SheelGanatra - Fukaya categories of A_m singularities

Note Title 6/17/2011

Case: $\mathfrak{g} = sl_m(\mathbb{C})$

e is a subregular nilpotent element

Prop: (Slodowy): S_e^0 is the singular $\{x^2+y^2+z^{m+1}=0\} \subseteq \mathbb{C}^3$

(cf. Travis' talk).

Case $sl_2(\mathbb{C})$.

$0 \in$ subregular

$$S_e^0 = \mathcal{N} = \left\{ \begin{pmatrix} a & b \\ c & -a \end{pmatrix} \middle| \ a^2+bc=0 \right\} \xrightarrow[\substack{b=x+iy \\ c=x-iy}]{} \{a^2+x^2+y^2=0\}$$

quadric

Deform : pick a deg-$(m+1)$ poly $P(z) = z^{m+1} + \dots + a_0$

$$\tilde{S}_P = \{x^2+y^2+P(z)=0\}. \longrightarrow \mathcal{S}$$

$$\downarrow$$

$P \in \mathrm{Sym}^{m+1}(\mathbb{C})$

(roots of poly)

$\tilde{S}_P$ smooth for roots of P distinct.

Restrict to
$$\begin{array}{ccc} \mathcal{S} & & S_{P_0} \\ \downarrow & & \downarrow \\ \mathrm{conf}(m+1, \mathbb{C}) & \ni & P_0 \end{array}$$

conf = dist. roots.

Rmk: $\mathcal{S} \to \mathrm{Conf}_{m+1}$ is a sympl. fibration. $\Rightarrow$ we have a parallel transport map.

$\Rightarrow \pi_1(\mathrm{Conf}_{m+1}, P_0) = Br_m$ acts on S_{P_0} by symplectos.

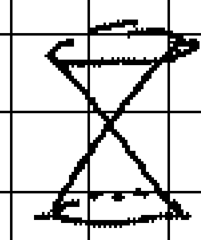
$\Rightarrow Br_m$ acts on $\mathrm{Fuk}(S_{P_0})$.

There's a Br_m action on W-algebras too.

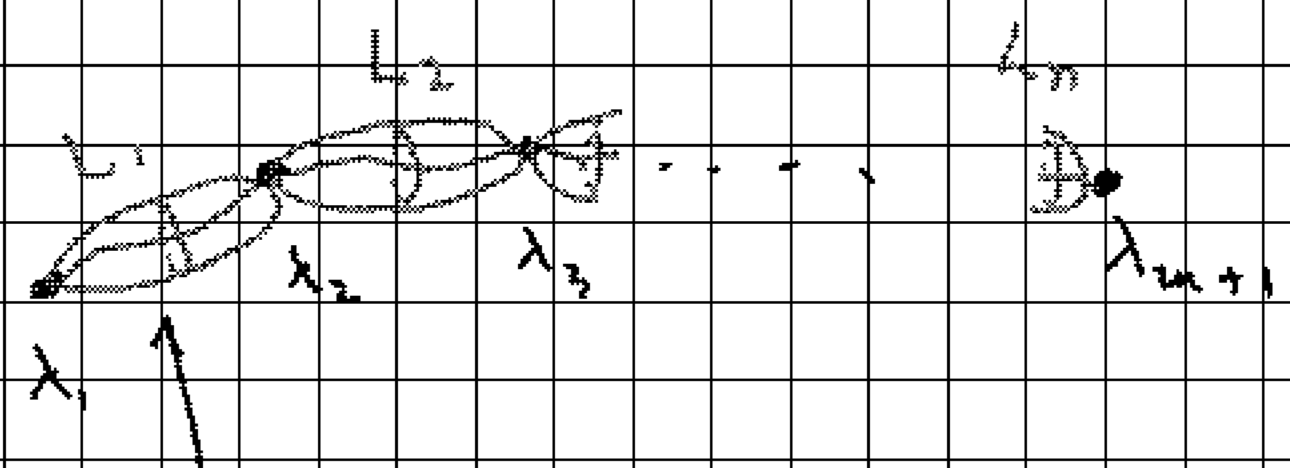
Look at $S_{P_0} = \{x^2+y^2+P_0(z)=0\} \subseteq \mathbb{C}^3$

$$\begin{array}{c} S_{P_0} \\ \big\downarrow \pi_z \\ \mathbb{C} \end{array} \quad \text{Lefschetz fibration.}$$

If $P_0(z) \neq 0$, $\pi_z^{-1}(z) = \{x^2+y^2+z=0\} \cong T^*S^1$

If $P_0(z) = 0$, $\pi_z^{-1}(z) = \{x^2+y^2=0\}$ 

Fix an ordering of roots $\lambda_1, \dots, \lambda_{m+1}$, and construct Lag. spheres by parallel transport:



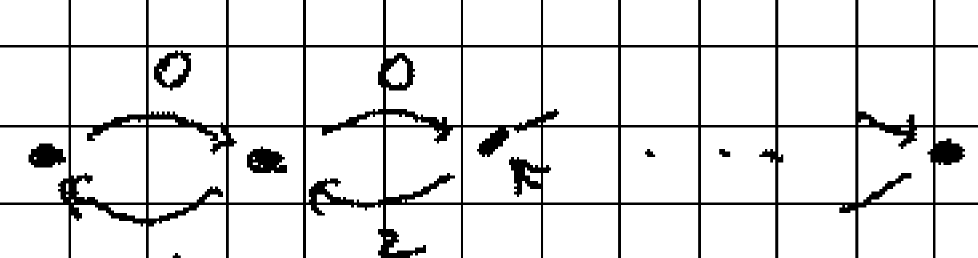
blue sphere in S_{P_0} fibres over red path.

$\Rightarrow$ spheres analogous to A_m chain from Travis' talk.

$\mathrm{Hom}(L_i, L_{i\pm1}) = \mathbb{C}$ (all in same degree)

$HF^*(L_i, L_i) \cong H^*(S^2)$.

$\Rightarrow HF^*(\oplus L_i, \oplus L_i)$ is path algebra of the quiver A_m:



with the relation $(i|i+1|i+2) = 0$

$(i+2|i+1|i) = 0$

$(i|i+1|i) = (i|i-1|i)$.

Rmk: • This algebra is formal $\Rightarrow$ up to quasi-isomorphism, there are no higher μ^d's.

• You can show these split-generate all other compact Lag's.

• A_m-mod is a model for Fuk.

Rmk: By theorems of ...

$$A_m\text{-mod} \hookrightarrow W^0(e)\text{-mod}.$$