<u>Kleinian singularities</u>

<u>A_m singularities</u>

<u>dim = 2</u>

$x^2 + y^2 + z^{m+1} = 0 \qquad m \geqslant 2 \qquad \mathbb{C}^3$

singularity $\Rightarrow$ replace z^{m+1} by $p_w(z) = z^{m+1} + w_m z^m + \dots + w_0$

$g_w = x^2 + y^2 + p_w(z) \qquad w = (w_0, \dots, w_m)$

Consider symplectic manifold $g_w = 0$, with $\omega_{\mathbb{C}^3}$, $\theta_{\mathbb{C}^3}$. This has $c_1 = 0$ $\eta \wedge dg = dz_1 \wedge dz_2 \wedge dz_3$

$\Rightarrow$ can choose grading.

$\bar{w} \in W \subseteq B^{2m+2}(\delta)$ (so it's close to $x^2 + y^2 + z^{m+1}$) W is the set such that E_w smooth.

$X = g_{\bar{w}}^{-1}(0) = E_{\bar{w}}$

$\bar{w}$ = fixed ref. pt.

$$\begin{array}{ccc} E = \{(E_w, w)\} & & \\ \downarrow & & \\ W & \hookrightarrow & \mathrm{Conf}^{m+1}(D^2, \partial D^2) \end{array}$$

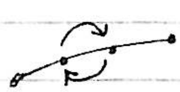
$m+1$ ~~points~~ distinct points in disc

$w \longmapsto$ zeroes of p_w

weak htpy equivalence

$\pi_1(\mathrm{Conf}^{m+1}) = \pi_1(W, \bar{w}) \hookrightarrow \pi_0(\mathrm{Symp}(X))$

$\parallel$

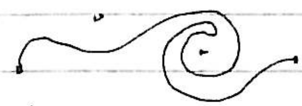
B_{m+1} via 

<u>Thm</u>: $B_{m+1} \overset{\rho}{\hookrightarrow} \pi_0(\mathrm{Symp}(X))$ injective.

$\mathfrak{Fuk}(X)$. What are Lagrangians?

$$\begin{array}{ccc} X = \{x^2+y^2+p(z)=0\} & & \\ \downarrow & \quad \downarrow \pi & \\ \mathbb{C} & \quad z & \end{array}$$

$\Delta = p^{-1}(0) =$ pts



matching cycle

Look at paths $\Rightarrow$ give Lag. spheres

$z_0 \quad \pi^{-1}(z_0) = Q_z \qquad z \notin \Delta \quad Q_z \simeq T^*S^1$

$p(z_0) = -1$

$Q_z : \quad x^2+y^2 = 1$

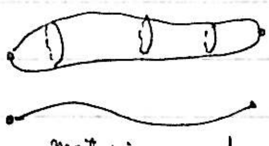
$\qquad (x,y) \longmapsto \left(\frac{x}{\|x\|}, \; y\|x\|\right) \in T^*S^1 = \left\{(u,v) : \|u\|=1, \; \langle u,v\rangle = 0\right\}$

$\qquad$ Symp. iso.

$z \notin \Delta \qquad Q_z \cong T^*S^1 \qquad \Sigma_z = S^1 \qquad (\Sigma_z \longleftrightarrow$ zero section$)$

$z \in \Delta \qquad Q_z = T^*S^1/S^1 \qquad \Sigma_z =$ pt.

$$L_\gamma = \bigcup_{z\in\gamma} \Sigma_z$$



matching cycle

<u>Prop</u>: (1) L_γ Lag. spheres

(2) $\gamma \underset{\text{iso.}}{\sim} \gamma' \Rightarrow L_\gamma \underset{\text{ham iso.}}{\sim} L_{\gamma'}$

$$\operatorname{rk} HF(L_\gamma, L_{\gamma'}, \mathbb{Z}_2) = 2\, I(\gamma,\gamma')$$

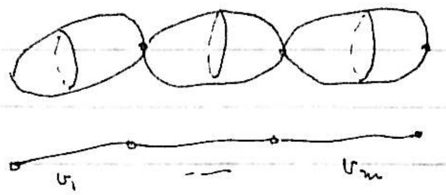
$$\downarrow$$

$$\sum_{z\in\gamma\cap\gamma'} |\gamma\cap\gamma' \setminus D| + \frac{1}{2}\underbrace{|\gamma\cap\gamma'\cap D|}_{\text{end points}}$$

Get S^1 of intersection

$\Rightarrow$ 2 pts by Morse Bott

Now choose paths

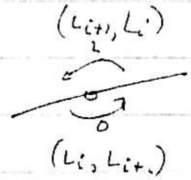


A_m chain of spheres.

$\mathcal{F}_m = \langle (v_1, \dots, v_m) \rangle \subseteq \mathcal{F}uk(X)$.

$H^*(\mathcal{F}_m)$

Gradings



$$HF^*(V_i^\#, V_i^\#) \overset{PSS}{=} H^*(S^2)$$

$$HF^*(V_i^\#, V_j^\#) = \begin{cases} 0 & |i-j| \geq 2 \\ \mathbb{Z} & j = i \pm 1 \end{cases}$$

$$\begin{tikzcd} V_1^\# \arrow[r, bend right, "\beta_i"', "0"'] & V_2^\# \arrow[l, bend right, "2"', "\alpha_i"'] \arrow[r, bend right, "0"'] & V_3^\# \arrow[l, bend right, "2"'] \end{tikzcd}$$

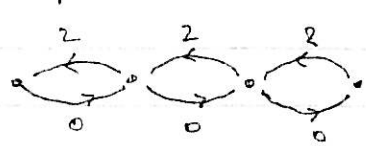
Poincaré duality: $HF^d(L, L') \otimes HF^{n-d}(L', L) \longrightarrow HF^n(L, L) \cong H^n(L) = \mathbb{Z}$.

nondegenerate

$\alpha_i \beta_i = k_i \beta_{i-1} \alpha_{i-1}$ (≠ 0 by nondegeneracy)

$k_i \in \mathbb{C}^*$

A_m quiver



path algebra

$(i \mid i+1 \mid i+2) = 0$

$(i \mid i-1 \mid i-2) = 0$

$(i \mid i-1 \mid i) = (i \mid i+1 \mid i)$

Path of length 0 = id in S^1

Path = top cohomology of S^1

Path = int. point.

$\mathbb{C}^m$ acts on algebra by i'th $\mathbb{C}$ acts on i'th spot.

① $V_i^\#$ split generate

② This A_m algebra is intrinsically formal

$\Pi\, Tw\, \mathcal{F}(X) \simeq \mathrm{mod}(A_m)$

Suppose $\pi : X \to X$

$\pi^2 = \pi$

$X \underset{i}{\overset{k}{\rightleftarrows}} Z \qquad Z = \operatorname{im} \pi$

$k \circ i = id_Z$

$i \circ k = \pi$

We call a category split closed if Z exists for any π.

A_∞ category split closed $\underset{(\infty)}{}$ $\Leftrightarrow$ $H^0(A)$ split closed $\underset{(usual)}{}$

Given A, define

$A \hookrightarrow \Pi A$

$\downarrow$

minimal split closed category containing A.

(Karoubi completion of A).

$V_i^\#$ split generate $\Leftrightarrow$ $\Pi\, Tw\, (V_i^\#) = \Pi\, Tw\, \mathcal{F}uk(X)$.

$T_{V^\#}(L) = T_{V^\#}(L)$

If $T_{Y_1} T_{Y_2} \cdots T_{Y_m}(X) = X[\sigma] \qquad \sigma \neq 0$

then $Y_1, \dots, Y_m$ split generate.

$\phi = \tau_{V_1} \cdots \tau_{V_m}$

$\phi^{2m+2}(L) = L[\sigma]$ (L = cpct Lag.)

depends on m, $\neq 0$. for m large enough

<u>Intrinsic formality:</u>

Thm: $HH^q(A_m, A_m[2-q]) = 0 \quad q \geq 3$ (from Sheel's talk)

$\Rightarrow A_\infty \underset{q\text{-iso}}{\simeq} A_m$

$H^0 A_\infty \ (= A_m)$