

EXERCISES FOR LECTURE 1

- (1) Show that $C_n \cong \mu_n$. Why is this not a *natural* isomorphism?
- (2) Verify the order of $GL(2, \mathbb{F}_q)$ and $SL(2, \mathbb{F}_q)$. What is the order of $GL(n, \mathbb{F}_q)$?
- (3) Determine the conjugacy classes in C_n . (!!Yes, really!!)
- (4) Determine the conjugacy classes in D_4 and D_5 . What about D_n ?
- (5) Determine the conjugacy classes in $GL(2, \mathbb{F}_2)$.
- (6) Determine the conjugacy classes in S_4 and S_5 . What about S_n ?
- (7) Let N be the subgroup of S_4 generated by the even permutations of order 2. What group is S_4/N isomorphic to?
- (8) Show that if $H \leq G$ has index 2 then $H \trianglelefteq G$.
- (9) Prove that if $A, B \in GL(n, F)$ then $\text{Tr}(AB) = \text{Tr}(BA)$. Deduce that if A and B are conjugate then $\text{Tr}(A) = \text{Tr}(B)$.
- (10) Take π_U from Section 10 on Projections. Show that $\pi_U \in \text{End}(V)$, that $\text{im } \pi_U = U$, that $\ker \pi_U = W$ and that $\pi_U^2 = \pi_U$.