Mathematical Techniques III
(PHY 317)

Problem Set 10

Due on Wednesday, December 16th

Problem 1. (*15 points*)

Compute the Laplace transforms of the following functions:

(a) $f(t) = 3\cos 2t - 8e^{-2t}$

(b) $f(t) = \dfrac{1}{\sqrt{t}}$

(c) $f(t) = \begin{cases} 1 \ , & \text{for } t < 1, \text{ and} \\ 0 \ , & \text{for } t \geq 1. \end{cases}$

(d) $f(t) = (\sin t)^2$

(e) $f(t) = \begin{cases} 0 \ , & \text{for } t < 1, \\ 1 \ , & \text{for } 1 \leq t \leq 2, \text{ and} \\ 0 \ , & \text{for } t > 2. \end{cases}$

Make sure to specify as part of your answer the values of s for which the Laplace transform is valid.

Problem 2. (*15 points*)

Find the inverse Laplace transform of the following functions:

(a) $F(s) = \dfrac{1}{s^2 + 4}$

(b) $F(s) = \dfrac{4}{(s-1)^2}$

(c) $F(s) = \dfrac{s}{s^2 + 4s + 4}$

(d) $F(s) = \dfrac{1}{s^3 + 3s^2 + 2s}$

(e) $F(s) = \dfrac{s+3}{s^2 + 4s + 7}$

Problem 3. (*20 points*)

Use the Laplace transform to solve the following initial value problems:

(a) $$\frac{d^2 f(t)}{dt^2} - 5\frac{df(t)}{dt} + 6f(t) = 0 \ , \qquad f(0) = 1, \ f'(0) = -1 \ ,$$

(b) $$\frac{d^2 f(t)}{dt^2} - \frac{df(t)}{dt} - 2f(t) = e^{-t}\sin 2t \ , \qquad f(0) = f'(0) = 0 \ ,$$

(c) $$\frac{d^2 f(t)}{dt^2} - 3\frac{df(t)}{dt} + 2f(t) = \begin{cases} 0 \ , & \text{for } 0 \leq t < 3, \\ 1 \ , & \text{for } 3 \leq t \leq 6, \text{ and} \\ 0 \ , & \text{for } t > 6, \end{cases}$$

$$f(0) = f'(0) = 0 \ .$$

Does any one of these equations describe a stable system?