LES relating SH^* and CH^*

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$W =$ compact symp. mfd with convex $\partial W = M$.

$\iota_X \omega = \alpha$ is contact form M

coefficient ring $\Lambda = \{ \sum a_i \sigma_i \mid a_i \in \mathbb{Q},\ \sigma_i \in H_2(W, \mathbb{Z}) \}$

Assume

$\int_{T^2} u^* \omega = 0$ $\quad$ ($\Rightarrow \int_{S^2} u^* \omega = 0$)

rules out bubbling

SH^*	CH^*
generators = crit H crit pts of Morse funcs f_γ on Reeb orbits γ	generators = Reeb orbits
remove crit H (legal by action filtration) $\rightsquigarrow SH^+$	
grading = $\mu(\gamma) + \mathrm{ind}(f_\gamma)$	grading = $\mu(\gamma) + \mathrm{ind}(f_\gamma) + (n-3)$

Contact homology $\overset{\varepsilon}{\rightsquigarrow}$ linearised contact homology $:= A_1 / A_2$

augmentation

$$\partial \rightsquigarrow g \circ \partial \circ g^{-1}$$

$$g(x) = x + \varepsilon(x)$$

filtration A_1 / A_2

filtration on SH^+: $|e^A| = -2c_1(A)$

$|\gamma_M| = \mu(\gamma)$

filter by index.

(similarly on CH^{para}).

define Morse-Bott differential as in previous talk: define $d_{CH^{para}}$ analogously.

Get spectral sequence: $d_0 : \gamma_M \to \gamma_m$ (gets rid of bad orbits)

$d_1 : \gamma^1_M \to \gamma^2_M$

$d_2 : \gamma^1_m \to \gamma^2_m$

$$E_{p,1} = \bigoplus_{|e^A\gamma_M| = k} \langle e^A \gamma_M \rangle$$

$$E_{p,0} = \bigoplus \langle e^A \gamma_m \rangle$$

$$\begin{tikzcd}
E_{k-2,1} & E_{k-1,1} & E_{k,1} \arrow[l, "d_1"'] \arrow[d, "d_0"] & E_{k+1,1} \\
E_{k-2,0} & E_{k-1,0} & E_{k,0} \arrow[ul, "d_2"] & E_{k+1,0}
\end{tikzcd}$$

$$0 \to E^\infty_{k,0} \to E^2_{k,0} \to E^2_{k-2,1} \to E^2_{u} \to 0$$

$$0 \to E^\infty_{k+1,1} \to H^{tot} \to E^\infty_{k,0} \to 0$$

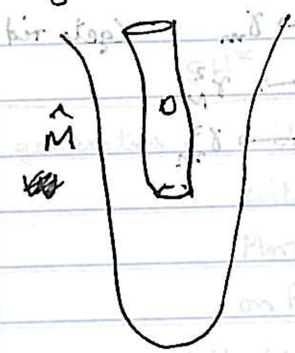
$$\cdots \longrightarrow E^2_{k-1,1} \longrightarrow H^{tot}_k \longrightarrow E^2_{k,0} \longrightarrow E^2_{k-2,1} \longrightarrow H^{tot}_{k+1}$$

$$\wr\!\downarrow$$

$$HC_{\sim} \longrightarrow SH_k \longrightarrow HC_{\sim} \longrightarrow HC_{\sim} \longrightarrow SH_{k-1} \cdots$$

this is our LES.

Why is $SH^+ \cong CH^{para}$? Set up isomorphism (idea of Tobias, conjectural):



count these objects

$\Rightarrow \Phi : C_*(\lambda) \longrightarrow C_*(H)$

$\langle \Phi(\gamma), \gamma' \rangle$ counts rigid curves from γ to γ'.

This gives $\Phi \circ d = d \circ \Phi$ chain homomorphism.
Now Φ is lower triangular by action argument, and we must check it has 1's on diagonal - show that (a reparametrisation of?) trivial cylinders solves some pseudoholomorphic equation where H becomes constant at ∞ (?).