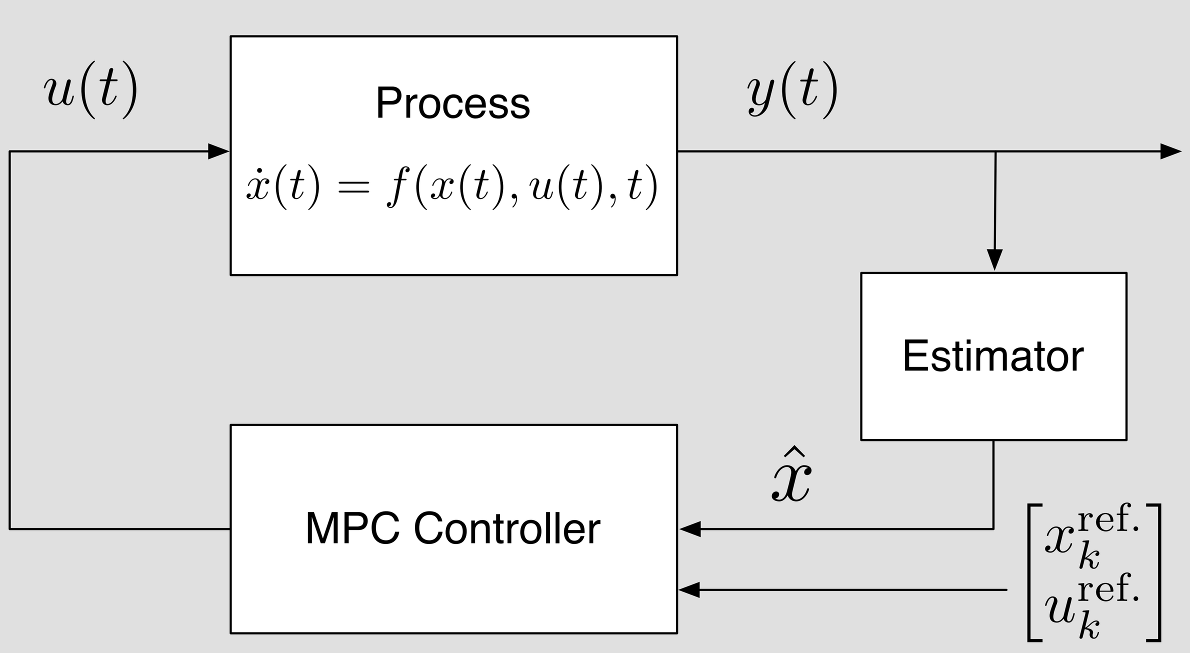
D. Kouzoupis*, H.J. Ferreau * *, H. Peyrl * *, M. Diehl *

*Laboratory of Systems, Control and Optimization, Department of Microsystems Engineering (IMTEK), University of Freiburg, Germany
** ABB Corporate Research, Baden-Dättwil, Switzerland

Summary

In embedded nonlinear MPC, optimization problems of several hundred variables must often be solved within few milliseconds. We assess the performance of first-order quadratic programming (QP) solvers that feature: simple algorithmic schemes that are suitable for parallelization, good warm-staring properties for using knowledge of previous solution and flexibility in trade-off between accuracy and speed.

Principle of Model Predictive Control



$$\begin{aligned} &\underset{\substack{u_0,\ldots,u_{N-1}\\ x_0,\ldots,x_N}}{\text{minimize}} \quad \sum_{k=0}^{N-1} \frac{1}{2} \begin{pmatrix} x_k - x_k^{\text{ref.}} \\ u_k - u_k^{\text{ref.}} \end{pmatrix}^T \begin{pmatrix} Q_k & 0 \\ 0 & R_k \end{pmatrix} \begin{pmatrix} x_k - x_k^{\text{ref.}} \\ u_k - u_k^{\text{ref.}} \end{pmatrix} \\ &\text{subject to:} \quad x_0 = \hat{x} \\ &\qquad x_{k+1} = A_k x_k + B_k u_k + c_k, \qquad \text{for } k = 0,\ldots,N-1 \\ &\qquad u_k^l \le u_k \le u_k^u \qquad \text{for } k = 0,\ldots,N-1 \\ &\qquad x_k^l \le x_k \le x_k^u \qquad \text{for } k = 0,\ldots,N \end{aligned}$$

At each sampling time:

- full state information of process is estimated.
- optimal inputs for horizon of N*Ts [s] are calculated (QP).
- only *first* input is applied to the process.
- controller starts preparing next QP until new estimate arrives.

Solving QP subproblems

1. Build structured QP

$$\begin{aligned} &\underset{Z}{\text{minimize}} \ \frac{1}{2} Z^T H Z + h^T Z \\ &\text{subject to: } AZ = b \\ &\qquad \underline{Z} \le Z \le \bar{Z} \end{aligned}$$

2. Dualize equality constraints

$$\overbrace{\underset{\Lambda}{\text{maximize}}}^{\text{unconstr. maximization}} \ \underbrace{\underset{\underline{Z}\le Z\le \bar{Z}}{\text{minimize}} \ \frac{1}{2} Z^T H Z + h^T Z + \Lambda^T (AZ - b)}_{\text{inner problem: } Z^* = H^{-1}(A^T\Lambda - h)}$$

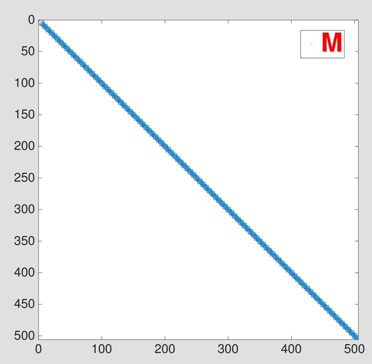
← If H diagonal

3. Solve with preconditioned fast gradient method

$$\begin{aligned} Z_{k+1} &= \underset{\underline{Z}\le Z\le \bar{Z}}{\text{argmin}} \ \frac{1}{2} Z^T H Z + h^T Z + Y_k^T (AZ - b) \\ \Lambda_{k+1} &= Y_k + M^{-1}(AZ_{k+1} - b) \\ Y_{k+1} &= \Lambda_{k+1} + \beta_k(\Lambda_{k+1} - \Lambda_k) \end{aligned}$$

Sparsity exploitation

Preconditioner M has a block tri-diagonal structure that we can exploit in both its factorization (sparse Cholesky decomposition) and the multiplication of its inverse with a vector.



Matrix	*non-zeros*
M	5705
R	3303
M^{-1}	255025

$$M = AH^{-1}A^T = R^T R$$

$$M^{-1}v = R\backslash(R^T\backslash v)$$

Control of inverted pendulum on a cart

Goal: Drive pendulum to upright position, respecting input and state constraints.

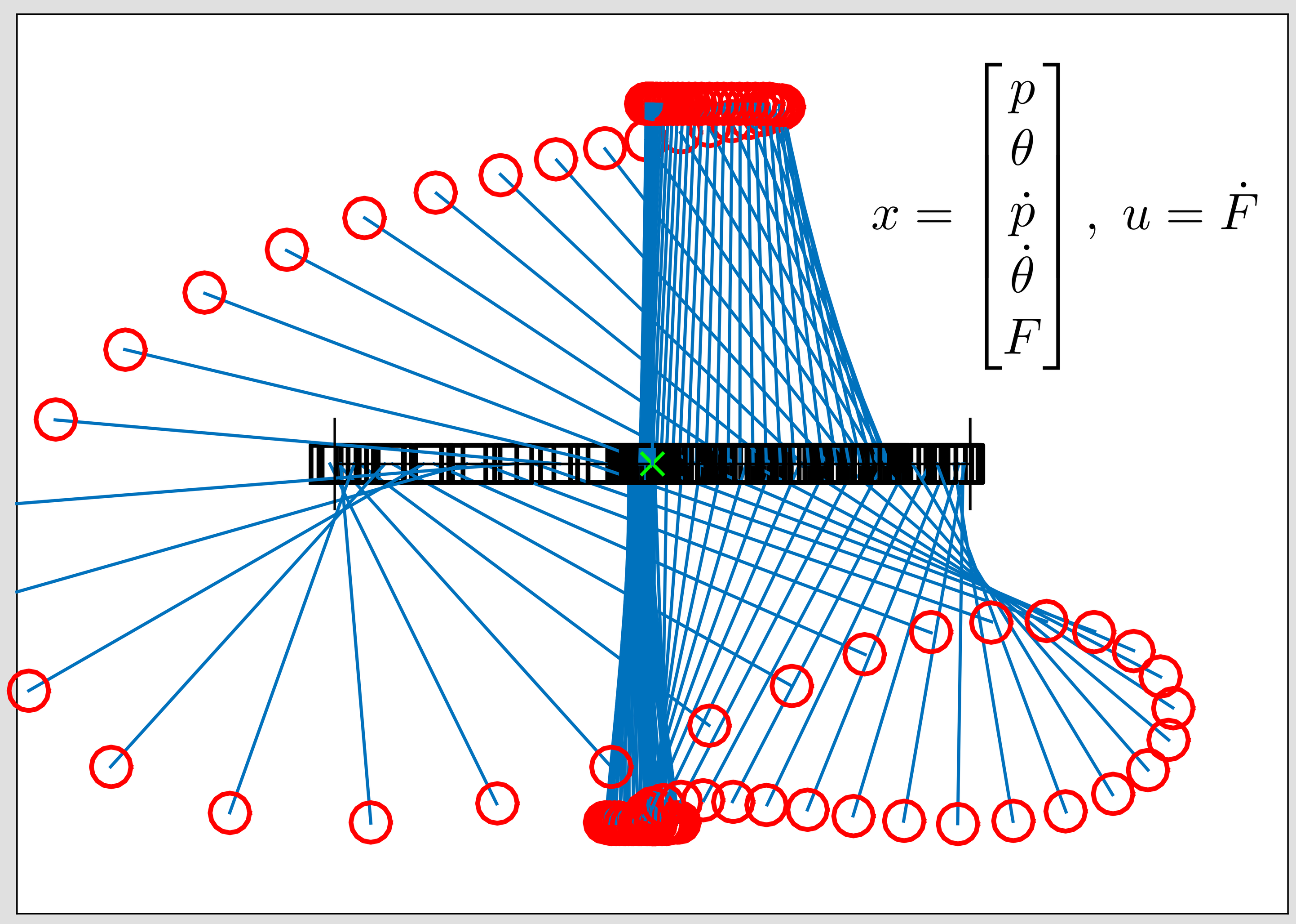


Figure 1: Closed-loop trajectory

- Sampling time T_s = 50 ms, horizon length N = 100 steps.
- 605 primal variables (states and controls).
- 505 dual variables (multipliers of equality constraints).
- 0.2 ms for integration and linearization of nonlinear dynamics
- 0.3 ms maximum execution time for QP solution.

Conclusions

The flexibility and simplicity of recently published first-order methods (often first proposed for image processing applications) may allow the use of nonlinear model predictive control at even higher sampling rates.

References

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[3] Y. Nesterov, *Introductory lectures on convex optimization: a basic course,* Kliwer Academic Publischers, 2004.

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