

REFERENCES FOR MINI-COURSE ON THE FUKAYA CATEGORY

1. LAGRANGIAN FLOER COHOMOLOGY

This lecture follows [Aur13, §1] closely.

We did not cover the analytical foundations (transversality, Gromov compactness, gluing) of Floer theory in the lecture. Here are some helpful references for learning this material: [MS04], [Sal99], lecture notes from Katrin Wehrheim's course on analysis of pseudoholomorphic curves (available from <https://piazza.com/berkeley/fall2013/berkeleymath278/resources>), and (for gluing theory especially) lecture notes from Katrin Wehrheim's course on regularization of moduli spaces of pseudoholomorphic curves (available from <https://math.berkeley.edu/~katrin/teach/regularization/lectures.shtml>). The generic regularity result that we cited in this lecture is from [FHS95].

The material about gradings is from [Sei99].

The proof that $HF^\bullet(L, L) \cong H^\bullet(L)$ can be found in [Flo89].

2. PRODUCT STRUCTURES

This lecture follows [Aur13, §2] closely.

One approach to resolving the issue of non-transverse intersections of Lagrangians when defining the Fukaya category is given in [Sei08].

The example of the three-punctured torus is from [Sei01, Proposition 3.2].

The observation that Floer cohomology can be defined in the presence of Maslov index 3 discs is due to [Oh93], and the extension to include Maslov index 2 discs is due to [Oh95].

The theorem about the form of the disc potential function for a torus fibre of a Fano toric variety can be found in [FOOO12, Theorem 5.2], building on [CO06]. The identification of the endomorphism algebra as a Clifford algebra is from [Cho05].

3. TRIANGULATED STRUCTURE

This lecture follows [Aur13, §3].

A preprint containing Fukaya, Oh, Ohta and Ono’s result about holomorphic discs with boundary on a Lagrangian connected sum is available at <https://www.math.kyoto-u.ac.jp/~fukaya/Chapter10071117.pdf>.

For the result of Abouzaid about the integral of a primitive for the symplectic form being the same for quasi-isomorphic objects, see [Abo08, §6] (Abouzaid’s result is for Fukaya categories of higher-genus surfaces, but the proof adapts to the case of the torus).

The result of Abouzaid, Fukaya, Oh, Ohta and Ono about the Lagrangian torus fibre split-generating its component of the Fukaya category is unpublished. It follows from an appropriate version Abouzaid’s split-generation criterion [Abo10], see for example [She15, Corollary 2.19].

4. OTHER HELPFUL RESOURCES

Helpful introductions to Fukaya categories include [Aur13], notes from the Talbot graduate student workshop on Fukaya categories mentored by Paul Seidel (available from <http://www.math.ias.edu/~nicks/talbot.html>), and notes from Denis Auroux’s topics course on mirror symmetry (available from <https://math.berkeley.edu/~auroux/277F09/index.html>).

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