

EXERCISES FOR LECTURES 14 AND 15

1. Let W be a representation of H . Prove that $\text{Ind}_H^G W$ is a subspace of the vector space of mappings from G to W .
2. Let $H = G$. Prove that $\text{Ind}_H^G W \cong W$.
3. Prove that $\text{Ind}_{A_4}^{A_5} \mathbb{C} = I_1 \oplus I_2$ (in the notation of Section 10 - the character table of A_5). (Here \mathbb{C} is the trivial representation of A_4 .)
4. Let W_1, W_2 be representations of H . Prove that $\text{Ind}_H^G(W_1 \oplus W_2) \cong \text{Ind}_H^G W_1 \oplus \text{Ind}_H^G W_2$.
5. Prove Theorem 11.6(c) (which states that if $H \leq K \leq G$ then $\text{Ind}_H^G W = \text{Ind}_K^G(\text{Ind}_H^K W)$) as follows:
 - (a) Note that an element of $\text{Ind}_K^G(\text{Ind}_H^K W)$ is a map from G to a map from K to W . Define a mapping $\psi : \text{Ind}_K^G(\text{Ind}_H^K W) \rightarrow \text{Ind}_H^G W$ by $(\psi(F))(g) = F(g)(e_K)$ for all $g \in G$. Check that this does indeed have $\psi(F) \in \text{Ind}_H^G W$.
 - (b) Check that ψ is G -intertwining linear mapping.
 - (c) By consider F such that $\psi(F) = 0$ show that ψ is injective.
 - (d) By considering the dimensions of the domain and codomain for ψ deduce that ψ is an isomorphism.
6. You have the character table of A_5 . Work out how to decompose $\text{Ind}_{A_3}^{A_5} \mathbb{C}$ into irreducible representations. (Here \mathbb{C} is the trivial representation of A_3 .)