Phase dependence of escape from star clusters on elliptic orbits

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Outline

In this short study the rate of escape from an idealised star cluster is investigated, by means of small N-body simulations. The cluster is subject to the tidal field of a galaxy modelled as a point mass. It is shown that the dependence of the lifetime on the initial orbital phase is negligible, even for an eccentricity $e = 0.5$. It is deduced that, for galactic orbits with a given semi-major axis, the lifetime is independent of e to first order in e. The results on the lifetime of a star cluster on an elliptical orbit agree with the formula given recently by Cai et al. (2015).

Method and initial conditions

Simulations are run with 8192 stars of mass $1M_\odot$, no primordial binaries, and no stellar evolution. The initial model is Plummer's model with a virial radius of 1pc, but the model is truncated at ten scale radii. It is in orbit about a galaxy represented by a point mass of mass $M_g = 10^{10}M_\odot$, and the semi-major axis is 1211pc. For a cluster on a circular orbit, it follows that the initial model has $r_h/r_J = 0.1$, where r_h, r_J are the half-mass and Jacobi radii, respectively. For models on an elliptic galactic orbit the eccentricity is $e = 0.5$, and the initial phase (strictly, the initial value of the eccentric anomaly) is $E_0 = 0, 90, 180$ or $270°$.

Simulations were run with NBODY6 Nitadori & Aarseth (2012), in a non-rotating frame initially centred at the barycentre of the cluster. All stars were retained in the simulation. The bound mass M was defined to be the mass of stars within a sphere, centred at the density centre, with radius r such that

$$r = R_g\left(\frac{M}{3M_g}\right)^{1/3},$$

where R_G is the instantaneous distance from the galactic centre. The lifetime $T_{0.1}$ is defined to be the time at which M is 0.1 times its initial value. In this poster quantities such as M and $T_{0.1}$ are expressed in Hénon units (Hénon, 1971).

Results of the N-body simulations – I

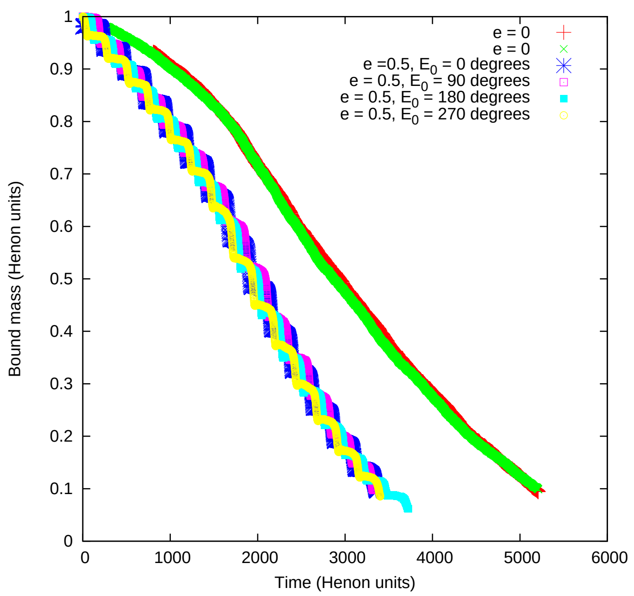

Fig.1 Bound mass against time for two models on circular orbits, and four on elliptic orbits in which the initial "phase" is $0°$ (perigalacticon), $90, 180$ and $270°$. Early data for one of the circular models is missing.

Immediate Conclusions

1. The mass-loss rate is greater on an elliptic orbit than on a circular orbit with the same semi-major axis
2. For a given eccentricity and semi-major axis, the mass-loss rate is almost independent of the starting phase (i.e. starting at apogalacticon, perigalacticon, or points in between).

These conclusions are quantified in the next panel.

Results of the N-body simulations – II

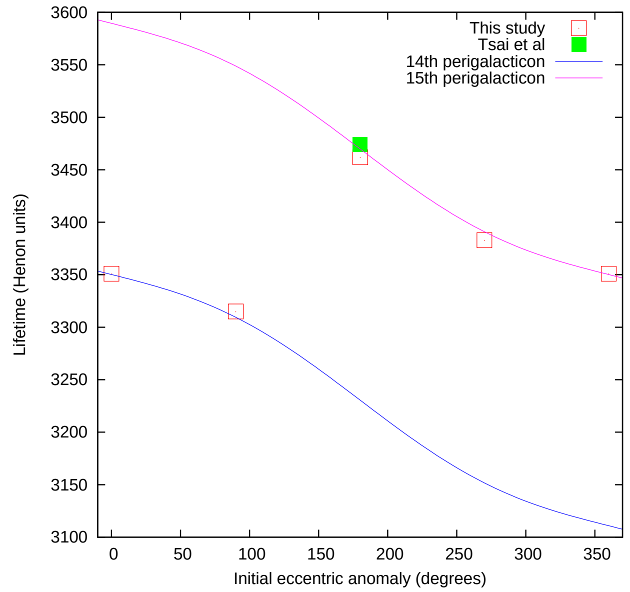

Fig.2 Lifetime $T_{0.1}$ as a function of the initial "phase" (eccentric anomaly, E_0.) The curves give the time of the 14th and 15th perigalactic passage; note that the 15th becomes the 14th as E_0 increases through 0 or $360°$. Also shown is the result obtained from the formula for $T_{0.1}(e)/T_{0.1}(0)$ given by Cai et al. (2015), where $T_{0.1}(e)$ is the lifetime on an orbit of eccentricity e and given semi-major axis. It is plotted at $E_0 = 180°$, which was their initial setup, and the value of $T_{0.1}(0)$ was obtained from the average of the two models in Fig.1 of this poster.

Immediate Conclusions

1. For the setup in this study the lifetime on an elliptical orbit of eccentricity 0.5 is 0.670 times the lifetime on a circular orbit with the same semi-major axis.
2. The lifetime is consistent with an escape rate which is independent of the initial "phase" E_0. The lifetime is reached close to the first perigalacticon where the remaining mass of the cluster falls below 0.1 times its initial value.

Implications

1. **Dependence on eccentricity**
   Let us suppose that the lifetime is independent of E_0, i.e. $T_{0.1}(e, E_0) = T_{0.1}(e)$. The equations of Kepler motion
   $$x = a(1 - e\cos E), y = b\sin E$$
   are invariant under the transformation
   $$e \to -e, E \to \pi - E.$$
   Thus motion on a galactic orbit of eccentricity $-e$ is the same as motion on an orbit of eccentricity e, but with a different value of E_0. From the above assumption it follows that $T_{0.1}(-e) = T_{0.1}(e)$, i.e. $T_{0.1}$ is an even function of e. This simplification was adopted in the fitting formula (for the dependence of $T_{0.1}$ on e) presented by Cai et al. (2015).
2. **The role of perigalacticon**
   For small eccentricity e, it follows that $T_{0.1}(e) = T_{0.1}(0) + O(e^2)$. Thus the lifetime on a slightly elliptical orbit is almost the same as the lifetime on a circular orbit with the same semi-major axis. This is an orbit with radius almost equal to the average of the peri- and apo-galactic radii of the elliptical orbit. Thus mass loss from a cluster on a mildly elliptical orbit is not determined by conditions at perigalacticon. It is already known that the half-mass radius is not set at perigalacticon either (Küpper et al., 2010).

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References

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