Effect of Legendrian Surgery

Max.

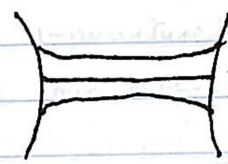
1) Legendrian Surgery
2) Effect on Reeb flow
3) Effect on CH.

Topology: $S^n \hookrightarrow M$ trivial normal bundle

$$\partial(D^{k+1} \times S^n) = S^n \times S^k = \partial(D^{n+1} \times S^k)$$

cut out nbhd $S^n \times D^{k+1}$ of S^n, then glue back in $D^{n+1} \times S^k$.

E.g. connect sum:



If $M = \partial W$, you can glue $D^{n+1} \times D^{k+1}$ to W along nbhd of S^n in M, this gives a cobordism from M to M after surgery.

We can do this symplectically, so that ~~the~~ if W is Liouville, the new W' is also Liouville $\Rightarrow$ new M' has contact structure. We require that our $S^n := \Lambda$ be Legendrian and come with a framing. There is a canonical framing:

$$\begin{aligned} N(\Lambda) &\cong \text{Reeb} \oplus J(T\Lambda) \\ &\cong \mathbb{R} \oplus TS^n \\ &\cong \mathbb{R}^{n+1} \quad \text{trivialised.} \end{aligned}$$

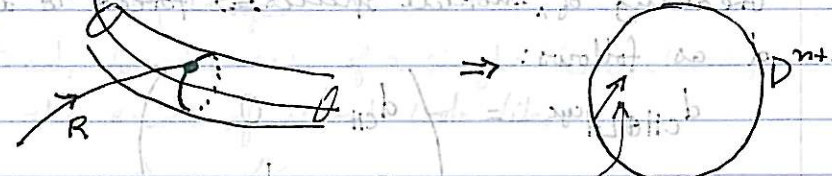
Then use a neighbourhood theorem and standard model.

NB. 1) This can be done for Λ isotropic. ($N(\Lambda) = \mathbb{R} \oplus J(T\Lambda) \oplus CSN(\Lambda)$
need to trivialise)

2) X Weinstein
h proper, Z gradient-like for h Morse.
can get h s.t. Legendrian handles are attached last.
handles of dim $< n$ are called subcritical.
$X' =$ (subcritical handle attachments) + Legendrian surgery
easily understood (Cieliebak)

2) Reeb orbits:
What are the Reeb orbits in the manifold after surgery? Assume Reeb orbits below certain energy are discrete and miss a nbhd of Λ. Do surgery along Λ in this nbhd. Then original Reeb orbits remain, but what about orbits going through the handle?

View the handle as ~~$D^*(D^{n+1})$~~ $D(T^*D^{n+1})$. Reeb orbits may pass through $S(T^*D^{n+1})$. Reeb flow looks like geodesic flow at unit speed. A Reeb flowline enters the handle along $S_x(T^*_x D^{n+1})$:



corresponding direction.

Then you do geodesic flow until you exit, then continue along Reeb flow.

To get an orbit you need this to close up.

$\Rightarrow$ new Reeb orbits $\overset{\cong}{\longleftrightarrow}$ old Reeb orbits $\cup$ words of Reeb chords.

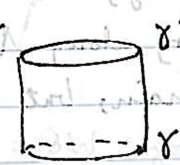
Thus

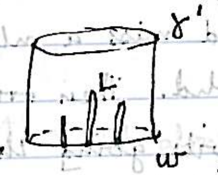
$$CH(X') = CH(X) \oplus LH^{cyc}$$

$\uparrow$ words on Reeb chords up to cyclic reordering, excluding empty word.

$$F^W : CH(X') \longrightarrow CH(X) \oplus LH^{cyc}$$

counts

$\#\Big($ 

$\Big)\, \gamma + \#\Big($ 

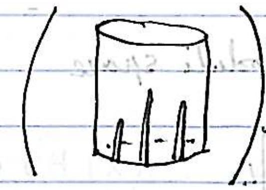
$\Big)\, w$ / weight

L = core of handle

word of Reeb chords.

Want this to be ~~This is~~ a chain homomorphism, by considering breaking of 1-dim moduli spaces $\Rightarrow$ forced to define d as follows:

$$d_{CH \oplus LH^{cyc}} = \begin{pmatrix} d_{CH} & 0 \\ \beta & d_{LH^{cyc}} \end{pmatrix}$$

where β counts $\#$ 

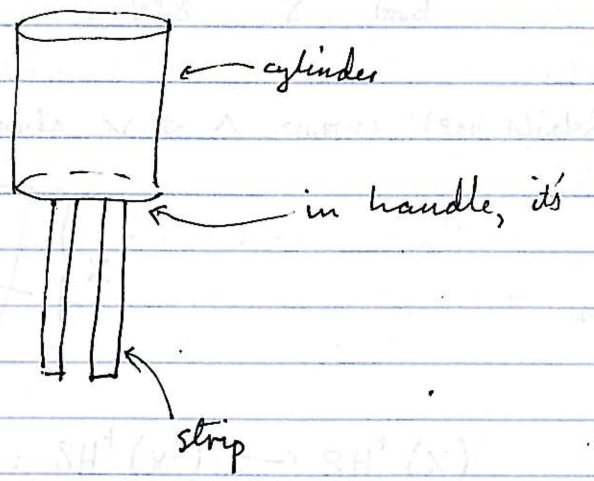
To prove isomorphism:

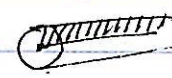
1) Filter by action $\rightarrow F^W$ lower triangular

2) Diagonal entries are 1 (up to certain action)
$\Updownarrow$ curves of very small energy.

over old Reeb orbits get trivial cylinders

over new orbits: would like to say it's





you can't do this: the estimates get screwed up because second derivative of Reeb chord when it enters the handle is very large.

Instead, for the case of a single Reeb chord, arrange the ends of chord look like

study this moduli space