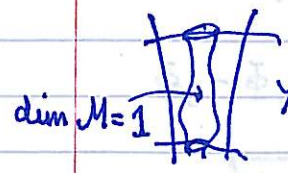
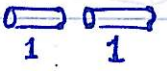
Invariance of Contact Homology

Tobias

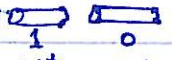
In the same way that



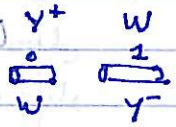
differential in CH

$\partial^2 = 0$: 

homomorphism $\Phi : CH^- \to CH^+$

$\Phi\partial + \partial\Phi = 0$ 

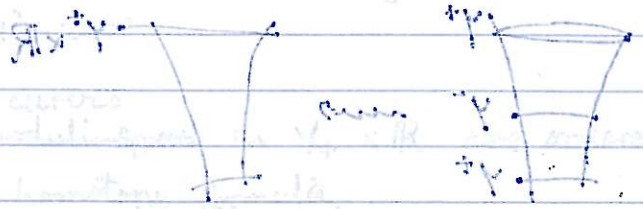
Similarly for Hamiltonian Floer Homology, we obtain a homomorphism between $CF(H^0, J^0)$ and $CF(H^1, J^1)$.



$$d\Phi = \Phi d$$

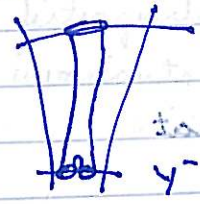
$$\Phi_{\tau=0} - \Phi_{\tau=1} = dK - Kd$$

equivalent ? 



In contact homology:

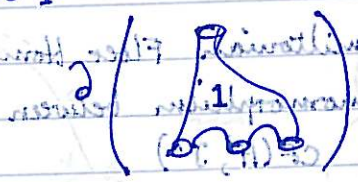
cobordism W



$$\Phi(\gamma) = \sum_{\bar{\beta}} |\mathcal{M}(\gamma, \bar{\beta})| \, \bar{\beta}$$

(word $\bar{\beta}$ of Reeb orbits)

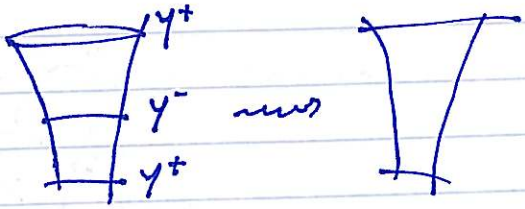
Then $\Phi\partial = \partial\Phi$ by counting



Isotopies of ξ induce cobordisms

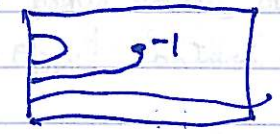
ξ changing

You can compose with inverse isotopy to get $\Phi \circ \Psi$, then deform to get $Y^+ \times \mathbb{R}$, show $\Phi \circ \Psi \simeq \mathbb{1} = dK + Kd$



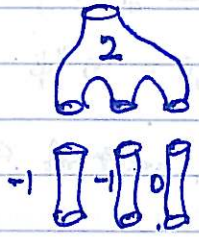
Given a 1-param. family of cobordisms $\mathcal{M}^\tau(\gamma, \vec{\beta})$ may degenerate to rational (-1)-curves at particular values of τ. Use these to define K, then consider the moduli space $\mathcal{M}^{[0,1]}(\gamma, \vec{\beta}) = \bigcup_{\tau \in [0,1]} \mathcal{M}^\tau(\gamma, \vec{\beta})$

$$\Phi_0 - \Phi_1 = dK + Kd$$



boundary of moduli space includes -1 breaking curves $\Rightarrow dK + Kd$

Even if we ensure only -1 curves appear, and only one appears at each time $\tau \in [0,1]$, how can we rule out



We rule this out by making a ~~domain-dependent~~ perturbation which depends on the domain and the point in the moduli space $\Rightarrow$ only one end at a time can get a (-1)-curve

If ends of ~~moduli space~~ curves in $Y_+ \times \mathbb{R}$ are ordered there seems to be a homotopy formula.

$$\Phi_1 = \Phi_0 e^{dK+Kd}$$

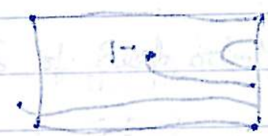
↑ power series expansion.

Given a 1-param. family (ω_t, J_t) on M, $(\bar{J}, \Phi)$

Ex! $\Phi_1 - \Phi_0 = d\alpha \pm \alpha d$.

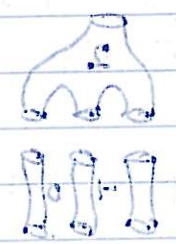
may degenerate to a (−1)-curve at particular values of τ. Use these to define K, then consider the moduli space $\mathcal{M}^{[0,1]}(\beta, \bar{J}) = \bigcup_{\tau \in [0,1]} \mathcal{M}^{\tau}(\beta, \bar{J})$

$\Phi_0 - \Phi_1 = dK + Kd$



boundary of moduli space includes −1 breaking curves on $dK + Kd$

Even if we ensure only −1 curves appear and only one appears at each time $\tau \in [0,1]$, how can we rule out



We rule this out by making a ~~domain-dependent~~ perturbation which depends on the domain and the point in the moduli space ⇒ only one end at a time can get a (−1)-curve

If ends of moduli spaces in $Y_\phi \times \mathbb{R}$ are ordered there seems to be a homotopy formula.

$\Phi_1 = \Phi_0 e^{dK + Kd}$

power series expansion.