

Tobias:

Handle attachment and product in SH

1. Reformulation of SH \rightarrow LH
2. Compute product

Handle attachment: subcritical handles can be isotoped anywhere by h-principle. \Rightarrow just topology.

New, more natural version of LH^{Ho}:

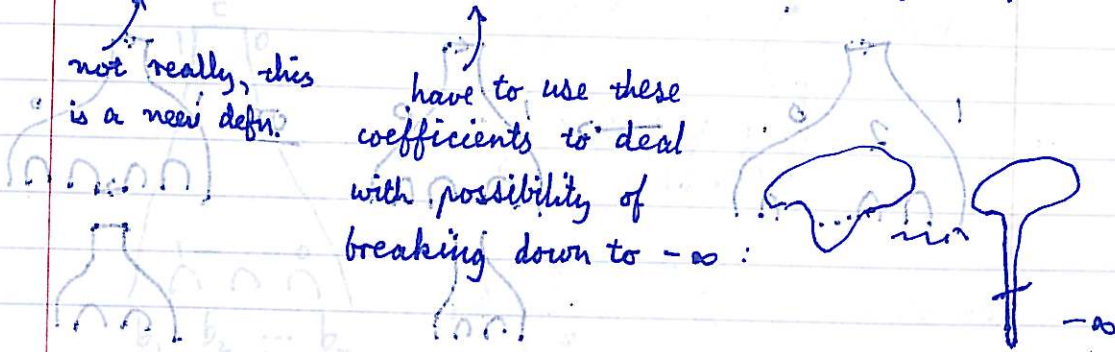
Take Λ with L core disk of handle attached along Λ . push off itself along R , ~~or~~ extend to push L off itself (one critical point in centre of L)



$$FH^*(L_0, L_1) = LHA(\Lambda) \langle \mathbb{Z}, \text{Reeb chords } \Lambda_0 \rightarrow \Lambda_1 \rangle$$

not really, this is a new defn.

have to use these coefficients to deal with possibility of breaking down to $-\infty$:



chords where in here you jump from L_0 to L_1 . This is why $S(d)$ appears.

chords $\Lambda_0 \rightarrow \Lambda_1$

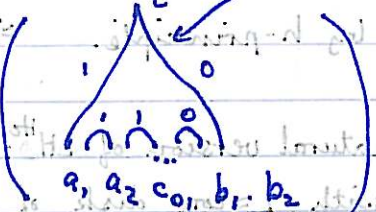
min x

max y

Reeb chord c of $\Lambda \rightarrow C_0$

$d: FH(L_0, L_1) \rightarrow \mathbb{Z}$

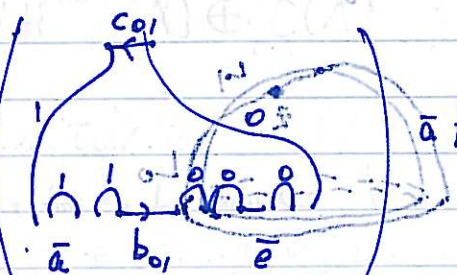
$$d_z = \sum \#$$



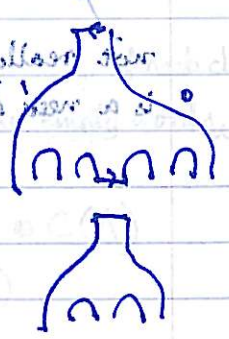
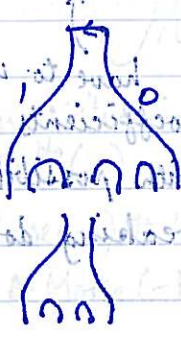
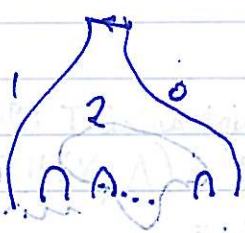
in $X - X_0 =$ handle with infinite -ve end

a, a_2, c_0, b_1, b_2

$$d_{C_0} = \sum \#$$



$\langle \Lambda \rightarrow \Lambda \text{ chords } \Lambda \rangle \rightarrow (\Lambda) \Lambda \Lambda = (1, 1, 1) \dots$

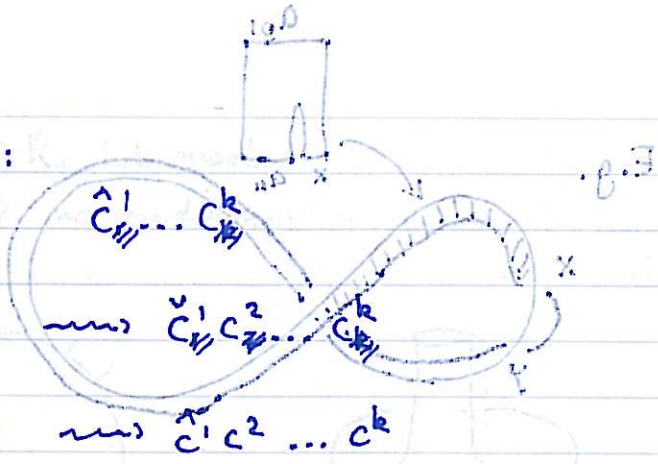


Compare LH^{H_0} :

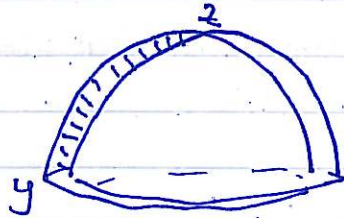
$$\hat{c}_1^1 \dots \hat{c}_k^k$$

$$\times c_1^1 \dots c_k^k$$

$$c_0^1, c^2 \dots c^k \rightsquigarrow \hat{c}_1^1 c^2 \dots c^k$$



what happened to y and z ? $w_0 - w_0 \leftarrow$
 There's a thin strip giving $dz = y$



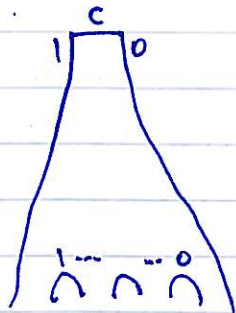
$$(X)H_2 \oplus (1)H_1 = (1)H_1$$

$(X)H_2$ not relevant

\Rightarrow cancel z, y . $(1)H_1 \leftarrow (X)H_2 : \mathbb{Z}$

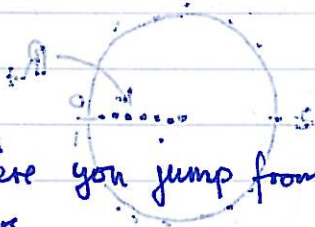
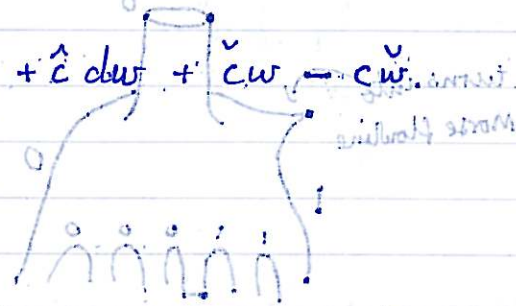
Why does this correspond to old LH^{H_0} ?

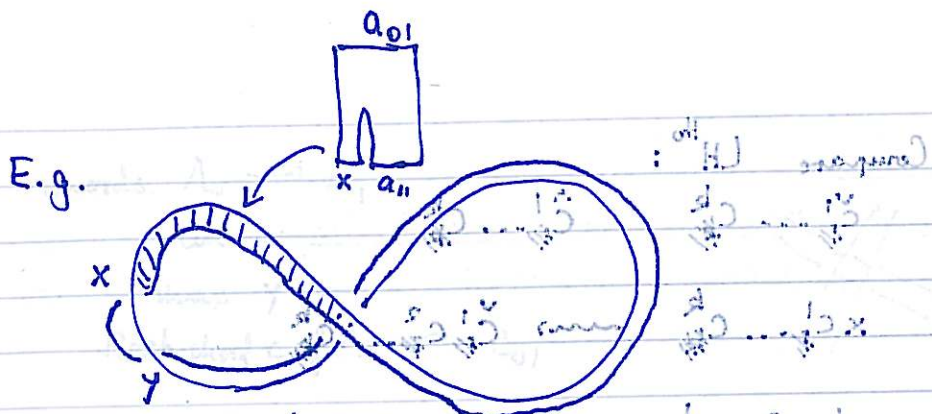
$$d \cdot \hat{c} w = S(dc) w + \hat{c} dw + \check{c} w - c \check{w}$$



b_1, b_2, \dots, b_m

choose where in here you jump from 1 to 0. This is why $S(dc)$ appears.

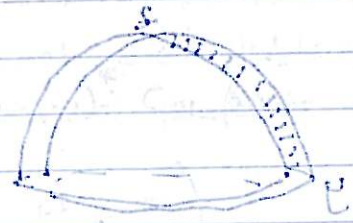




$\Rightarrow \check{C}W - CW$ term, has μ at boundary ends
 $\mu = \pm 1$ giving signs with a descent

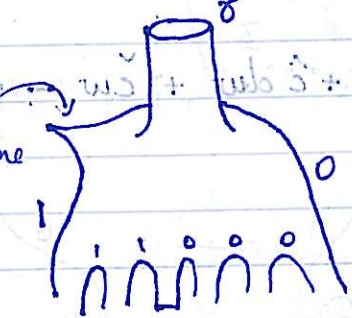
$$MH(L) = FH(L) \oplus SH(X_0)$$

model for $SH(X)$

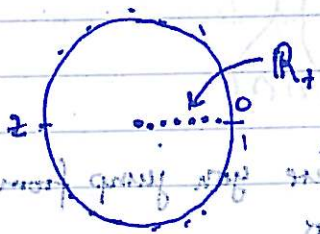


$$\Phi: SH(X) \rightarrow MH(L)$$

turns into
Morse flowline



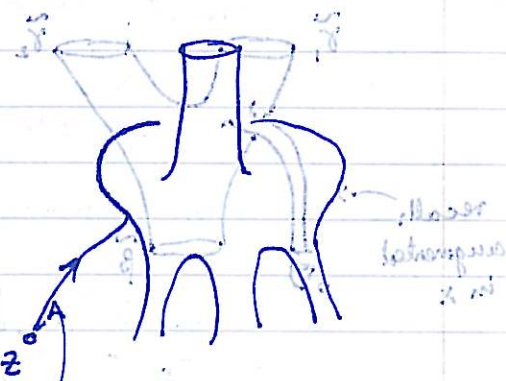
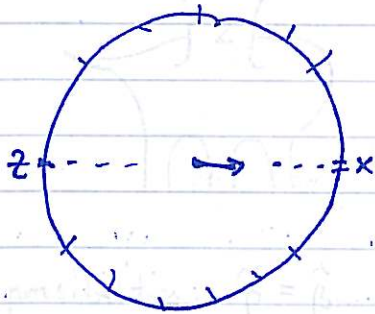
Domain:



is will 0 at 1 mark group exp need in space around
 maybe (b)2 what

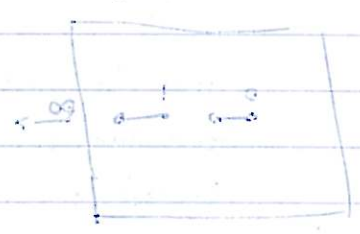
$$\tilde{\gamma} = \begin{cases} \gamma & \Rightarrow R_+ \text{ hits marker} \\ \hat{\sigma} & \Rightarrow \text{no condition} \end{cases}$$

Product on \mathbb{H}^2 as boundary
 $\mathbb{H}^2 : \mathbb{C} \setminus \mathbb{R}$
 Part II

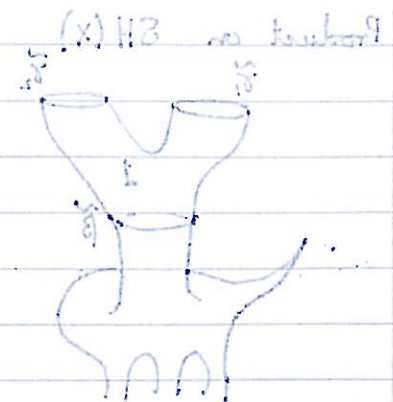
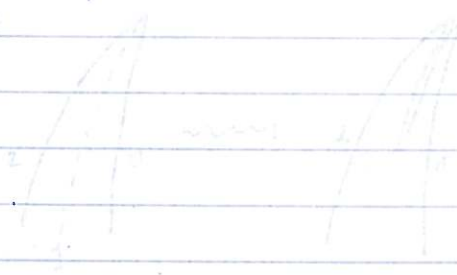


Masses
 latitudes
 x, y

Represent flowline with segments marked along R_+



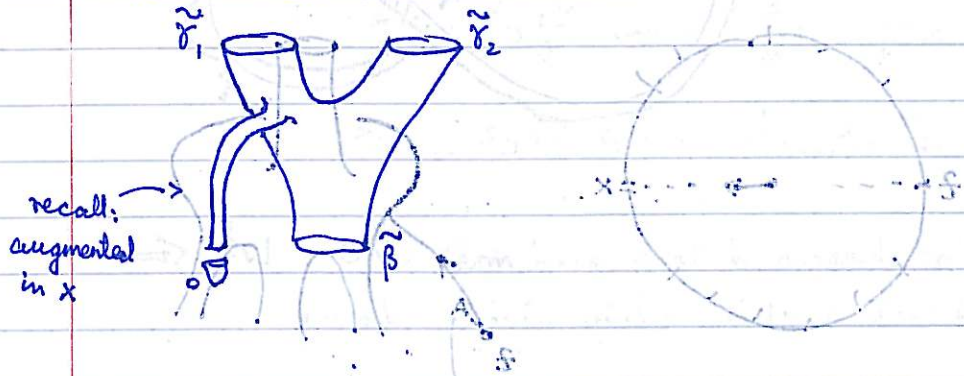
Try to flow for you need 3 conditions
 always take into account
 by lit



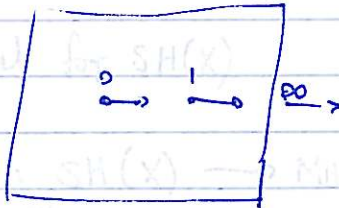
Part II:

Use $\Phi: SH(X) \rightarrow MH(X)$ where $\Phi(\gamma) = \tilde{\gamma}$

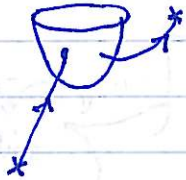
Product on SH counts



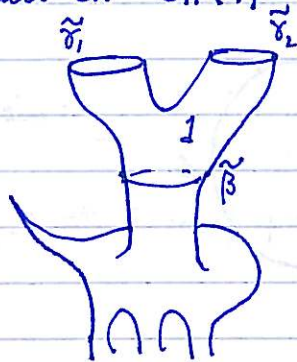
Represent $\mathbb{C}P^1$ with asympt. markers along \mathbb{R}_+



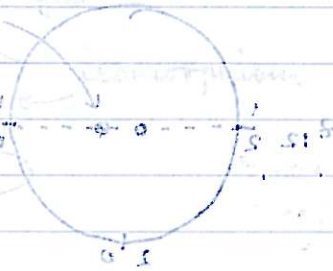
E.g.



Product on SH(X)



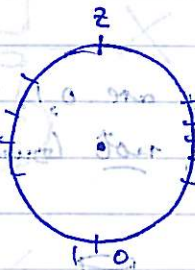
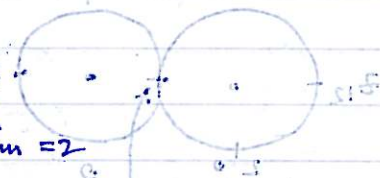
SFT glue:



2 possibilities: $\tilde{\beta} = \hat{\beta}$ or $\check{\beta}$.

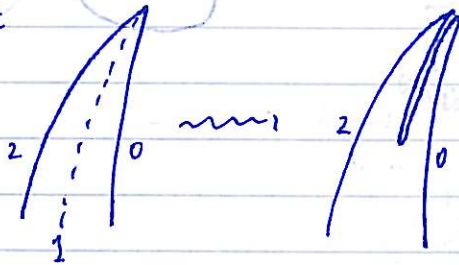
dim=2

dim=2

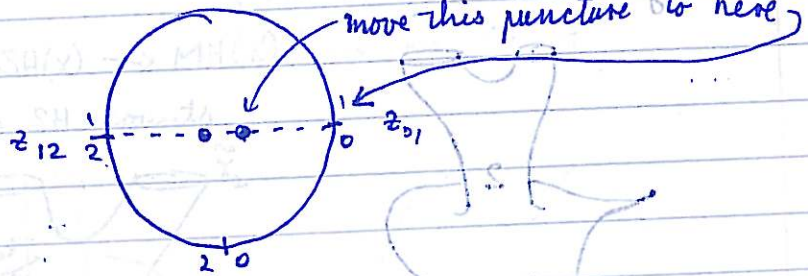


To get a flow tree you need 3 Morse functions
 \Rightarrow now take L_0, L_1, L_2 .

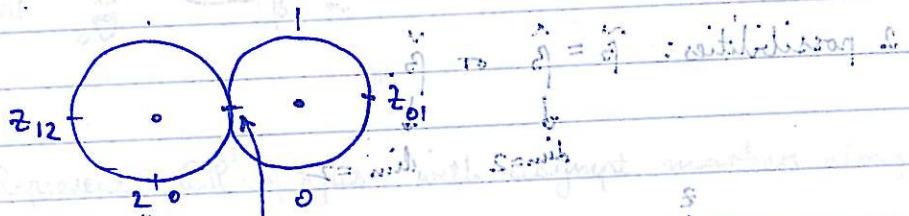
Split



Count

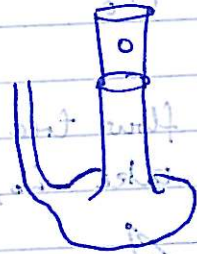


as puncture approaches z_{01} you get



since boundary comp's are 0,1, this is breaking at a chord, not boundary breaking.

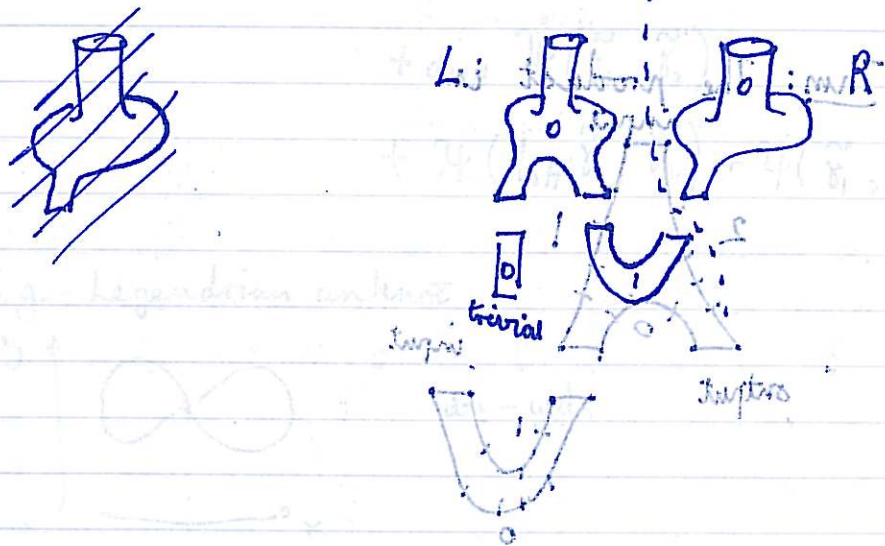
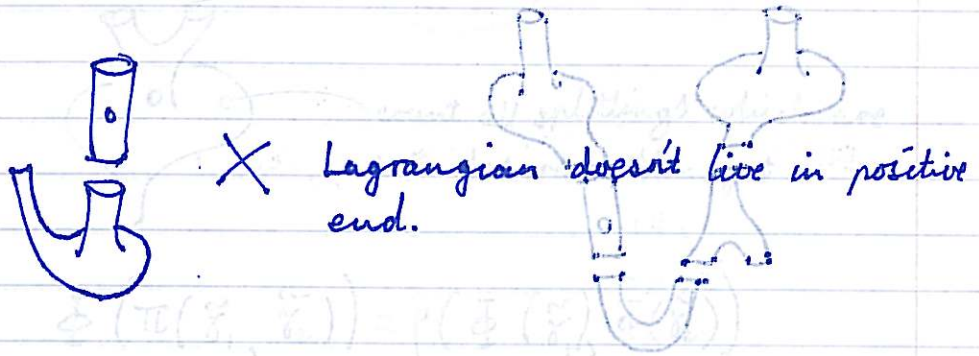
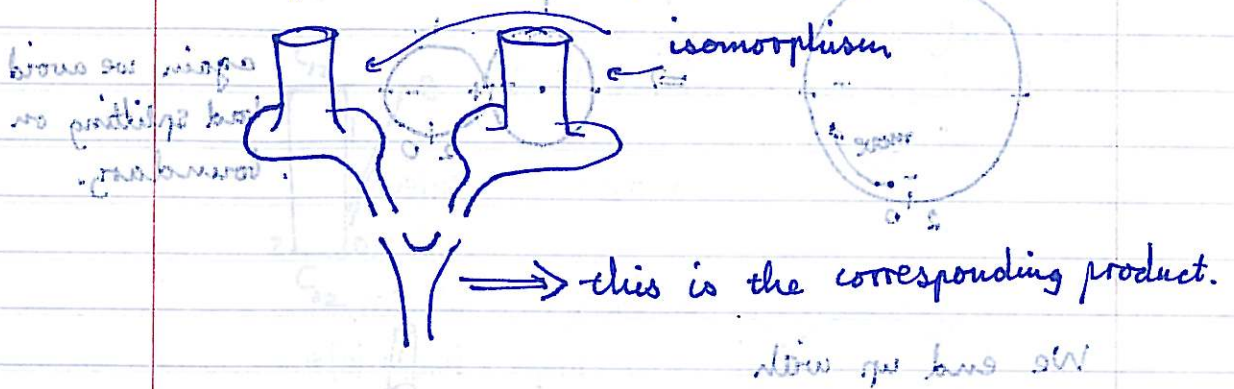
cases:



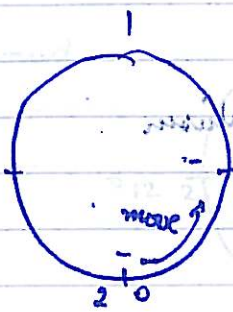
or

breaking chord
is pos or neg.

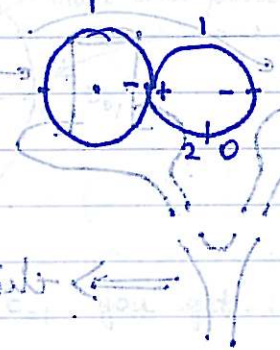
Main goal: show you get



All your present books



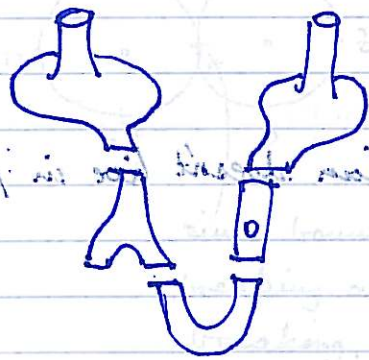
top map circle: loop shift



again we avoid bad splitting on boundary.

boundary pinching process also as circle ←

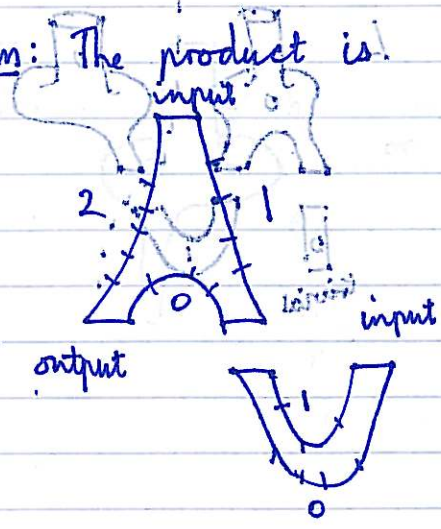
We end up with



utilizing in this case the principal laws

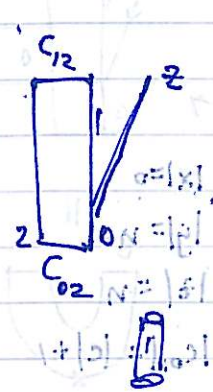


Thm: The product is



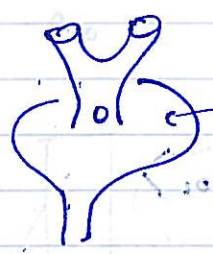
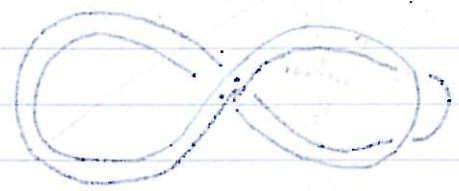
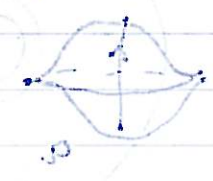
"extend linearly over LHA"

For every chord you also get a nice (submanifold)



$$1 - \sigma = |a|$$

$$0 \leq \sigma \leq 1$$



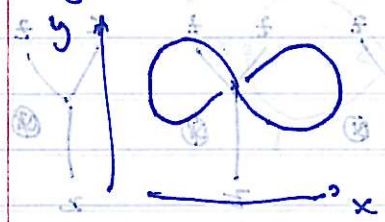
count all splittings which are not desired. You get H^1

$$\Phi(\pi(\tilde{\gamma}_1, \tilde{\gamma}_2)) = \rho(\Phi(\tilde{\gamma}_1), \Phi(\tilde{\gamma}_2))$$

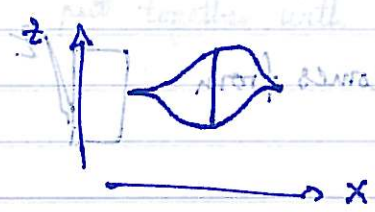
$$+ d_{MH} \Psi(\tilde{\gamma}_1, \tilde{\gamma}_2)$$

$$+ \Psi(d_{SH} \tilde{\gamma}_1, \tilde{\gamma}_2) + \Psi(\tilde{\gamma}_1, d_{SH} \tilde{\gamma}_2)$$

E.g. Legendrian unknot



$$dz - y dx$$



$$y = \frac{\partial z}{\partial x}$$

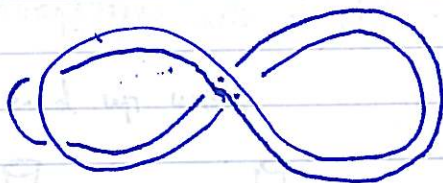
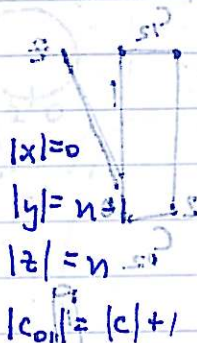


Generalise: spin around the top handle gives π



$$|a| = n - 1.$$

$$\partial a = 0$$



one handle \rightarrow $\pi_1(L) = \mathbb{Z}$

$$FH(L) = LHA(\Lambda) \langle z, x, y, a_0 \rangle$$

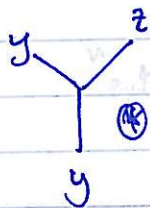
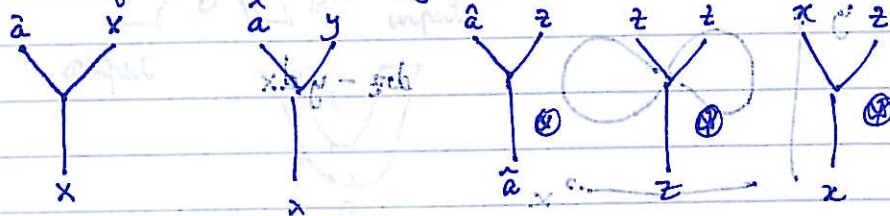
$$\partial z = y$$


$$\begin{pmatrix} \partial x \\ \partial y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} \partial z \\ \partial a_0 \end{pmatrix} = \begin{pmatrix} x a_0 - a x + y \\ 0 \end{pmatrix}$$

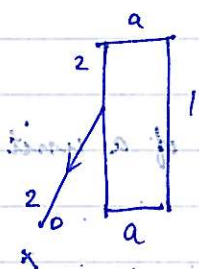
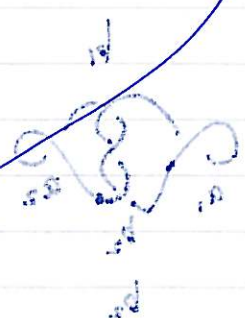
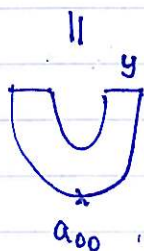
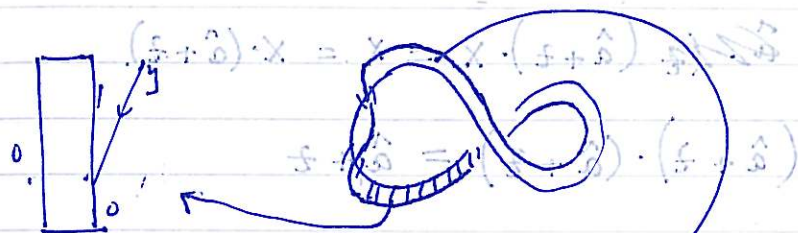
$$\partial a_0 = x a_0 - a x + y$$

(since left, right don't matter)

In degree n we have one elt $z + a_0$. Since unit of SH lives in deg n , this must be our unit.



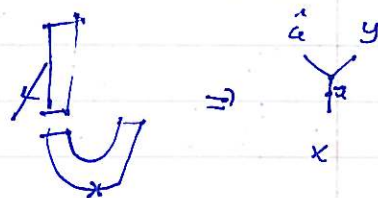
$\frac{y^2}{x^2} = \textcircled{R}$ comes from  term.



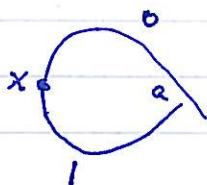
simply a function has finite number of points

$$\dots + \hat{a} + \hat{a} + \hat{b} = \text{lines}$$

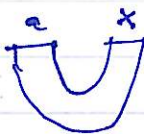
Stack these together you get



$$\partial a = y$$



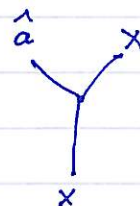
⇒



put together with



to get



$$\hat{a} + z \cdot X = X = X \cdot (\hat{a} + z)$$

$$(\hat{a} + z) \cdot (\hat{a} + z) = \hat{a} + z$$

EX:



$$\text{unit} = z + \hat{a}_1 + \hat{a}_2 + \dots$$

signed commutativity and existence of a unit are very non-obvious.

