Daniel Rowe - Examples

Note Title 6/13/2011

Unstratified space X, $\mathcal{F}$ loc. syst. on X.

$\Big[$  $\Big] \longleftrightarrow [F_x \xrightarrow{\sim} F_y]$

homotopy class of paths

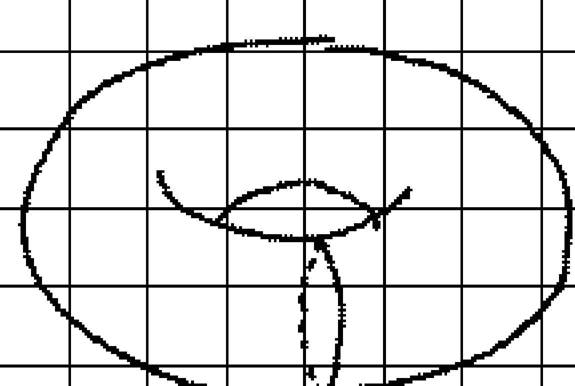
$$\mathrm{Loc}(X) \longleftrightarrow \mathrm{Rep}(\Pi_1 X) \longleftrightarrow \mathrm{Rep}(\pi_1(X))$$

fund. groupoid

Fix stratification $\mathcal{S}$, $X = \coprod X_\alpha$

$F \in Sh_{c,\mathcal{S}}(X) \Leftrightarrow F|_{X_\alpha}$ is a local system $\forall\, \alpha$

E.g.



Study not <u>all</u> paths, but <u>exit paths</u>

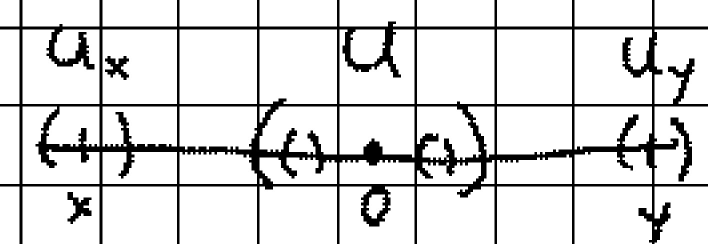
$EP(X, \mathcal{S})$ = category with objects = paths
morphisms = paths that only move from lower to higher-dimensional strata

$\left\{\begin{matrix}\mathcal{S}\text{-constructible} \\ \text{sheaves on } X\end{matrix}\right\} \longleftrightarrow \left\{\text{Representations of } EP(X, \mathcal{S})\right\}$

($\rightsquigarrow$ A_∞ functors, with higher homotopies of exit paths, if you talk about complexes of sheaves)

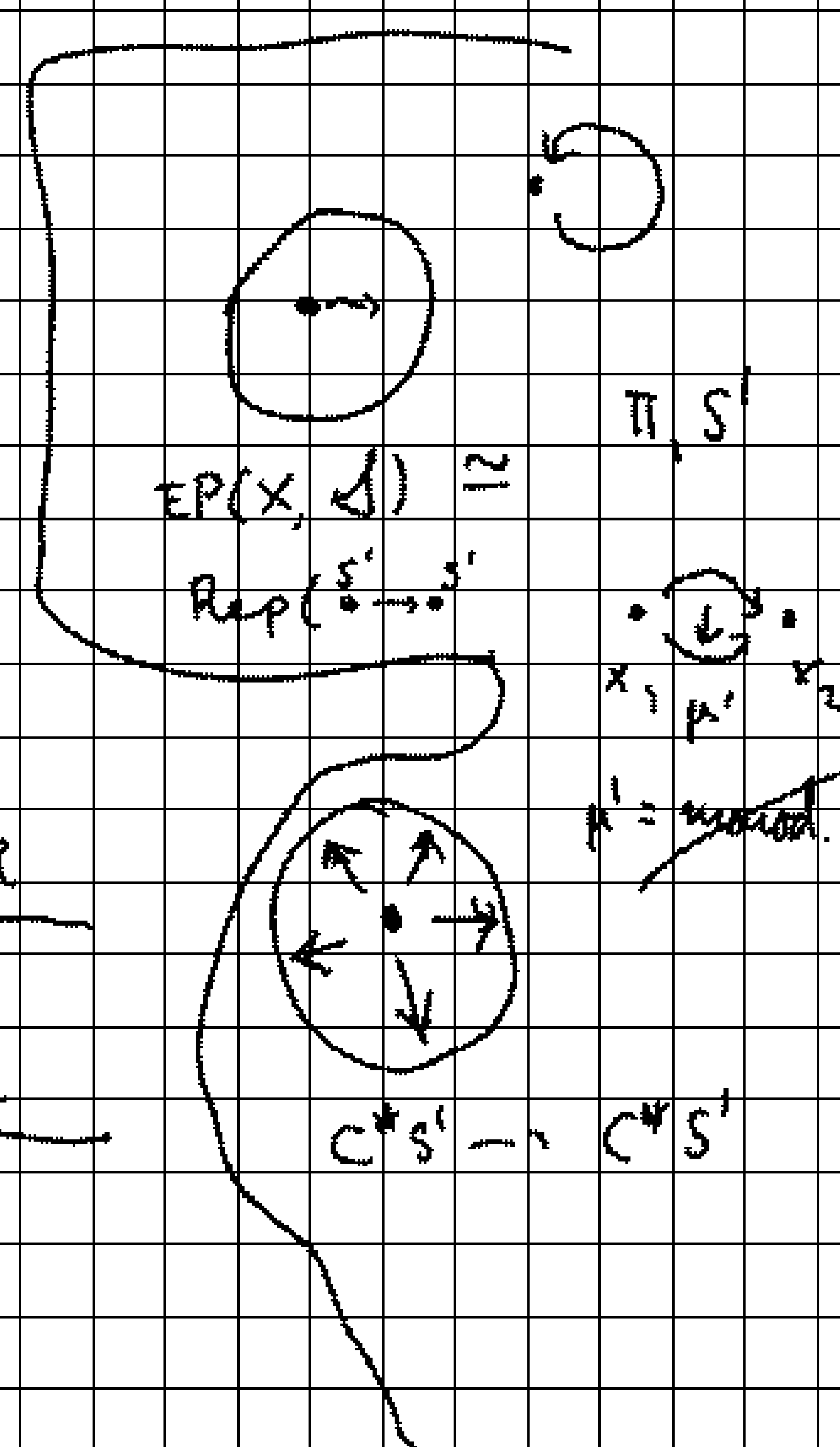
E.g.: $X = \mathbb{R}$, $S = 0 \subset \mathbb{R}$

$F \in Sh_c(X)$



$$\begin{tikzcd} & F_0 = F(U) \arrow[dl] \arrow[dr] & \\ F(U_x) = F_x & & F_y = F(U_y) \end{tikzcd}$$

$Sh_{c,\mathcal{S}}(\mathbb{R}) = Rep\left(\bullet \rightrightarrows \bullet \ \bullet \right)$



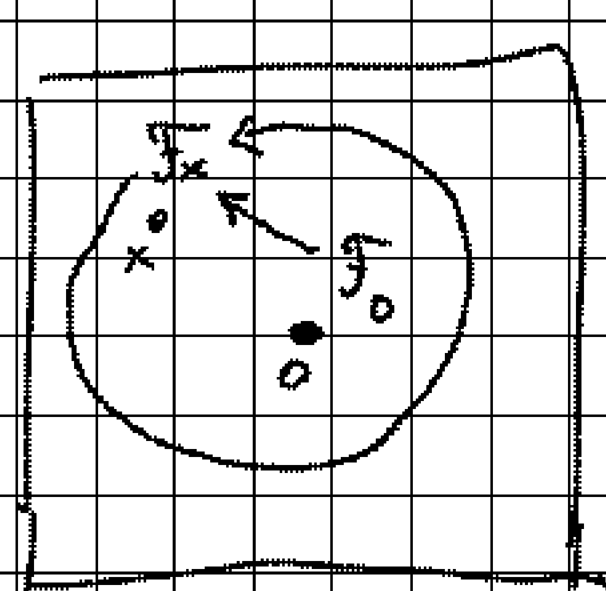
E.g. $j: \mathbb{R}_{>0} \hookrightarrow \mathbb{R}$

what is $j_* \tilde{k}$? $\quad 0 \xleftarrow{\circ} k \xrightarrow{\sim} k$

$j_! \tilde{k}$? $\quad 0 \xleftarrow{\circ} 0 \xrightarrow{\circ} k$

E.g. $X = \mathbb{C}$ $\quad \mathcal{S} = \{0, \mathbb{C}^*\}$

$\mathcal{F} \in Sh_{c,\mathcal{S}}$:



$F_0 \longrightarrow F_x \circlearrowleft m \qquad m =$ monodromy

$F_0 \longrightarrow F_x^m := \ker(m-1) \qquad m$-invariants

This is because $EP(\mathbb{C}, \mathcal{S}) = \left\{ \bullet \xrightarrow{a} \bullet \circlearrowleft m, m^{-1} \;\middle|\; \begin{matrix} ma = a \\ mm^{-1} = 1 \end{matrix} \right\}$

$Sh_{c,\mathcal{S}}(\mathbb{C}) = Rep\left[\bullet \xrightarrow{a} \bullet \circlearrowleft m, m^{-1} : ma = a \right]$

Alternate quiver

$$\tilde{O} \underset{q}{\overset{p}{\rightleftarrows}} x$$

related to previous quiver via

$\tilde{O} = \mathrm{Cone}(a) \qquad X = X.$

$1 - qp$ inv.

$1 - pq$ inv.

5 favourite objects

1) k_o
2) $k_{\mathbb{C}}$ — these are all the indecomposables
3) $j_* k_{\mathbb{C}^*}$
4) $j_! k_{\mathbb{C}^*}$
5) ?

In our two quivers, these look like:

$$\bullet \xrightarrow{a} \bullet \circlearrowleft m \qquad\qquad \bullet \underset{q}{\overset{p}{\rightleftarrows}} \bullet$$

1) k_o $\qquad k \longrightarrow 0 \circlearrowleft \qquad\qquad k \rightleftarrows 0$

2) $k_{\mathbb{C}}$ $\qquad k \xrightarrow{\sim} k \circlearrowleft \qquad\qquad 0 \rightleftarrows k$

3) $j_* k_{\mathbb{C}^*}$ $\qquad \begin{array}{ll} 1 & k \\ 0 & k \xrightarrow{\sim} k \end{array} \qquad\qquad k \underset{0}{\overset{\sim}{\rightleftarrows}} k$

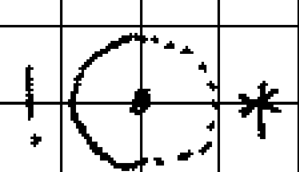
(NB: Rj_*)

4) $j_! k_{\mathbb{C}^*}$ $\qquad 0 \longrightarrow k \circlearrowleft ? \qquad\qquad k \underset{\sim}{\overset{0}{\rightleftarrows}} k$

5) ? $\qquad k[-1] \qquad k \qquad\qquad k \oplus k \xrightarrow{\pi_1} k, \quad k \oplus k \xleftarrow{\iota_2} k$

tilting sheaf.

Sheaf ? looks like this:



$j_{!/*} k_{\mathbb{C}^*}$ - sections can approach 0 from one side but not the other.

E.g. $X = S^1$ $\quad \mathcal{S} = \{0, S^1 \setminus 0\}$

$\mathcal{F}_0 \rightleftarrows \mathcal{F}_*$.