Tobias:

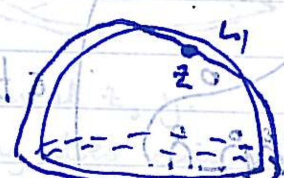
Handle attachment and product in SH

1. Réformulation of SH $\longrightarrow$ LH
2. Compute product

Handle attachment: subcritical handles can be isotoped anywhere by h-principle. $\Rightarrow$ just topology.

New, more natural version of $LH^{Ho}$:

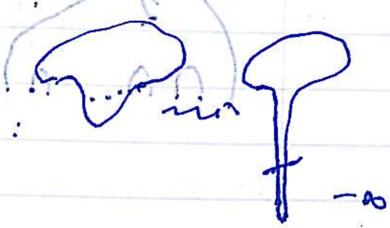
Take $\Lambda$ with $L$ = core disk of handle attached along $\Lambda$. push $\Lambda$ off itself along R, ~~one~~ extend to push L off itself (one critical point in centre of L)




$F_H^*(L_0, L_1) = LHA(\Lambda)\langle z, \text{Reeb chords } \Lambda_0 \longrightarrow \Lambda_1\rangle$

not really, this is a new defn.

have to use these coefficients to deal with possibility of breaking down to $-\infty$ :



chords $\Lambda_0 \longrightarrow \Lambda_1$ :

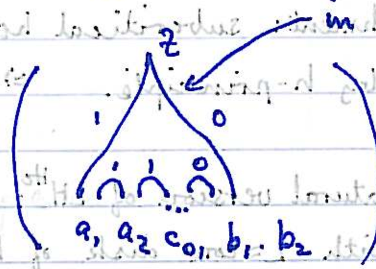
min $x$

max $y$

Reeb chord $c$ of $\Lambda \longrightarrow c_{01}$

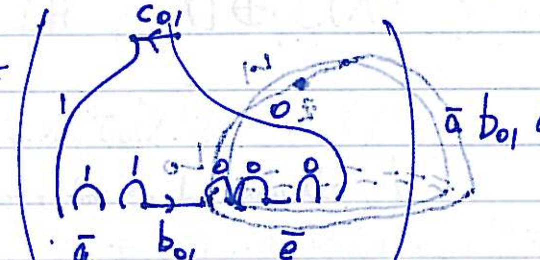
$d : FH(L_0, L_1) \circlearrowleft$

in $X - X_0$ = handle with infinite -ve end

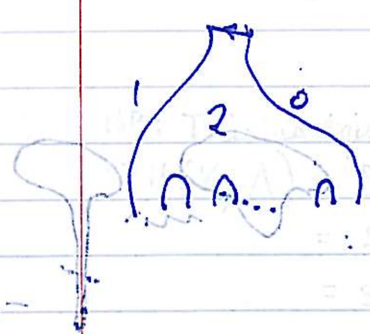
$dz = \sum \#$ 

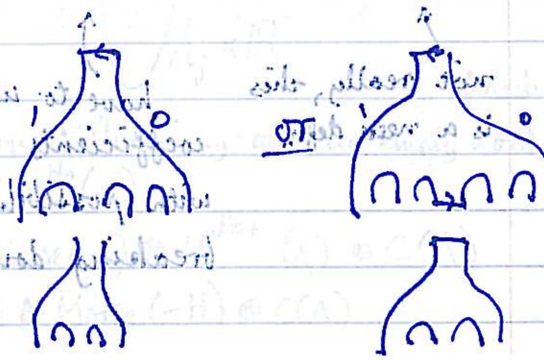
$a_1 a_2 c_{01} b_1 b_2$

set $\alpha c_{01} \beta := (-1)^* c_{01} \beta \alpha$

$d c_{01} = \sum \#$ 

$\bar{a} b_{01} \bar{e}$



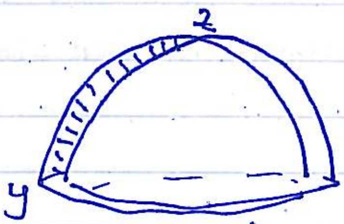
Compare $LH^{Ho}$:

$\check{c}^1_{\cancel{//}} \dots c^k_{\cancel{//}}$ $\hat{c}^1_{\cancel{//}} \dots c^k_{\cancel{//}}$

$x\, c^1_{\cancel{/}} \dots c^k_{\cancel{///}} \rightsquigarrow \check{c}^1_{\cancel{/}} c^2_{\cancel{/}} \dots c^k_{\cancel{/}}$

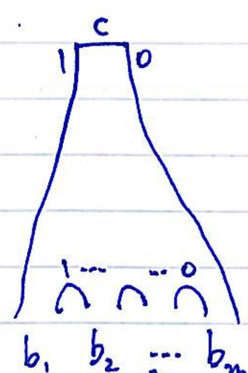
$c^1_{01}\, c^2 \dots c^k \rightsquigarrow \hat{c}^1 c^2 \dots c^k$

what happened to $y$ and $z$?
There's a thin strip giving $dz = y$



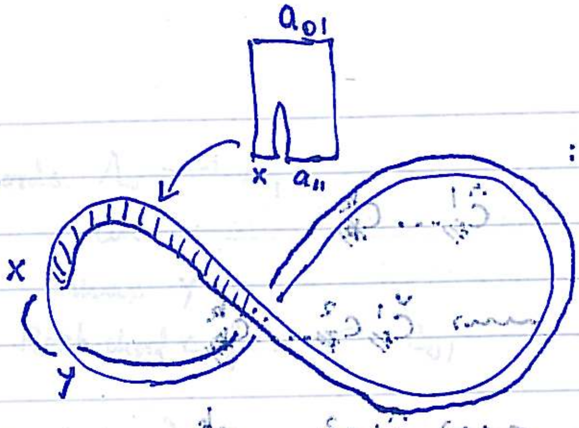

$\Rightarrow$ cancel $z$, $y$.
~~///~~ Why does this correspond to old $LH^{Ho}$?

$$d\,\hat{c}w = S(dc)\,w + \hat{c}\,dw + \check{c}w - c\check{w}.$$



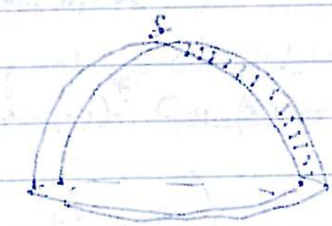

choose where in here you jump from 1 to 0. This is why $S(dc)$ appears.

E.g.



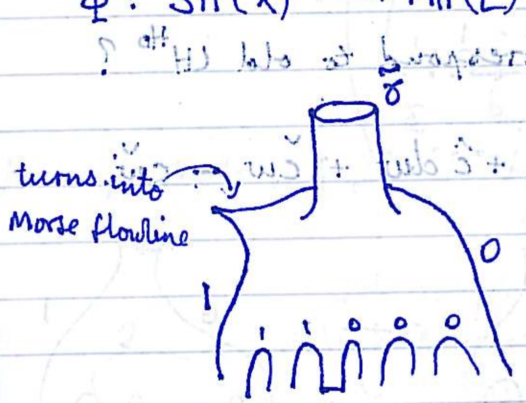

$\Rightarrow \check{c}w - c\check{w}$ term.

$MH(L) = FH(L) \oplus SH(X_0)$

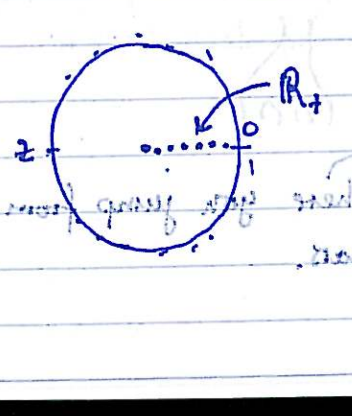


model for $SH(X)$

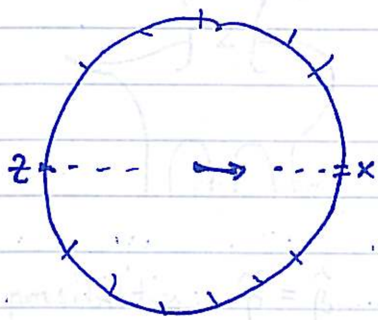
$\Phi : SH(X) \longrightarrow MH(L)$

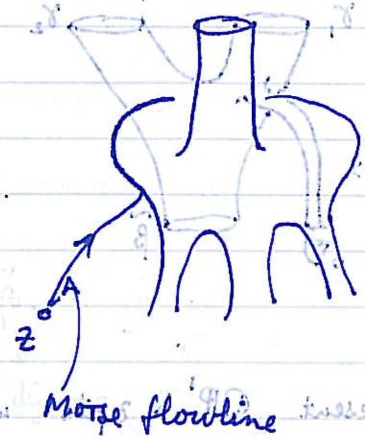



Domain :



$$\tilde{\gamma} = \begin{cases} \check{\gamma} \Rightarrow R_+ \text{ hits marker} \\ \hat{\gamma} \Rightarrow \text{no condition} \end{cases}$$

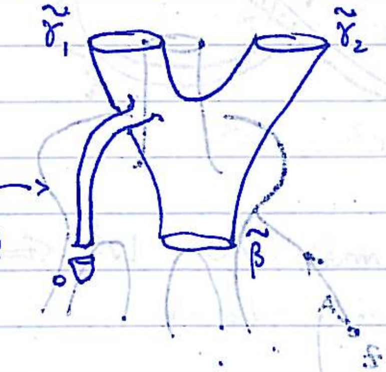


## Part II:

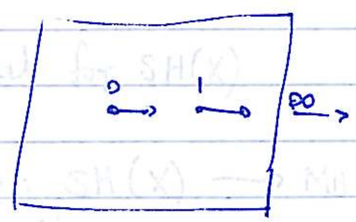
Use $\Phi : SH(X) \longrightarrow MH(X)$

Product on SH counts

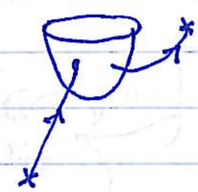



recall: augmented in X

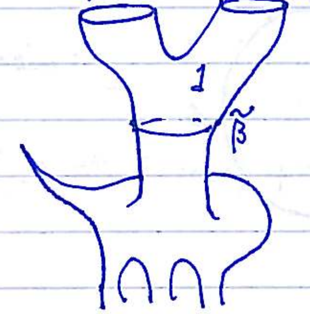
Represent $\mathbb{CP}^1 \setminus \{3 \text{ pts}\}$ with asympt. markers along $\mathbb{R}_+$




E.g.



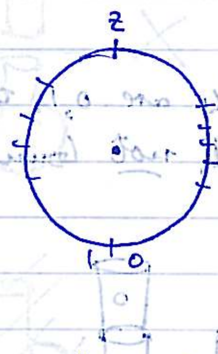
Product on SH(X)



SFT glue:


2 possibilities: $\tilde{\beta} = \hat{\beta}$ or $\check{\beta}$.

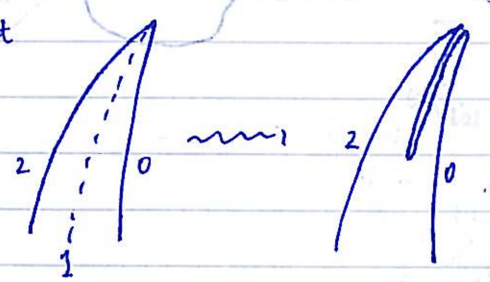
$\hat{\beta}$: dim=2 $\qquad$ $\check{\beta}$: dim =2




To get a flow tree you need 3 Morse functions

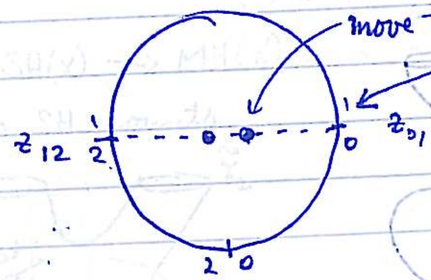
$\Rightarrow$ now take $L_0, L_1, L_2$.

Split

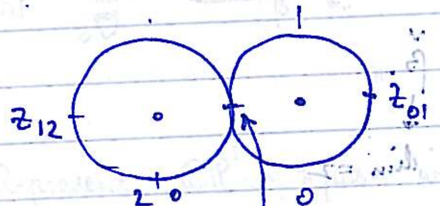


Count

move this puncture to here



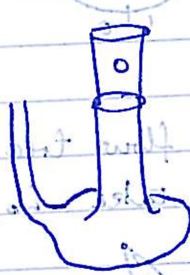

as puncture approaches $z_{01}$ you get




since boundary comp's are 0, 1, this is breaking at a chord, <u>not</u> boundary breaking.

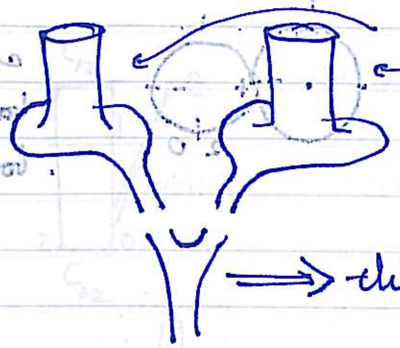
cases:

or



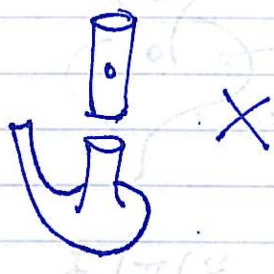

breaking chord is pos or neg.

Main goal: show you get

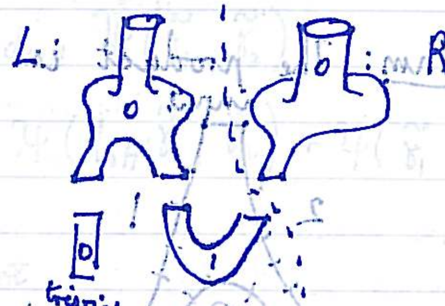


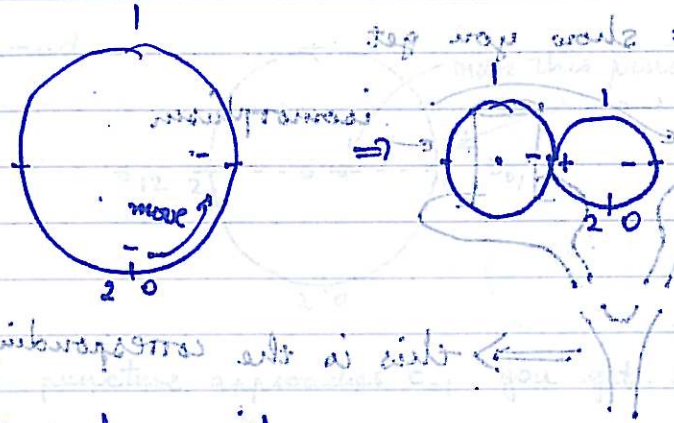
isomorphism

this is the corresponding product.



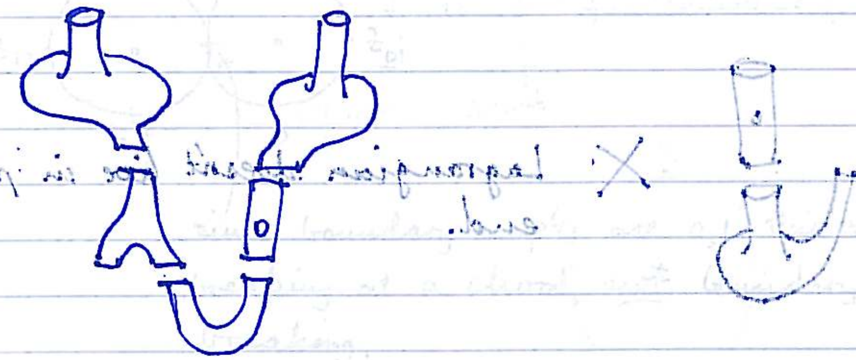

Lagrangian doesn't live in positive end.



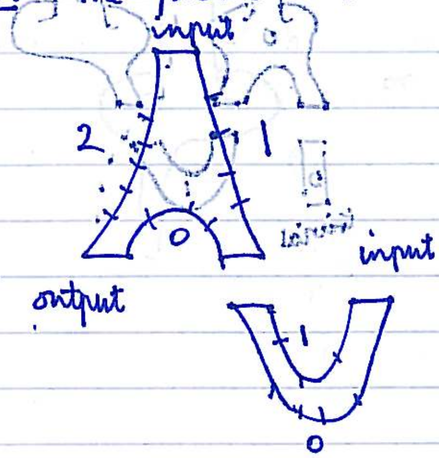



again we avoid bad splitting on boundary.

We end up with

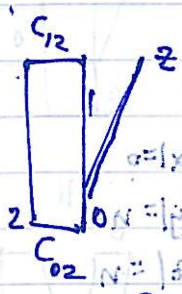


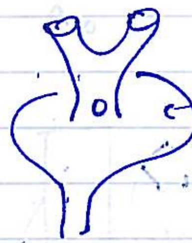

Thm: The product is:




"extend linearly over LHA".

For every chord you also get



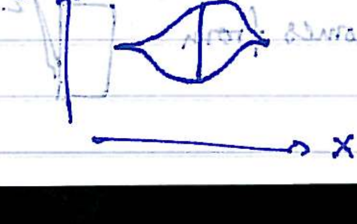




count all splittings which are not desired. You get

$$\Phi\left(\pi(\tilde{\gamma}_1, \tilde{\gamma}_2)\right) = \rho\left(\Phi(\tilde{\gamma}_1), \Phi(\tilde{\gamma}_2)\right)$$
$$+ d_{MH} \Psi(\tilde{\gamma}_1, \tilde{\gamma}_2)$$
$$+ \Psi(d_{SH}\tilde{\gamma}_1, \tilde{\gamma}_2) + \Psi(\tilde{\gamma}_1, d_{SH}\tilde{\gamma}_2)$$

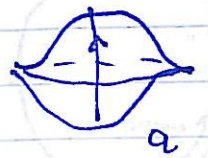
E.g. Legendrian unknot


$dz - y\,dx$



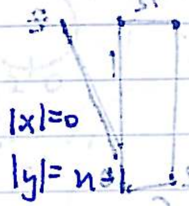

$y = \frac{\partial z}{\partial x}$

Generalise: spin around




$|a| = n-1.$

$\partial a = 0$

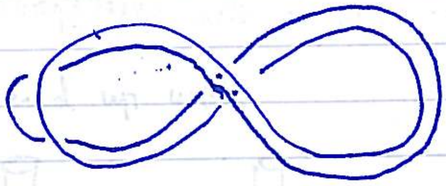


$|x| = 0$

$|y| = n-1$

$|z| = n$

$|c_{01}| = |c|+1$

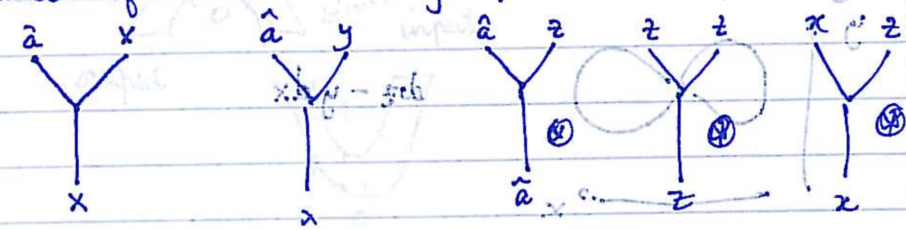


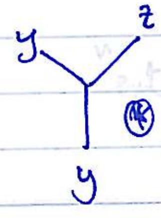
$$FH(L) = LHA(\Lambda)\langle z, x, y, a_{01}\rangle$$

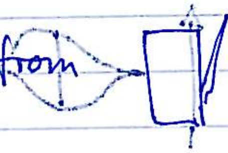
$$\partial z = y$$
$$\partial x = 0$$
$$\partial y = 0$$
$$\partial a_{01} = \underbrace{x a_0 - a x}_{0} + y$$

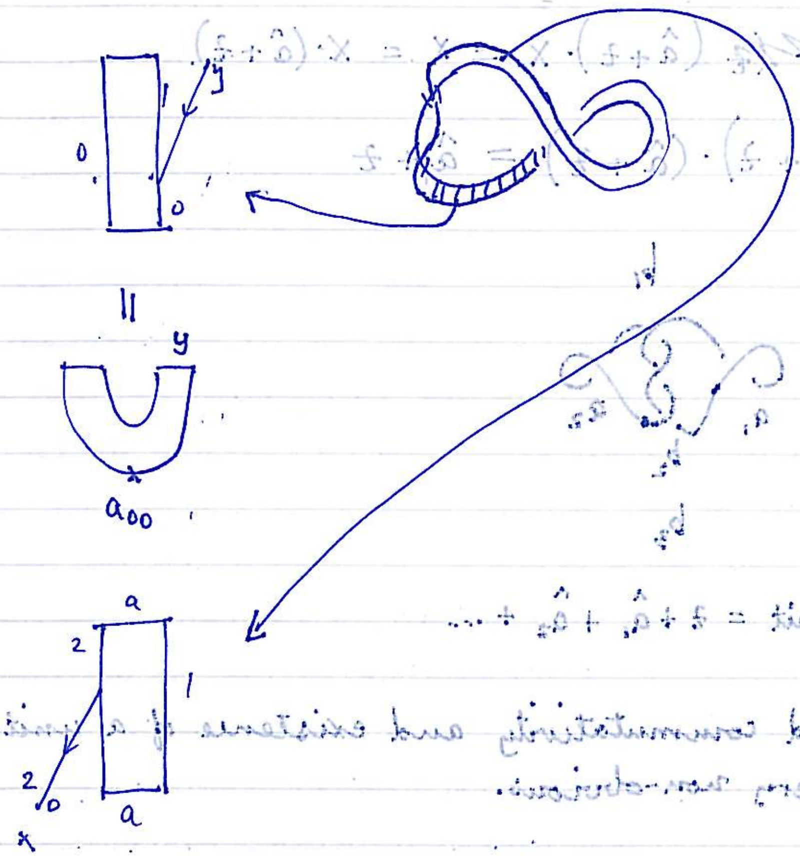
(since left, right don't matter)

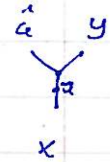
In degree $n$ we have one elt $z + \hat{a}_{01}$. Since unit of SH lives in deg $n$, this must be our unit.



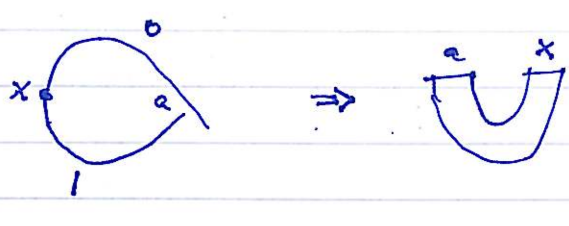



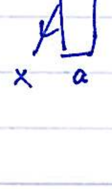

★ comes from  term.




Stack these together you get  $\Rightarrow$ 


$\partial a = y$



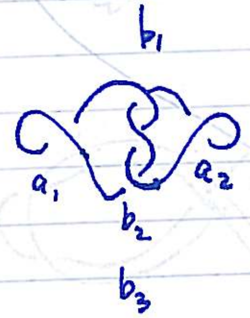

put together with 


to get 

$\cancel{\hat{a}\,z}\ (\hat{a}+z)\cdot x = x = x\cdot(\hat{a}+z).$

$$(\hat{a}+z)\cdot(\hat{a}+z) = \hat{a}+z$$

<u>Ex</u>:




unit $= z + \hat{a}_1 + \hat{a}_2 + \dots$

signed commutativity and existence of a unit are very non-obvious.