

EXERCISES FOR LECTURE 16

- (1) Let G be a nilpotent group.
 - (a) Show that if H is a subgroup of a nilpotent group G then H is nilpotent.
 - (b) Show that if K is a normal subgroup of a nilpotent group G then G/K is nilpotent.
 - (c) Is the converse true? i.e. Suppose that K is a normal subgroup of G and that both G/K and K are nilpotent. Is G necessarily nilpotent?

- (2) Let G be the group of matrices

$$G = \left\{ \left(\begin{array}{ccc} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 0 \end{array} \right) : a, b, c \in \mathbf{F}_3 \right\}.$$

Using the fact that G is nilpotent, or otherwise, calculate the character table of G .

- (3) Prove that the subgroup constructed in the proof of Lemma B is normal.
- (4) $|D_4| = 8$ so D_4 is nilpotent. Describe all of the representations of D_4 as representations induced from one-dimensional representations of appropriate subgroups.
- (5) $|Q| = 8$ so Q is nilpotent. Describe all of the representations of Q as representations induced from one-dimensional representations of appropriate subgroups. (Q was first defined in Question 2 of the exercises for lectures 5 and 6.)