

MACDONALD POSITIVITY VIA THE HARISH-CHANDRA D -MODULE

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ABSTRACT. Using the Harish-Chandra D -module, we give a proof of Haiman's theorem on the positivity of Macdonald polynomials. Ginzburg's work on the connection between this D -module and the isospectral commuting variety is fundamental to this approach.

1. INTRODUCTION

The (transformed) Macdonald polynomials $\tilde{H}_\mu(z; q, t)$ are symmetric functions with coefficients that are rational functions of two parameters q and t . They have remarkable specialisations to important families of symmetric functions including Hall-Littlewood polynomials, Jack polynomials and Schur functions.

Expanding the Macdonald polynomials in terms of Schur functions,

$$\tilde{H}_\mu(z; q, t) = \sum_{\lambda} \tilde{K}_{\lambda, \mu}(q, t) s_{\lambda}(z),$$

Macdonald conjectured that the coefficients $\tilde{K}_{\lambda, \mu}(q, t)$ belong to $\mathbb{N}[q, t]$. In a wonderful paper, [7], Haiman confirmed this conjecture by proving the $n!$ theorem. This showed the existence of a vector bundle $\tilde{\mathcal{P}}$ on $\text{Hilb}^n \mathbb{C}^2$, the Hilbert scheme of points on the plane, with many remarkable properties. In particular, the fibres of $\tilde{\mathcal{P}}$ at the torus fixed points of $\text{Hilb}^n \mathbb{C}^2$ are bigraded representations of \mathfrak{S}_n encoding the Macdonald polynomials. Haiman's proof of the $n!$ theorem is a remarkable blend of sophisticated algebraic geometry and subtle combinatorics.

In this note we give a different proof of Macdonald positivity using recent work of Ginzburg, [4]. This proof again displays a vector bundle on $\text{Hilb}^n \mathbb{C}^2$ whose fibres at torus fixed points carry the Macdonald polynomials. The bundle is constructed from a degeneration of the Harish-Chandra D -module on the Grothendieck-Springer resolution of type A_{n-1} ; to describe its fibres requires only standard constructions from D -module theory and the Springer correspondence. It should be noted that in [4] Ginzburg showed that this bundle is isomorphic to $\tilde{\mathcal{P}}$ if one assumes Haiman's results. We do not know if it is possible to give a new proof of the $n!$ theorem along similar lines.

Following Haiman's pioneering work there have been two recent proofs of generalisations of Macdonald positivity, [1] and [5]. These are of a different flavour to this note.

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2. POSITIVITY

Let V be an n -dimensional complex vector space, $G = GL(V)$ with Lie algebra $\mathfrak{g} = \mathfrak{gl}(V)$, and set \mathfrak{t} to be the subalgebra of \mathfrak{g} consisting of diagonal matrices. Let $B \leq G$ be the Borel subgroup of upper triangular matrices, with Lie algebra \mathfrak{b} . The Weyl group, $W = \mathfrak{S}_n$, acts on \mathfrak{t} . We will identify \mathfrak{g} and \mathfrak{t} with \mathfrak{g}^* and \mathfrak{t}^* via the trace pairing.

Let $\kappa : \mathfrak{g} \times \mathfrak{g} \rightarrow \mathfrak{g}$ be the commutator. The commuting variety, \mathfrak{C} , is the scheme-theoretic fibre $\kappa^{-1}(0)$. Set $\mathfrak{T} = \mathfrak{t} \times \mathfrak{t}$. Simultaneous conjugation provides an action of G on \mathfrak{C} such that the algebraic geometric quotient \mathfrak{C}/G is isomorphic to \mathfrak{T}/W , see [2, Theorem 1.3]. Let $\mathfrak{X} = [\mathfrak{C} \times_{\mathfrak{T}/W} \mathfrak{T}]_{\text{red}}$, the reduced *isospectral commuting variety*, and let $\mathfrak{X}_{\text{norm}}$ be its normalisation with morphism $\psi : \mathfrak{X}_{\text{norm}} \rightarrow \mathfrak{X}$. There is a projection morphism $p_{\mathfrak{C}} : \mathfrak{X} \rightarrow \mathfrak{C}$ and an induced morphism on the normalisations $p : \mathfrak{X}_{\text{norm}} \rightarrow \mathfrak{C}_{\text{norm}}$.

There is an action of G on \mathfrak{X} induced from \mathfrak{C} , of $\mathbb{C}^* \times \mathbb{C}^*$ by dilation in both sets of variables, and of W from the diagonal action on \mathfrak{T} . All these lift to $\mathfrak{X}_{\text{norm}}$.

Let $\tilde{\mathfrak{g}} = G \times_B \mathfrak{b}$ be the Grothendieck-Springer resolution. It admits morphisms $\mu : \tilde{\mathfrak{g}} \rightarrow \mathfrak{g}$ and $\nu : \tilde{\mathfrak{g}} \rightarrow \mathfrak{t}$ defined by $(g, x) \mapsto gxg^{-1}$, respectively $(g, x) \mapsto x \bmod [\mathfrak{b}, \mathfrak{b}]$. Let $\mathcal{M} = \int_{\mu \times \nu} \mathcal{O}_{\tilde{\mathfrak{g}}}$, the *Harish-Chandra $D_{\mathfrak{g} \times \mathfrak{t}}$ -module*. It is holonomic.

Theorem 1. [4, Theorem 1.3.3, Theorem 1.3.4, Theorem 1.5.2]

- (1) *There is a filtration on \mathcal{M} , the Hodge filtration, such that $\text{gr } \mathcal{M} \cong \psi_* \mathcal{O}_{\mathfrak{X}_{\text{norm}}}$.*
- (2) *$\mathfrak{X}_{\text{norm}}$ is Cohen-Macaulay and Gorenstein.*
- (3) *Set $\mathcal{R} = p_* \mathcal{O}_{\mathfrak{X}_{\text{norm}}}$. Over the smooth locus of \mathfrak{C} , \mathcal{R} is a $G \times W \times \mathbb{C}^* \times \mathbb{C}^*$ -equivariant vector bundle whose fibres carry the regular representation of W .*

Let $\mathcal{S} = \{(X, Y, v) \in \mathfrak{g} \times \mathfrak{g} \times V : [X, Y] = 0, \mathbb{C}\langle X, Y \rangle v = V\}$. The action of G on \mathcal{S} is free, and its quotient is $\text{Hilb}^n \mathbb{C}^2$, the Hilbert scheme of n points on the plane. The $\mathbb{C}^* \times \mathbb{C}^*$ -action on $\text{Hilb}^n \mathbb{C}^2$ has a finite number of fixed points, I_μ , labelled by partitions of n , see for instance [7, §3.2].

The projection morphism from \mathcal{S} to $\mathfrak{g} \times \mathfrak{g}$ has image \mathfrak{C}° , the set of pairs $(X, Y) \in \mathfrak{C}$ that have a cyclic vector. This makes \mathcal{S} a torsor over \mathfrak{C}° .

Since \mathfrak{C}° is smooth we may define an open set $\mathfrak{X}^\circ = p^{-1}(\mathfrak{C}^\circ)$ in $\mathfrak{X}_{\text{norm}}$ and then set $\mathfrak{W} = (\mathfrak{X}^\circ \times_{\mathfrak{C}^\circ} \mathcal{S})/G$. We have the following diagram, see [4, (8.2.1)]

$$\begin{array}{ccccccc}
 \mathfrak{X}^\circ & \xleftarrow{\delta} & \mathfrak{X}^\circ \times_{\mathfrak{C}^\circ} \mathcal{S} & \xrightarrow[h \cong]{\quad} & \mathfrak{W} \times_{\text{Hilb}^n \mathbb{C}^2} \mathcal{S} & \xrightarrow{\tilde{\rho}} & \mathfrak{W} \\
 & \searrow p & & \searrow \tilde{\rho} & \swarrow \tilde{\eta} & & \swarrow \eta \\
 & & \mathfrak{C}^\circ & \xleftarrow[\delta]{\quad} & \mathcal{S} & \xrightarrow[\rho]{\quad} & \text{Hilb}^n \mathbb{C}^2
 \end{array}$$

Set $\mathcal{P} = (\rho_* \delta^*(\mathcal{R}|_{\mathfrak{C}^\circ}))^G$. By [4, Corollary 8.1.3] this is a $W \times \mathbb{C}^* \times \mathbb{C}^*$ -equivariant vector bundle on $\text{Hilb}^n \mathbb{C}^2$ whose fibres carry the regular representation of W . It is shown in [4, §8.2] that \mathfrak{W} is isomorphic to the relative spectrum of \mathcal{P} , so $\mathcal{P} \cong \eta_* \mathcal{O}_{\mathfrak{W}}$.

The *transformed Macdonald polynomials* $\tilde{H}_\mu(z; q, t)$ are two parameter symmetric functions attached to partitions μ . They may be characterised by the following conditions in the ring of symmetric functions over the base field $\mathbb{Q}(q, t)$, [8, Definition 3.5.2].

- (Mi) $\tilde{H}_\mu[(1-q)Z; q, t] \in \mathbb{Q}(q, t)\{s_\lambda(z) : \lambda \geq \mu\}$
- (Mii) $\tilde{H}_\mu[(1-t)Z; q, t] \in \mathbb{Q}(q, t)\{s_\lambda(z) : \lambda \geq \mu^t\}$
- (Miii) $\tilde{H}_\mu[1; q, t] = 1$.

Here $s_\lambda(z)$ is the Schur function attached to the partition λ , \geq is the dominance ordering on partitions, and the $[\cdot]$ denotes plethystic substitution, see [8, §3.3].

The following theorem gives another proof of Macdonald positivity. This was proved first by Haiman in [7], and subsequently in [1] and [5]. We do not assert here that \mathcal{P} is the Procesi bundle, although that does follow from the work of Haiman and Ginzburg, see [4, Corollary 8.2.5]. Recall the Frobenius characteristic is the unique linear map from the representation ring of \mathfrak{S}_n to symmetric functions, sending the irreducible representation λ to the Schur function $s_\lambda(z)$, see [8, §3.2].

Theorem 2. *Let $\mathcal{P}(I_\mu)$ be the fibre of \mathcal{P} above $I_\mu \in \text{Hilb}^n \mathbb{C}^2$, which by the above carries a $W \times \mathbb{C}^* \times \mathbb{C}^*$ -action. The Frobenius characteristic $F_{\mathcal{P}(I_\mu)}(z; q, t)$ equals $\tilde{H}_\mu(z; q, t)$.*

The proof of this will occupy the rest of this note. It proceeds in a similar way to the tactic of Haiman's own proof, using however basic facts about D -modules.

Any function in $\mathcal{O}(\mathfrak{X})$ pulls back to a regular function on $\mathfrak{X}_{\text{norm}}$, and by construction these functions are invariant under the action of G . Thus the functions in $\mathcal{O}(\mathfrak{X})$ give rise to functions on \mathfrak{W} and hence an action on \mathcal{P} . Let y_1, \dots, y_n be a basis of linear functionals on $\mathfrak{t} \times \{0\} \subset \mathfrak{X}$.

Claim 1. *The elements y_1, \dots, y_n are a regular sequence at any point in \mathfrak{W} at which they vanish.*

Proof. Let $I = (y_1, \dots, y_n)$ be the ideal of $\mathcal{O}_{\mathfrak{W}}$ generated by the y_i 's. Thanks to [4, Proposition 3.2.4] \mathfrak{W} is Cohen-Macaulay. Hence it is enough to show that $\text{codim } I = n$. This follows just as in [7, Proposition 3.3.3], for instance. \square

In [4, Proposition 3.2.4] it is shown that $\mathfrak{W} \cong [\text{Hilb}^n \mathbb{C}^2 \times_{\mathfrak{T}/W} \mathfrak{T}]_{\text{red, norm}}$. Since the support of $I_\mu \in \text{Hilb}^n \mathbb{C}^2$ is concentrated at the origin of \mathfrak{T}/W , there is a unique point $(I_\mu, 0) \in [\text{Hilb}^n \mathbb{C}^2 \times_{\mathfrak{T}/W} \mathfrak{T}]_{\text{red}}$ lying above $I_\mu \in \text{Hilb}^n \mathbb{C}^2$ and we let \mathcal{J}_μ be the corresponding maximal ideal sheaf. Let $A = \mathcal{O}_{[\text{Hilb}^n \mathbb{C}^2 \times_{\mathfrak{T}/W} \mathfrak{T}]_{\text{red}}}$ and $B = \mathcal{O}_{\mathfrak{W}}$. We now know that (y_1, \dots, y_n) is a regular sequence in $(AB)_{\mathcal{J}_\mu}$. It follows that $(AB)_{\mathcal{J}_\mu}/(y_1, \dots, y_n)_{(AB)_{\mathcal{J}_\mu}}$ admits a Koszul resolution, and hence by [8, Proposition 3.3.1] that we have an equality of Frobenius characteristics

$$F_{(AB)_{\mathcal{J}_\mu}}([1-q]Z; q, t) = F_{(AB)_{\mathcal{J}_\mu}/(y_1, \dots, y_n)_{(AB)_{\mathcal{J}_\mu}}}(z; q, t).$$

Since $\eta : \mathfrak{W} \rightarrow \text{Hilb}^n \mathbb{C}^2$ factors through $[\text{Hilb}^n \mathbb{C}^2 \times_{\mathfrak{T}/W} \mathfrak{T}]_{\text{red}}$, the stalk \mathcal{P}_μ of \mathcal{P} at I_μ equals $(AB)_{\mathcal{J}_\mu}$. By freeness $F_{\mathcal{P}_\mu}(z; q, t) = F_{\mathcal{P}(I_\mu)}(z; q, t)p_\mu(q, t)$ where $p_\mu(q, t) \in \mathbb{Q}(q, t)$ is the bigraded Poincaré series for the local ring of $\text{Hilb}^n \mathbb{C}^2$ at the point I_μ . It follows that

$$F_{\mathcal{P}(I_\mu)}([1-q]Z; q, t) = F_{\mathcal{P}_\mu}([1-q]Z; q, t)p_\mu(q, t) = F_{(AB)_{\mathcal{J}_\mu}/(y_1, \dots, y_n)_{(AB)_{\mathcal{J}_\mu}}}(z; q, t)p_\mu(q, t).$$

Therefore to check (Mi), we need only show that

$$F_{(AB)\mathcal{J}_\mu/(y_1, \dots, y_n)(AB)\mathcal{J}_\mu}(z; q, t) \in \mathbb{Q}(q, t)\{s_\lambda(z) : \lambda \geq \mu\}.$$

By [6, Proposition 5.3] this is implied by the following.

Claim 2. *The λ isotypic component of $(AB)\mathcal{J}_\mu/(y_1, \dots, y_n)(AB)\mathcal{J}_\mu$ is zero unless $\lambda \geq \mu$.*

Proof. Since \mathfrak{C}° belongs to smooth locus of \mathfrak{C} , the restriction of $p : \mathfrak{X}_{\text{norm}} \longrightarrow \mathfrak{C}_{\text{norm}}$ to \mathfrak{X}° factors through \mathfrak{X} , that is $p|_{\mathfrak{X}^\circ} = (p_{\mathfrak{C}} \circ \psi)|_{\mathfrak{X}^\circ}$. It follows that

$$\mathcal{R}|_{\mathfrak{C}^\circ} = p_*(\mathcal{O}_{\mathfrak{X}_{\text{norm}}}|_{\mathfrak{X}^\circ}) = (p_{\mathfrak{C}})_*\left((\text{gr } \mathcal{M})|_{p_{\mathfrak{C}}^{-1}(\mathfrak{C}^\circ)}\right).$$

Now let (X_μ, Y_μ) be an element in the principal nilpotent pair orbit corresponding to μ , see [3, (0.1)]. We deduce that the stalk of \mathcal{R} above (X_μ, Y_μ) equals $(\text{gr } \mathcal{M})_{K_\mu}$ where K_μ is the maximal ideal of $(X_\mu, Y_\mu, 0, 0)$, the unique point in \mathfrak{X} lying over (X_μ, Y_μ) .

Let $\pi : \mathfrak{g} \longrightarrow \mathfrak{g} \times \mathfrak{t}$ be the inclusion that sends X to $(X, 0)$. Define

$$T^*(\mathfrak{g}) = \mathfrak{g} \times \mathfrak{g}^* \xleftarrow{\rho_\pi} \mathfrak{g} \times_{\mathfrak{g} \times \mathfrak{t}} T^*(\mathfrak{g} \times \mathfrak{t}) = \mathfrak{g} \times \mathfrak{g}^* \times \mathfrak{t}^* \xrightarrow{\varpi_\pi} T^*(\mathfrak{g} \times \mathfrak{t}) = \mathfrak{g} \times \mathfrak{g}^* \times \mathfrak{t} \times \mathfrak{t}^*$$

by $\rho_\pi(X, Y, w) = (X, Y)$ and $\varpi_\pi(X, Y, w) = (X, Y, 0, w)$. We set $T_{\mathfrak{g}}^*(\mathfrak{g} \times \mathfrak{t}) = \rho_\pi^{-1}(T_{\mathfrak{g}}^*(\mathfrak{g})) = \mathfrak{g} \times \{0\} \times \mathfrak{t}^*$. The characteristic variety of \mathcal{M} is $\text{Ch}(\mathcal{M}) = [\mathfrak{X}]$, [4, Corollary 2.4.1]. Now

$$\begin{aligned} \varpi_\pi^{-1}(\mathfrak{X}) \cap T_{\mathfrak{g}}^*(\mathfrak{g} \times \mathfrak{t}) &= \{(X, Y, w) : [X, Y] = 0, X \text{ nilpotent, e-vals}(Y) = w\} \cap \{(X, 0, w)\} \\ &= \{(X, 0, 0) : X \text{ nilpotent}\} \subset \mathfrak{g} \times \{0\} \times \{0\} = \mathfrak{g} \times_{\mathfrak{g} \times \mathfrak{t}} T_{\mathfrak{g} \times \mathfrak{t}}^*(\mathfrak{g} \times \mathfrak{t}). \end{aligned}$$

Thus π is non-characteristic with respect to \mathcal{M} . In particular we deduce from [10, Theorem 4.7] that $\text{Ch}(\pi^*\mathcal{M}) = \rho_\pi \varpi_\pi^{-1}(\text{Ch}(\mathcal{M})) = \{(X, Y) : [X, Y] = 0, X \text{ nilpotent}\} \subset \mathfrak{C}$. In fact, the y_1, \dots, y_n form a regular sequence for $\text{gr } \mathcal{M}$ by [4, Proposition 9.1.3], so multiplication by each y_i on $\text{gr } \mathcal{M}/(y_1, \dots, y_{i-1}) \text{gr } \mathcal{M}$ is injective, and iterating the proof of Step 1 of [10, Theorem 4.7] shows that $(\rho_\pi)_* \varpi_\pi^*(\text{gr } \mathcal{M})$ is isomorphic to $\text{gr } \pi^*\mathcal{M}$.

The support of $(\rho_\pi)_* \varpi_\pi^*(\text{gr } \mathcal{M})$ is $\{(X, Y) : [X, Y] = 0, X \text{ nilpotent}\}$. Since \mathcal{M} is holonomic this space is lagrangian in $T^*(\mathfrak{g})$, a union of conormal bundles $\bigcup_\lambda \overline{T_{\mathcal{O}_\lambda}^*(\mathfrak{g})}$, where \mathcal{O}_λ denotes the nilpotent orbit in \mathfrak{g} of type λ . The D -module \mathcal{M} carries a W -action, [9, §5] and this induces the W -action that is inherited by \mathcal{R} in the statement of Theorem 1(3). The λ -isotypic component of the stalk of $\mathcal{R}|_{\mathfrak{C}^\circ}/(y_1, \dots, y_n)\mathcal{R}|_{\mathfrak{C}^\circ}$ at (X_μ, Y_μ) is non-zero if and only if (X_μ, Y_μ) is in the support of the λ -isotypic component of $(\rho_\pi)_* \varpi_\pi^*(\text{gr } \mathcal{M})$.

We have a decomposition $\pi^*\mathcal{M} = \bigoplus_\lambda (\pi^*\mathcal{M})_\lambda$. We've seen above that the support of $\text{gr}(\pi^*\mathcal{M})_\lambda$ equals the support of the λ -isotypic component of $(\rho_\pi)_* \varpi_\pi^*(\text{gr } \mathcal{M})$. By [9, Proposition 4.8.1 and Theorem 5.3(3)], $(\pi^*\mathcal{M})_\lambda$ is supported on the closure of the nilpotent orbit \mathcal{O}_λ , and so $\text{Ch}((\pi^*\mathcal{M})_\lambda) \subseteq \bigcup_{\nu \leq \lambda} \overline{T_{\mathcal{O}_\nu}^*(\mathfrak{g})}$. Thus the λ -isotypic component of the stalk of $\mathcal{R}|_{\mathfrak{C}^\circ}/(y_1, \dots, y_n)\mathcal{R}|_{\mathfrak{C}^\circ}$ at (X_μ, Y_μ) is non-zero only if $(X_\mu, Y_\mu) \in \overline{T_{\mathcal{O}_\nu}^*(\mathfrak{g})}$ for some $\nu \leq \lambda$. But since $X_\mu \in \mathcal{O}_\mu$, this in turn requires that $\mu \leq \nu$. So we deduce that the stalk at (X_μ, Y_μ) of the λ -isotypic component of $\mathcal{R}|_{\mathfrak{C}^\circ}/(y_1, \dots, y_n)\mathcal{R}|_{\mathfrak{C}^\circ}$ is non-zero only if $\mu \leq \lambda$.

Given any $s \in \mathcal{S}$ we have by definition

$$(\mathcal{P}/(y_1, \dots, y_n)\mathcal{P})_{\rho(s)} \otimes_{\text{Hilb}^n \mathbb{C}^2, \rho(s)} \mathcal{O}_{\mathcal{S}, s} \cong (\mathcal{R}|_{\mathfrak{e}^\circ}/(y_1, \dots, y_n)\mathcal{R}|_{\mathfrak{e}^\circ})_{\delta(s)} \otimes_{\mathcal{O}_{\mathfrak{e}^\circ, \delta(s)}} \mathcal{O}_{\mathcal{S}, s}.$$

If $s \in \delta^{-1}(X_\mu, Y_\mu)$ then $\rho(s) = I_\mu$ and it follows that the λ -isotypic component of $\mathcal{P}_\mu/(y_1, \dots, y_n)\mathcal{P}_\mu$ is non-zero only if $\mu \leq \lambda$. Since $(AB)_{\mathcal{J}_\mu}/(y_1, \dots, y_n)(AB)_{\mathcal{J}_\mu} = \mathcal{P}_\mu/(y_1, \dots, y_n)\mathcal{P}_\mu$, this proves our claim. \square

To deal with (Mii) we argue similarly, reducing the calculations about \mathcal{P} to ones on \mathfrak{X} . We need to factor out a basis z_1, \dots, z_n of \mathfrak{t}^* . To see this is a regular sequence observe first that there is an automorphism of \mathfrak{X} induced by interchanging $\mathfrak{g} \times \mathfrak{t}$ with $\mathfrak{g}^* \times \mathfrak{t}^*$. This induces an automorphism of the normalisation $\mathfrak{X}_{\text{norm}}$ and we see that z_1, \dots, z_n is a regular sequence since y_1, \dots, y_n is. Now recall that $(Y_\mu, X_\mu) = (X_{\mu^t}, Y_{\mu^t})$. Thus we deduce that the λ -isotypic component of $\mathcal{P}_\mu/(z_1, \dots, z_n)\mathcal{P}_\mu$ is non-zero only if $\mu^t \leq \lambda$. This implies (Mii).

Condition (Miii) states that the trivial representation appears in $\mathbb{C}^* \times \mathbb{C}^*$ -bidegree $(0, 0)$ and nowhere else. But since \mathcal{P}_μ carries the regular representation of W and the trivial isotypic component is spanned by the constant functions, this is immediate.

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