Luis h-Principles

V^{2n}, W^{2m} smooth manifolds

Def $\mathcal{S}_{symp}(V) = \{\omega \in \Omega^2(V) : \omega^n \neq 0\}$ almost symplectic structures

$\mathcal{\$}_{symp}(V) \subseteq \{\omega \in \mathcal{S}_{symp}^2(V) : d\omega = 0\}$

Q: Does the existence of $\omega \in \mathcal{S}_{symp} \Rightarrow \exists\, \tilde{\omega} \in \mathcal{\$}_{symp}$

Thm: V open $\Rightarrow$ $\mathcal{\$}_{symp} \subset \mathcal{S}_{symp}$ homotopy equivalence

Vague formulation: "structures" $\mathcal{\$}$ on a manifold on mfld satisfy an h-principle if every "almost structure" $\omega \in \mathcal{S}$ can be deformed in $\mathcal{S}$ to actual structure $\tilde{\omega} \in \mathcal{\$}$

Parametric form: given $\varphi_0 : (D^k, S^{k-1}) \to (\mathcal{S}, \mathcal{\$})$
$\exists\, \varphi_t$ s.t. $\varphi_1(D^k) \subset \mathcal{\$}$

Restriction form: $A \subset V$, if $\omega \in \mathcal{S}$ s.t. $\omega|_U \in \mathcal{\$}(U)$ $A \subset U$ open, then can fix ω_t on $A \subset \tilde{U} \subset U$ open.

In fact
$$\mathcal{S}^a_{symp} \hookrightarrow \mathcal{S}_{symp} \text{ h.e.}, \quad a \in H^2(V)$$

$$\mathcal{S}^a_{symp} = \{\omega \in \mathcal{S}_{symp} \mid [\omega] = a\}$$

<u>Will "prove"</u>: Any $\omega \in \mathcal{S}$ can be htped in $\mathcal{S}_{symp}$ to $\tilde{\omega}$ exact symplectic.

<u>Def</u>: $\begin{matrix} X \\ \downarrow \\ V \end{matrix}$ fiber bundle, $\begin{matrix} X^{(r)} \\ \downarrow \\ V \end{matrix}$ is the r-jet bundle

Given $v \in V$, the fibre of $X^{(r)}$ is
$$\{(s(v), Ds(v), \dots, D^r s(v)) \mid s \text{ local section } X \to V\}$$

<u>Def</u>: Given $X \underset{s}{\rightleftarrows} V$, $\exists\, J^r_s(v) \in X^{(r)}_s$

Say that J^r_s is a holonomic section of $X^{(r)}$

h-Principle for PDR on open manifolds

V open mfd $\qquad \begin{matrix} X \\ \downarrow \pi \\ V \end{matrix}$ natural fiber bundle $\Leftrightarrow$ ($\exists$ hom $j : \mathrm{Diff}\, V \to \mathrm{Diff}_V X$) s.t. $\pi(j(\varphi)) = \varphi \quad \forall \varphi$

Thm V open, $\begin{matrix} X \\ \downarrow \\ V \end{matrix}$ nfb, then a open Diff V invariant differential relation $R \subset X^{(r)}$ satisfies a parametric h-principle

relative h-principle holds for $B \subset V$ closed s.t. all connected comp of $V \setminus B$ have an exit to ∞.