

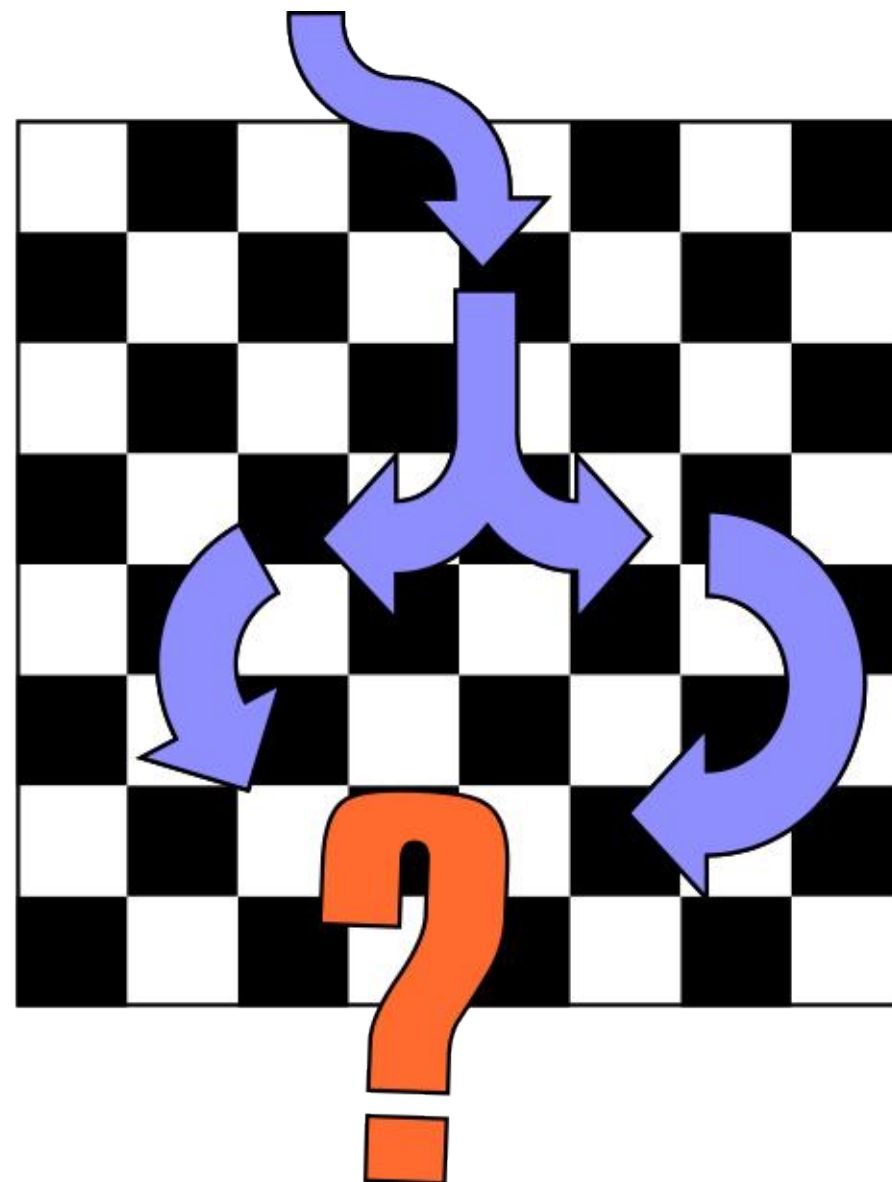
FINDING

YOUR

WAY

ON A

CHESSBOARD



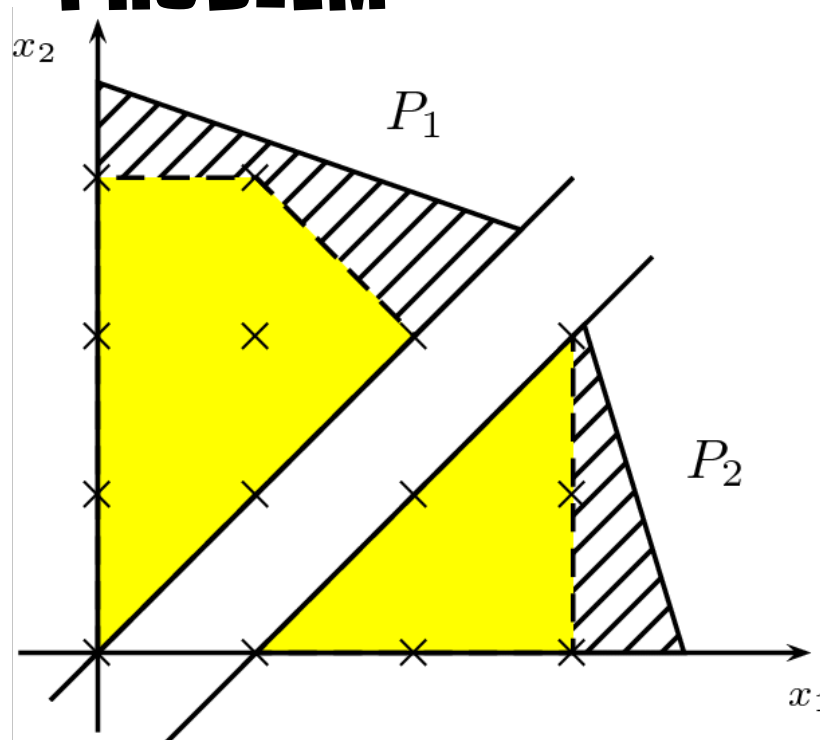
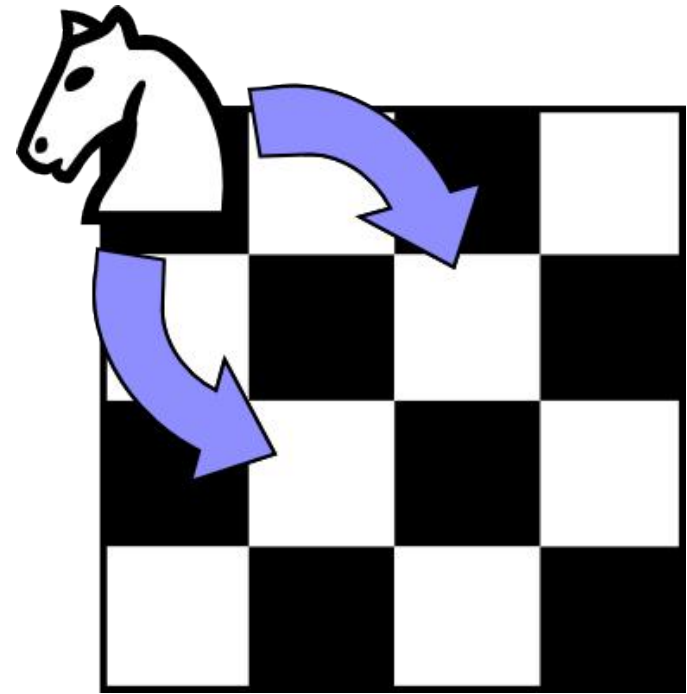
MARCO COLOMBO

26 JANUARY 2006

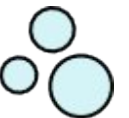


TODAY'S TWO COURSES DEAL

1. THE KNIGHT'S TOUR PROBLEM



2. MATHS FORMULATION AND SOLUTION

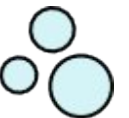


THE KNIGHT'S TOUR PROBLEM

**EULER POSED THE
FOLLOWING PROBLEM
(1759):**



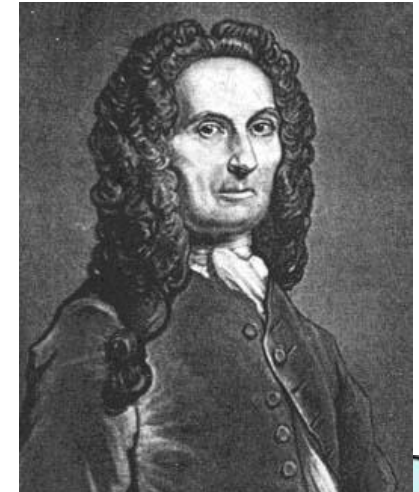
**“GIVEN A CHESSBOARD OF SIZE N ,
IS IT POSSIBLE TO FIND A PATH
FOR THE KNIGHT THAT TOUCHES
EVERY SQUARE IN SUCCESSION?”**



THE ORIGINS OF THE PROBLEM

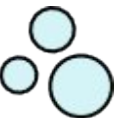
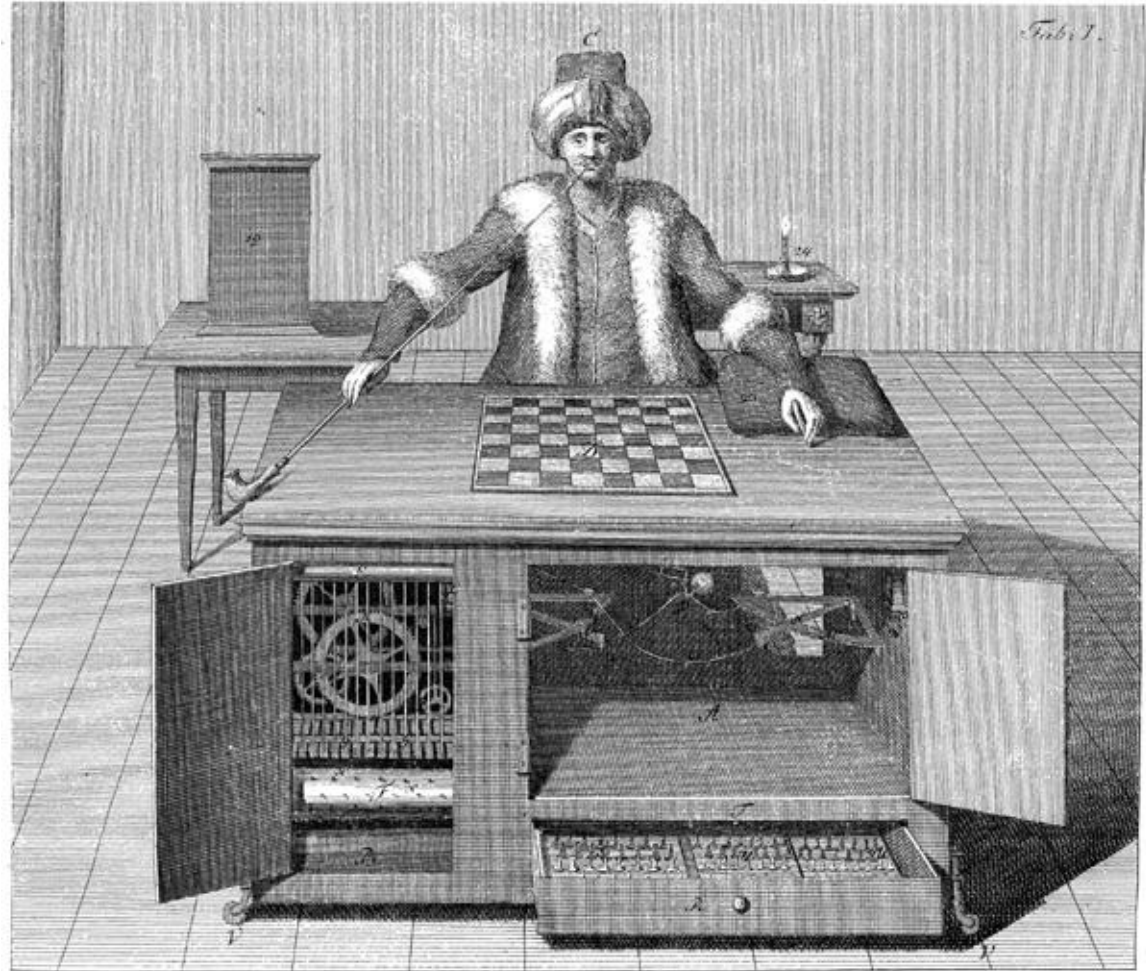
**SOME SOLUTIONS BY ARAB
MATHEMATICIANS (~800) SURVIVED
CITED IN SOME LATER TEXTS (~1300)**

**TAYLOR, MONTMORT, DE MOIVRE,
(~1700)**

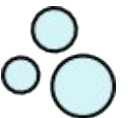
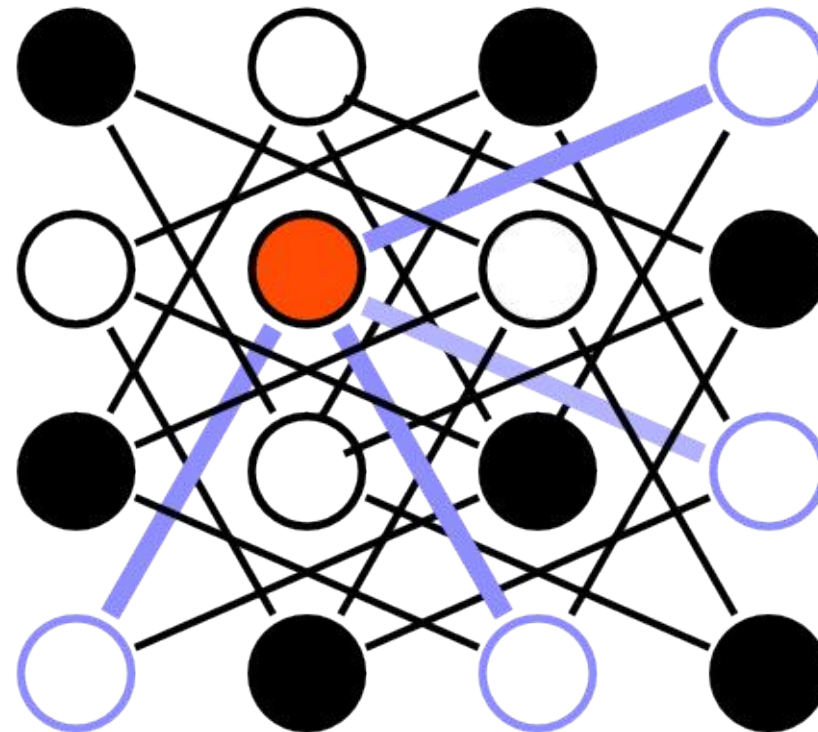
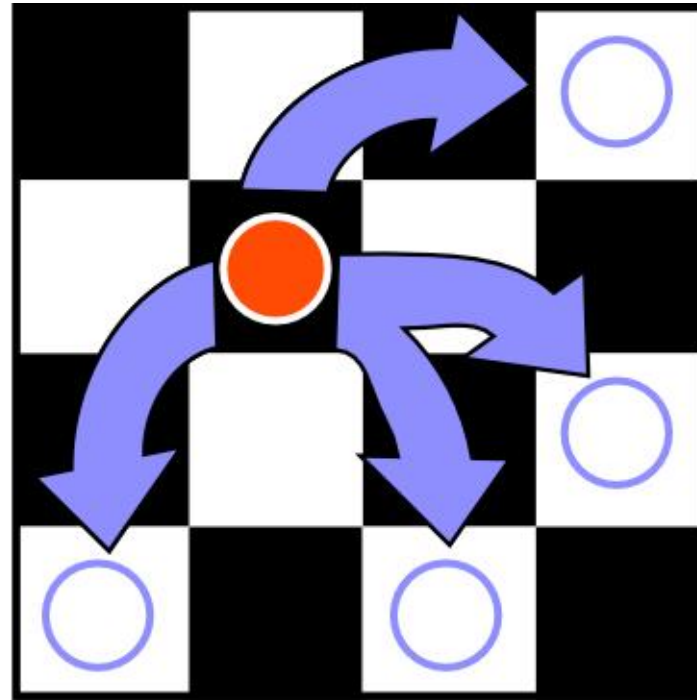


**THE
KNIGHT'S
TOUR
AS A
CONJURING
TRICK**

VON KEMPELEN BUILDS THE TURK (1769):

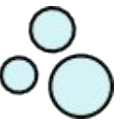
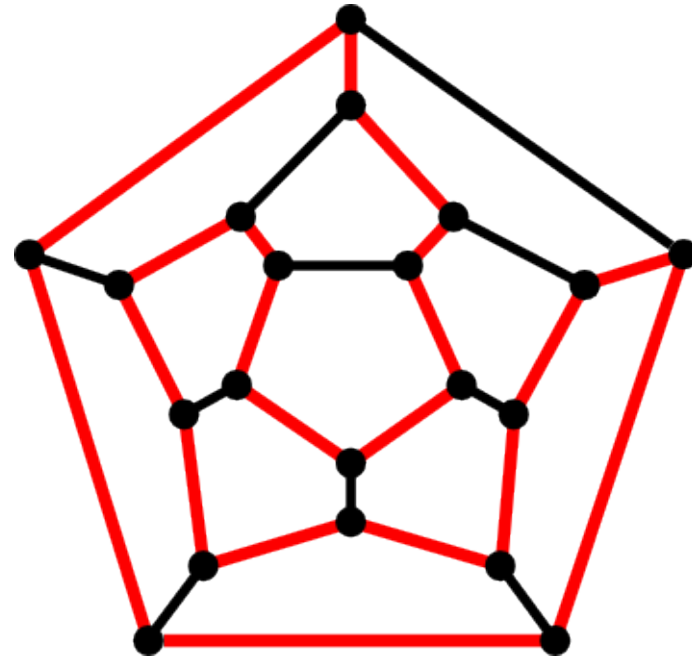


FROM A CHESSBOARD TO A GRAPH



**THE
ICOSIAN
GAME
AND
HAMILTONIAN
CYCLES**

GAME DESCRIBED BY HAMILTON (1857):



HAMILTONIAN

CYCLE

AND

TRAVELLING

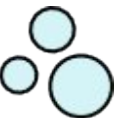
SALESMAN

H.C.P.:

**FIND A H.C. IN A GRAPH THAT HAS FEW
(USUALLY A DIRECTED GRAPH)**

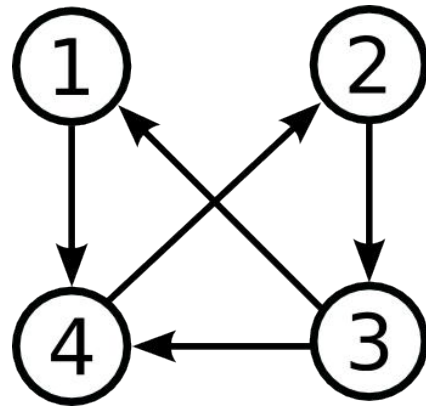
T.S.P.:

**FIND THE MINIMUM DISTANCE H.C. IN A
GRAPH THAT HAS MANY
(USUALLY A COMPLETE GRAPH)**



ON THE ROAD TOWARDS THE SOLUTION

INTEGER PROGRAMMING FORMULATION



$$A = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 & 0 \\ 1 & 0 & 0 & 1 & -1 \end{bmatrix}$$

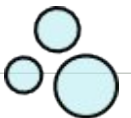
$$x_{ij} = \begin{cases} 1 & \text{if arc } (i, j) \text{ is used in the HC} \\ 0 & \text{otherwise} \end{cases}$$

One arc enters and one arc leaves each node:

$$Ax = 0$$

There are m edges in a cycle:

$$\sum_{i,j \in V} x_{ij} = m$$



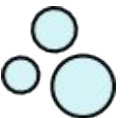
**BUT
IT'S NOT
THAT
EASY!**

**THE PROBLEM IS INTRACTABLE FOR
NONTRIVIAL SIZES
(COMBINATORIAL EXPLOSION)**

RELAX INTEGRALITY CONSTRAINTS

$$0 \leq x_{ij} \leq 1$$

DEAL WITH FRACTIONAL SOLUTIONS

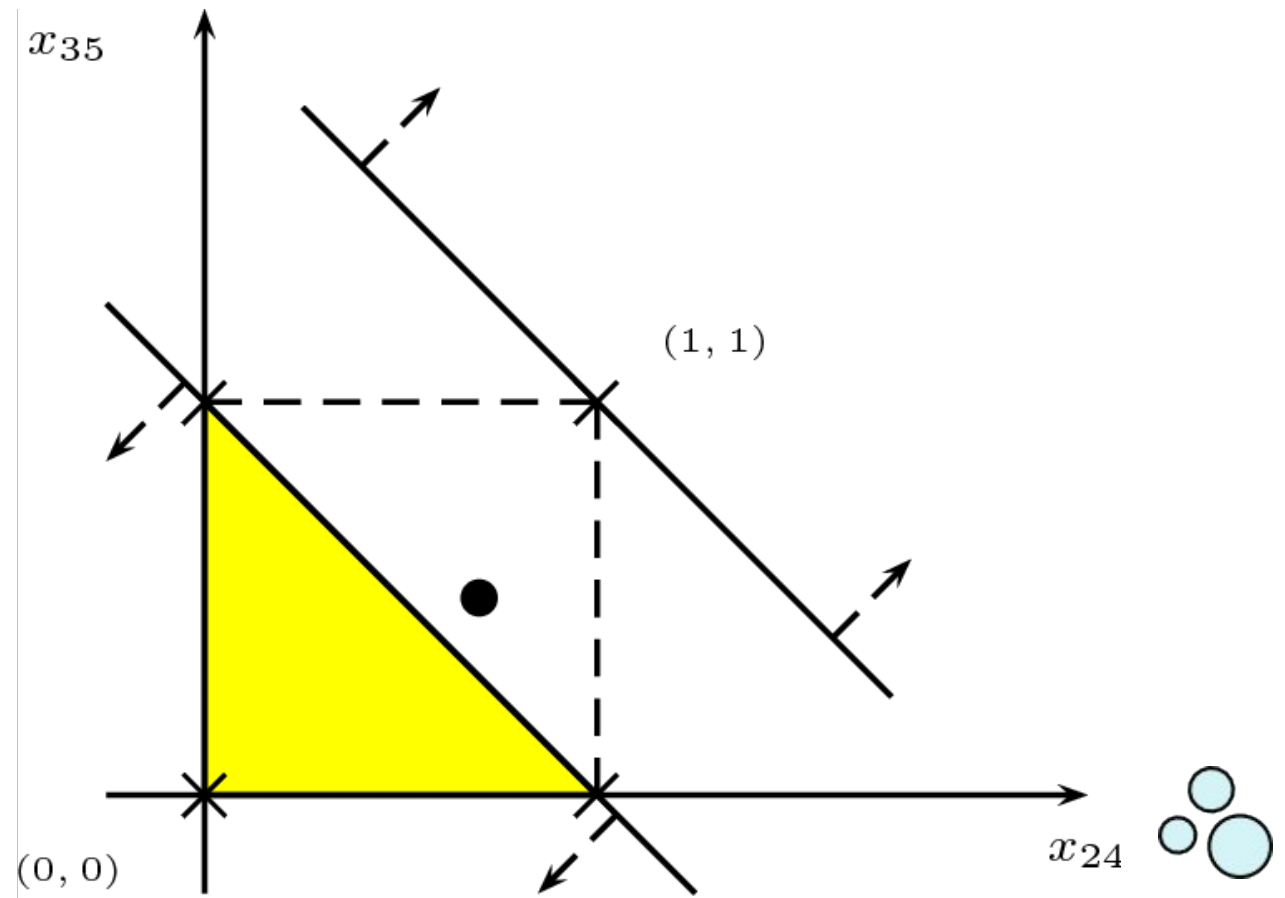


**THE
IDEA
OF
BRANCH
AND
CUT**

**USE CUTTING PLANES TO REMOVE THE
FRACTIONAL SOLUTIONS**

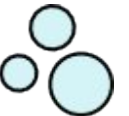
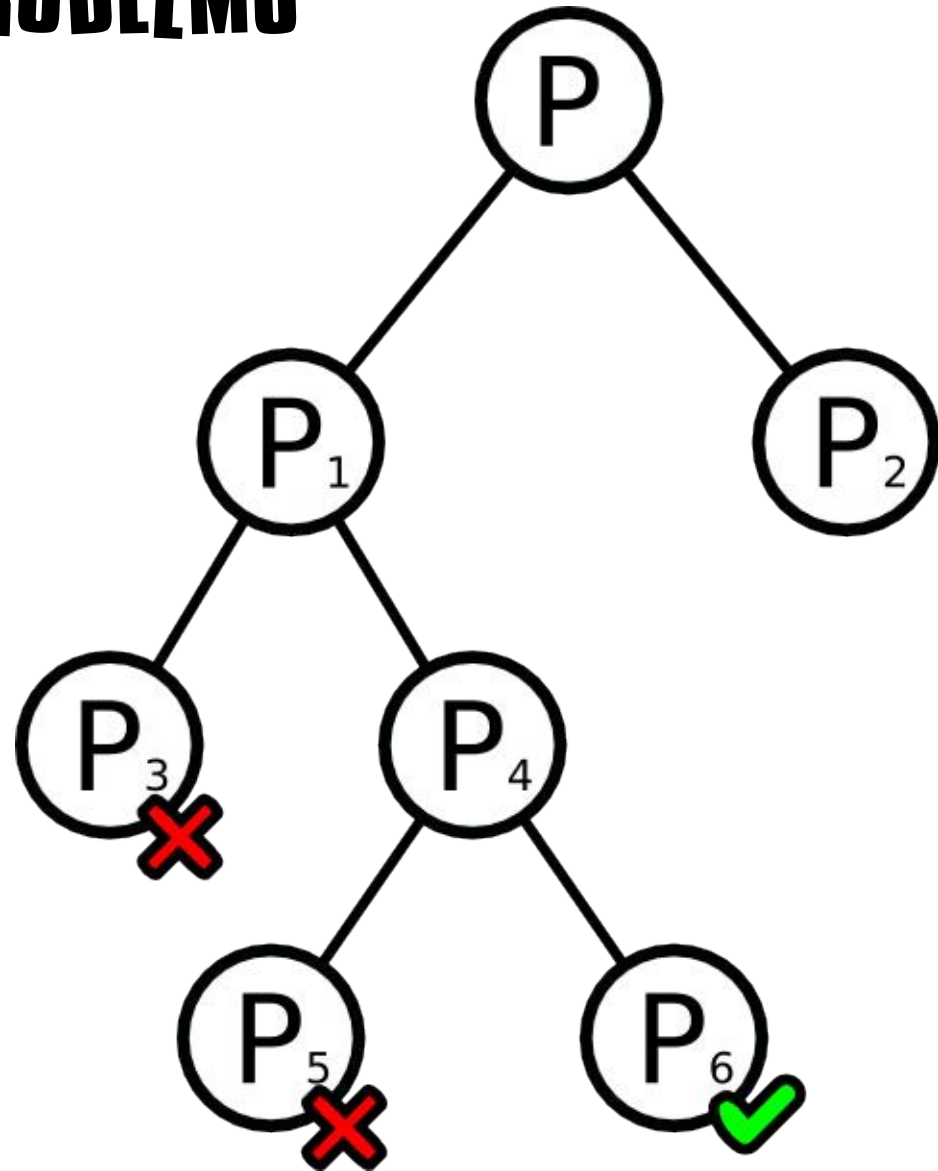
$$x_{24} = 0.7 \quad x_{35} = 0.5$$

$$x_{24} + x_{35} \leq 1 \quad \vee \quad x_{24} + x_{35} \geq 2$$



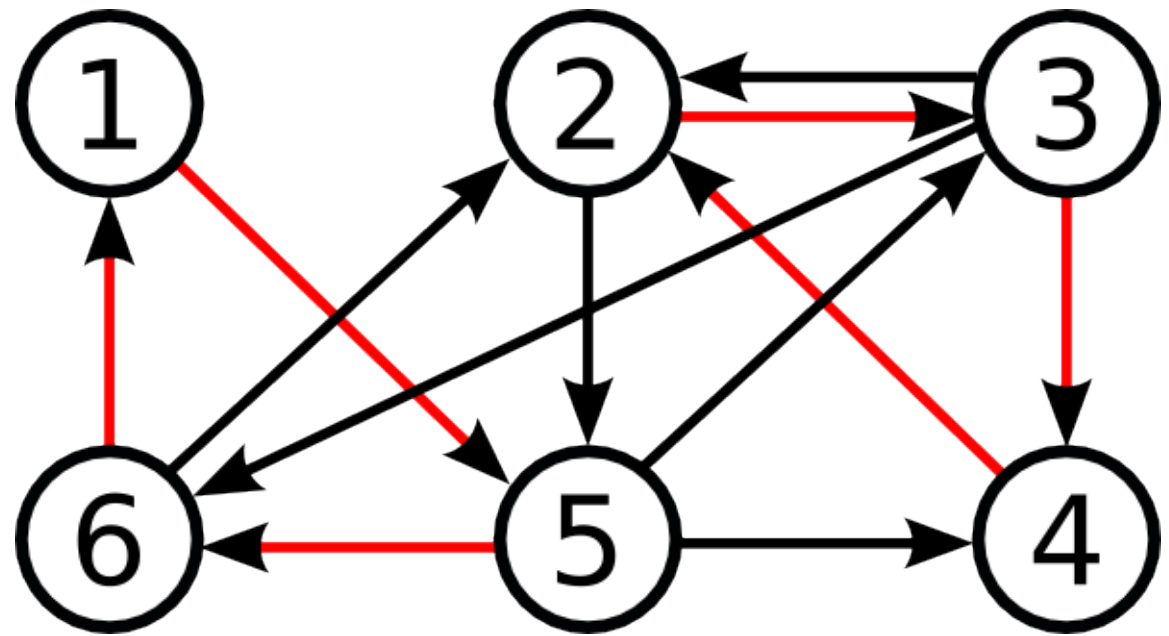
**THE
IDEA
OF
BRANCH
AND
CUT**

**DIVIDE THE PROBLEM INTO SMALLER
SUBPROBLEMS**

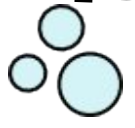


**IS IT
THE
END
OF THE
STORY?**

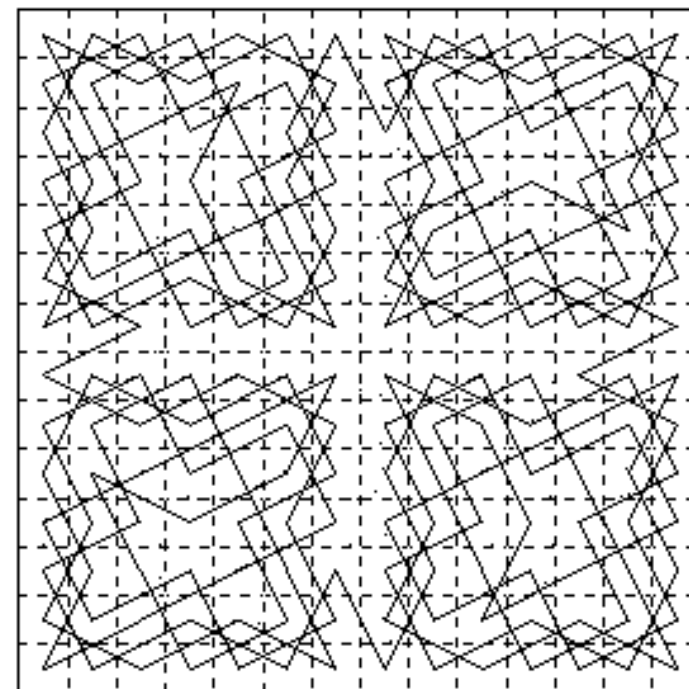
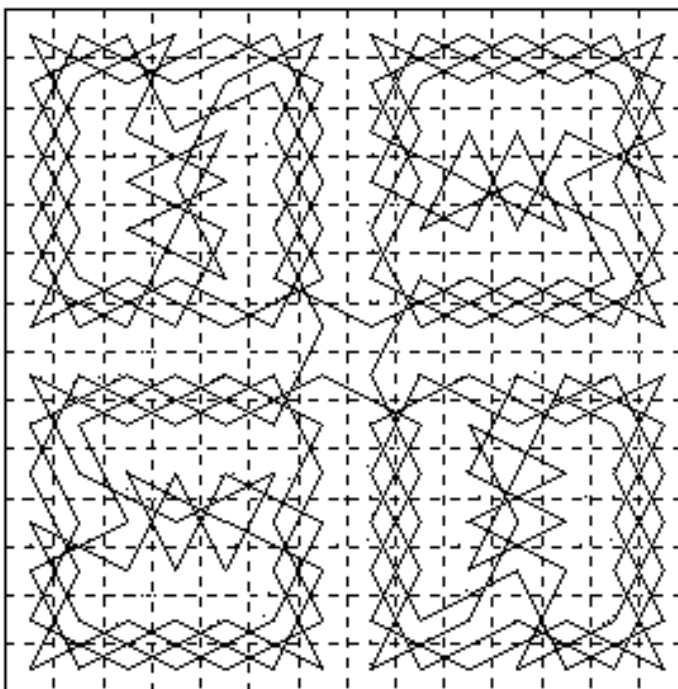
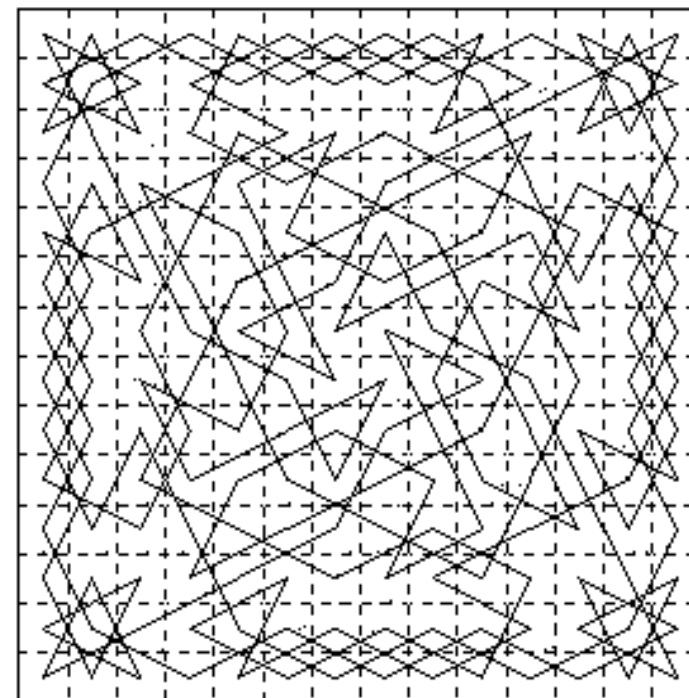
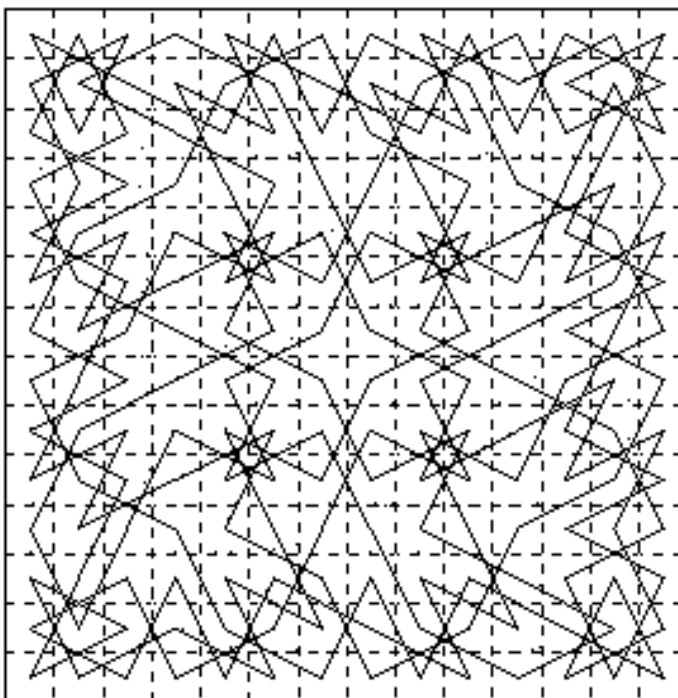
**WHEN ALL FRACTIONAL SOLUTIONS HAVE
GONE, WE MAY STILL NOT HAVE FOUND
A HAMILTONIAN CYCLE!**



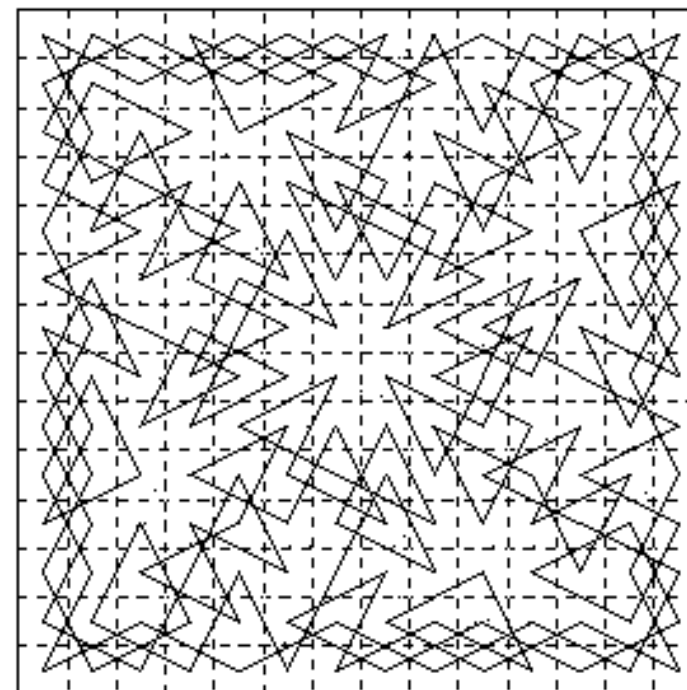
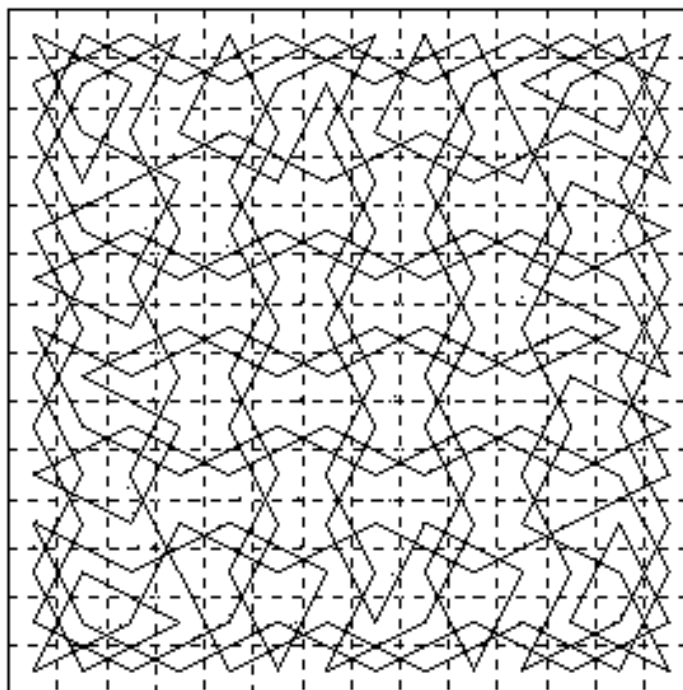
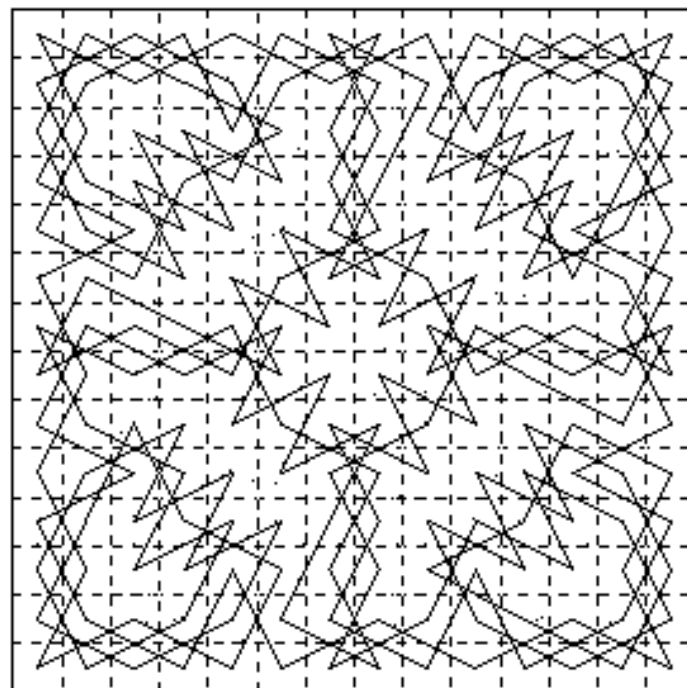
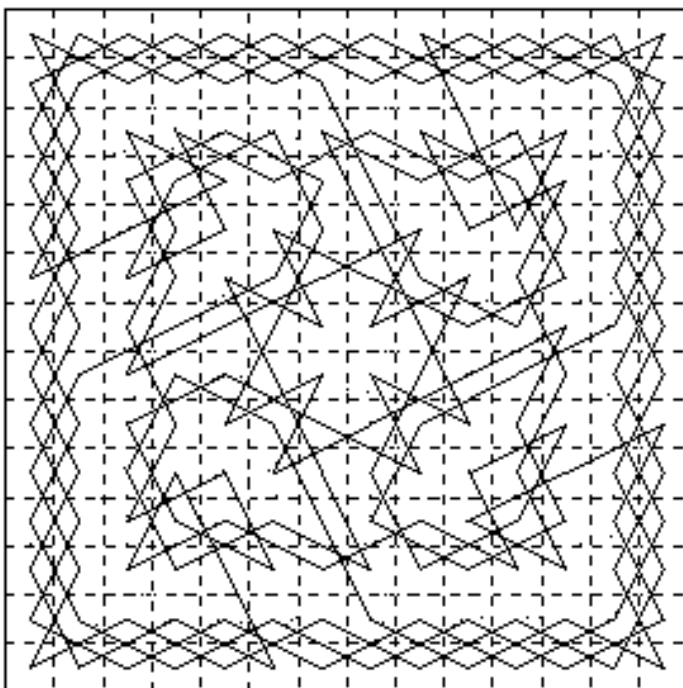
ADD SUBCYCLE REMOVING CONSTRAINTS



**SOME
SOLUTIONS
FOR
BOARD
SIZE 14**



**MORE
SOLUTIONS
FOR
BOARD
SIZE 14**



**A
SOLUTION
FOR
BOARD
SIZE 18**

