Gromov Compactness in the Relative Case

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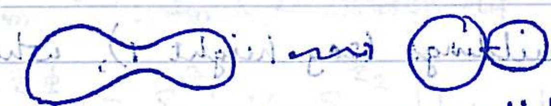
X symplectic, L Lagrangian

$(D^2, \partial D^2) \to (X, L)$

There are no compact exact Lagrangian submanifolds of $\mathbb{C}^n$ (Gromov) for $n \geq 2$.
Proven by showing there is a nontrivial holomorphic disk with boundary on L.

Disks with boundary can bubble in new ways, e.g.



two disks joined at a boundary point.

Recall: Given $u: S^2 \to M$
if $[u] \in H_2(M)$ indivisible
then bubbling can't happen.

Suppose our symplectic manifold is exact.
Then there are no sphere bubbles.

Similarly one can rule out disk bubbles in some cases.

$\Lambda \subset Y$

contact

Legendrian

$\Rightarrow (\mathbb{R}\times Y, \mathbb{R}\times \Lambda)$ call $L := \mathbb{R}\times\Lambda$

symplectic Lagrangian

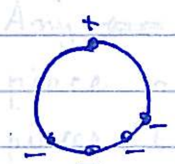
Consider $u : (D, \partial D \setminus Z) \longrightarrow (\mathbb{R}\times Y, \mathbb{R}\times\Lambda)$

$\partial D \supseteq Z$ a finite set of "boundary punctures"

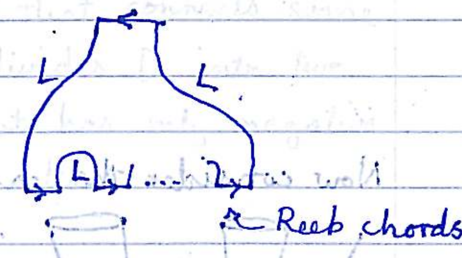
Near punctures, u is asymptotic to a Reeb chord = a ~~segment~~ flowline of the Reeb vec. field in Y with endpoints on $\Lambda \subset Y$.

(could also have interior punctures asymptotic to Reeb orbits as before).

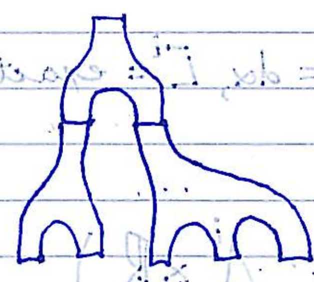
E.g.



Looks like



These can degenerate into multilevel curves, like



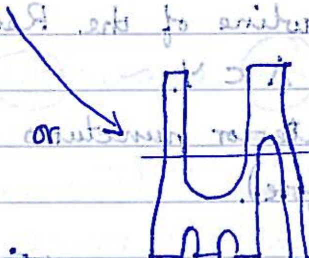
one positive puncture.

counts of these give "relative (Legendrian) contact homology".

cannot happen by exactness

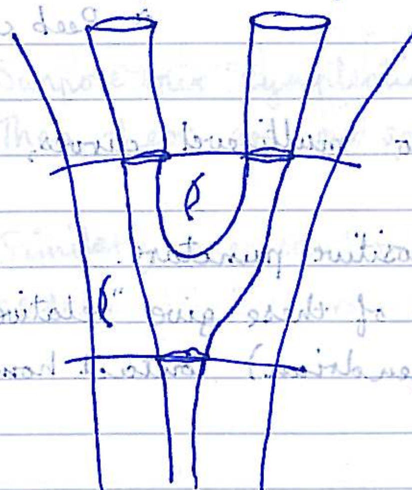
If you instead have multiple positive punctures

boundary bubble.



or multi-level curve.

Now consider the case $\Lambda = \coprod_i \Lambda_i$



$(Y_+ \times \mathbb{R}_+, \Lambda^i_+ \times \mathbb{R}_+)$

$\overline{X}, \overline{\omega} = d\alpha, \overline{L}^i =$ exact

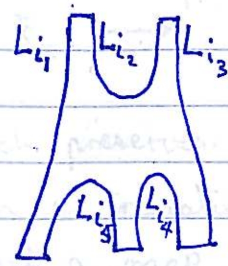
$(Y_- \times \mathbb{R}_-, \Lambda^i_- \times \mathbb{R}_-)$

$\rightsquigarrow (X, L^i)$

X: exact symplectic

L^i: exact Lagrangians with cylindrical ends

$\Rightarrow$ consider moduli spaces



Admissible disks (i.e. homotopy classes of maps $(D, \partial D \setminus Z) \longrightarrow (X, L)$ with specified asymptotics) are those ~~that~~ s.t.

i) ≥ 1 pos. punctures

ii) Any ~~arc~~ collapsing arc that connects some piece to itself then divides D into two pieces s.t. one component has only negative punctures or none at all.

Tobias: start with Λ, push off a bit by R. Then we deal with boundary degenerations