6. Why I love the 3-body problem

1. "Star-studded" history

   ... the great names of dynamics

2. A clean problem

   ... the only mess is in one's head

3. A cross-roads in science

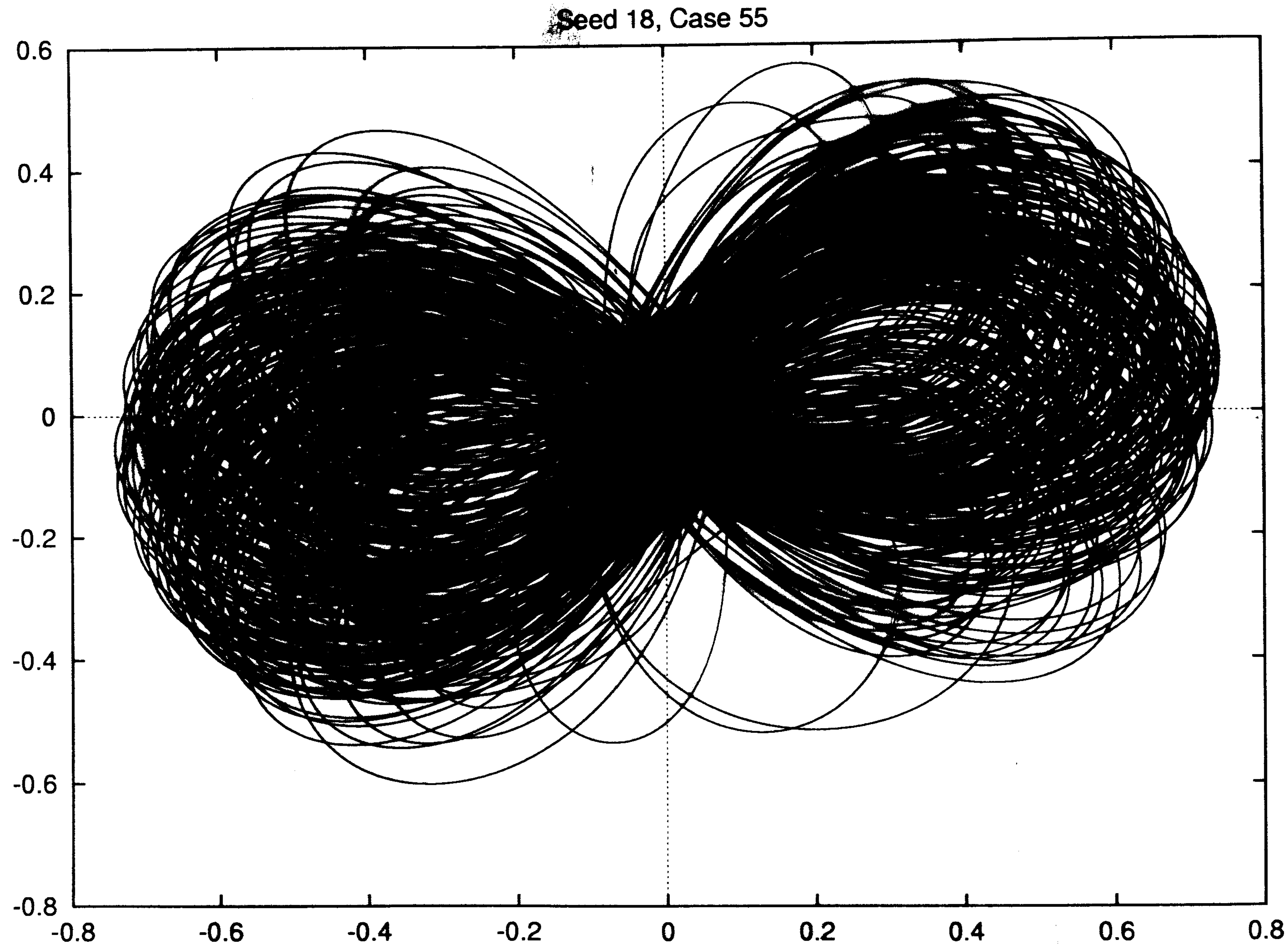
   ... where mathematics, physics and astronomy all meet.

4. Self-rejuvinating

   ... new problems always emerging from new applications and new insights.

'... every generation formulates and solves its own "fundamental questions in the three-body problem".'

[Wintner]

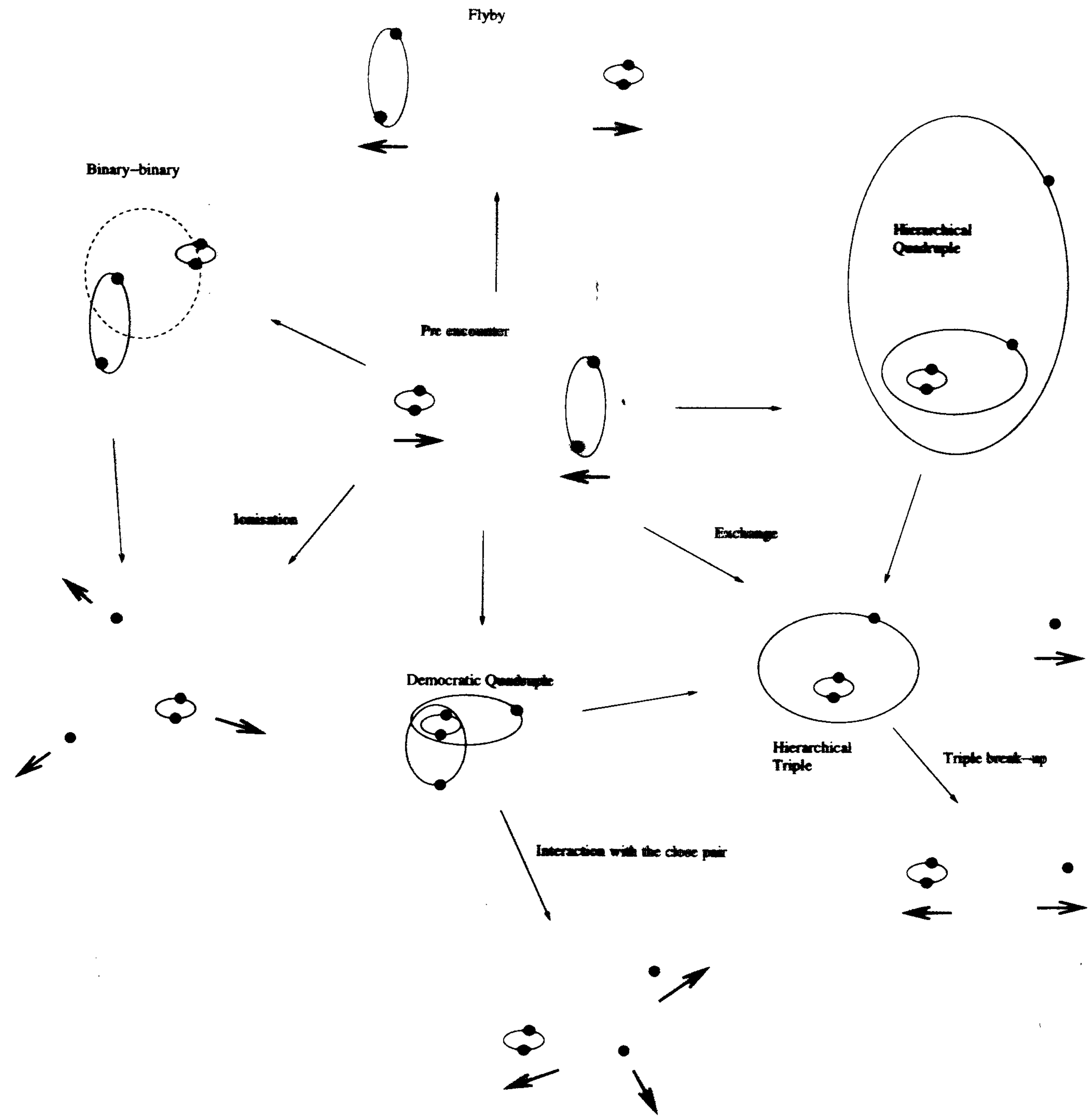

Seed 18, Case 55
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8

4. Proportion of 8-like systems

(a) select x_i, y_i, u_i, v_i uniformly on $(0,1)$
(b) shift to barycentre
(c) rotate to make $\sum m \underline{r}_i \times \underline{v}_i = \underline{0}$
(d) scale to virial equilibrium $2T - U = 0$.
(e) scale to standard units: $M = G = 4T = 2U = 1$.

Result
(i) $\simeq 5\%$ 8-like to $t = 10$
(ii) $\lesssim 1\%$ 8-like to $t = 100$
(iii) $\simeq 0.1\%$ 8-like to $t = 1000$

Conclusion: quite an improbable outcome of binary-binary scattering



Flyby
Binary–binary
Hierarchical
Quadruple
Pre encounter
Ionisation
Exchange
Democratic Quadruple
Hierarchical
Triple
Triple break–up
Interaction with the close pair

Binary-Binary Scattering

1. Outcome analysed in terms of asymptotic solutions

2. New possible outcome

3. Recognising an 8-like orbit automatically

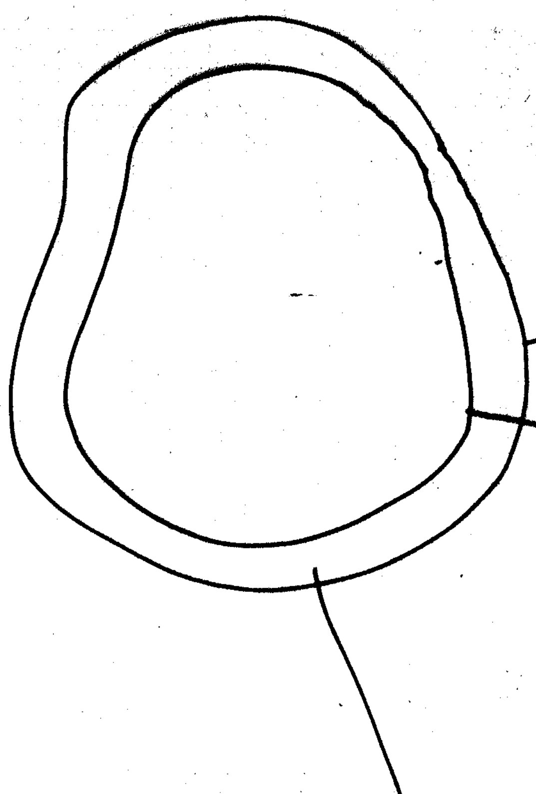
   (i) find change in orientation, i.e. sign change in $(\underline{r}_2-\underline{r}_1)\times(\underline{r}_3-\underline{r}_1)$, to detect collinearity

   (ii) determine central body at each collinear config

   (iii) look for sequence ... 123123123... or reverse (Such a system is described as "8-like")

The impossibility of permanent capture

Let R be some region in phase space with finite volume, e.g. $|\underline{r}_i - \bar{\underline{r}}| < L$, $i = 1, 2, 3$



I: all $x(0)$ s.t. $x(t) \in R \ \forall t \geq 0$

II: all $x(1)$ s.t. $x(t) \in R \ \forall t \geq 0$

$=$ all $x(0)$ s.t. $x(t) \in R \ \forall t \geq -1$

$\subset$ all $x(0)$ s.t. $x(t) \in R \ \forall t \geq 0$

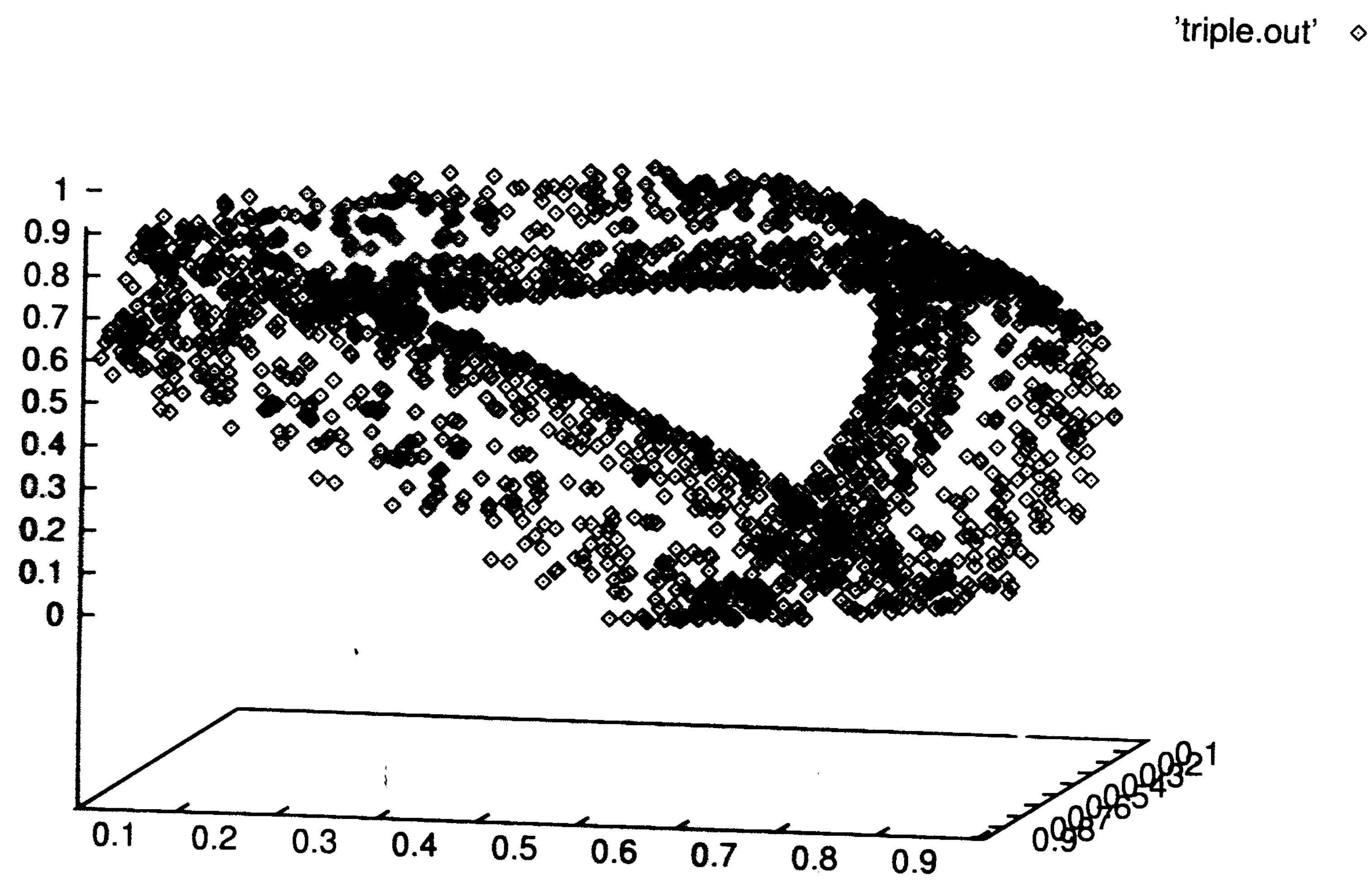
III: 0 volume –

all $x(0)$ s.t. $x(t) \in R \ \forall t \geq 0$

& $x(t) \notin R$ for some $t \in [-1$

$=$ all $x(0)$ s.t. $x(t)$ becomes trapped in $[-1, 0]$

Displaced central particle



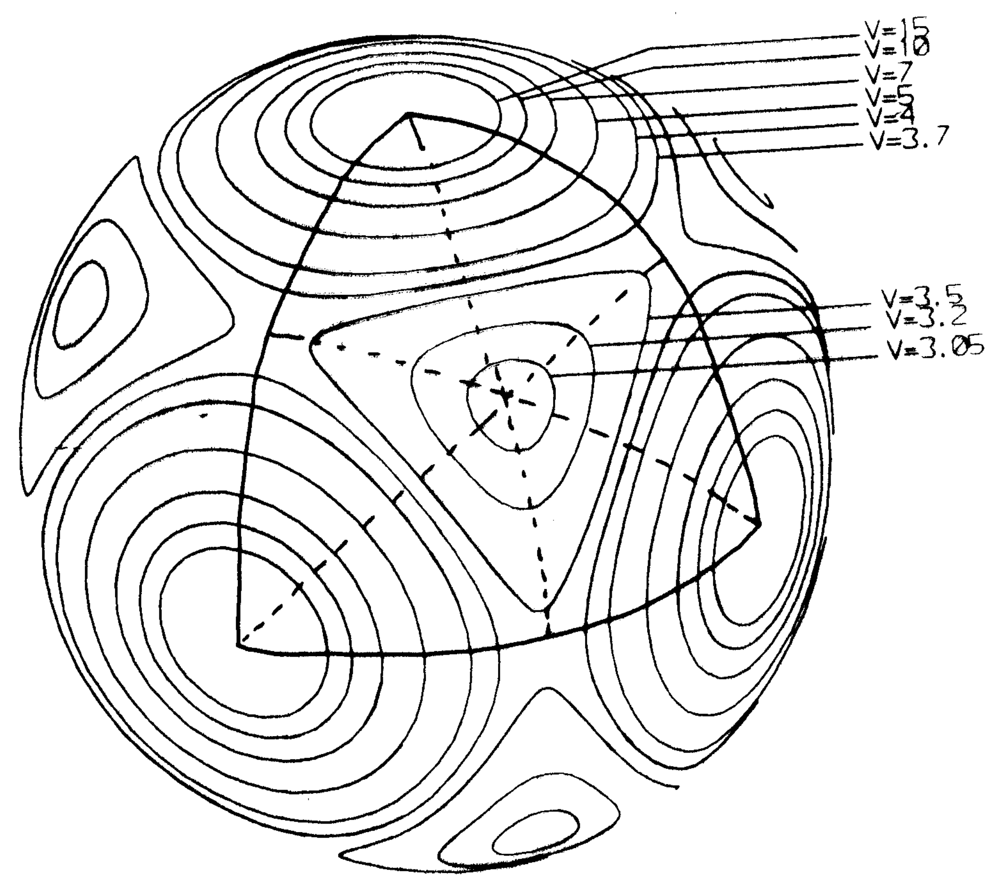


Fig. 4. The surface $\tilde{I}$ = const of Equation (58) in configuration space $(\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3)$. The lines are contours $\tilde{U}$ = const. Equal mass case $m_1 = m_2 = m_3$.

2. Exclude collisions

(a) Suppose collision between m_2 & m_3. Then

$$\int (T_{def} + U)\,dt > \int (\underset{\substack{\uparrow \\ T|_{m_1=0}}}{T^*_{def}} + \underset{\substack{\uparrow \\ U|_{m_1=0}}}{U^*})\,dt$$

$$\geq A_2 \equiv \inf_{\substack{\text{2-body} \\ \text{collision orbits}}} \int (T^*_{def} + U^*)\,dt$$

(b) Construct non-collision path C from $L_3 \cap I_3$ to I_1 along contour of constant U, described at constant speed; length $= \ell_0$.

Then $\inf \int (T_{def} + U)\,dt < \int_C (T_{def} + U)\,dt < A_2$

provided that $\ell_0 < \frac{\pi}{5}$.

Numerical quadrature shows

$$\ell_0 < \frac{\pi}{5.08255\ldots}$$

3. Show that minimiser joins smoothly

(to next segment from I_1 to $L_2 \cap I_2$)

Existence of Segment from $L_3 \cap I_3$ to I_1

1. Extremise action $\int \mathcal{L}\, dt$

where $\mathcal{L} = T + U$ ← $\sum_{i<j} \frac{m}{|r_i - r_j|}$

↑ kinetic energy

(a) $T = T_{rot} + T_{def}$ ← k.e. of deformation

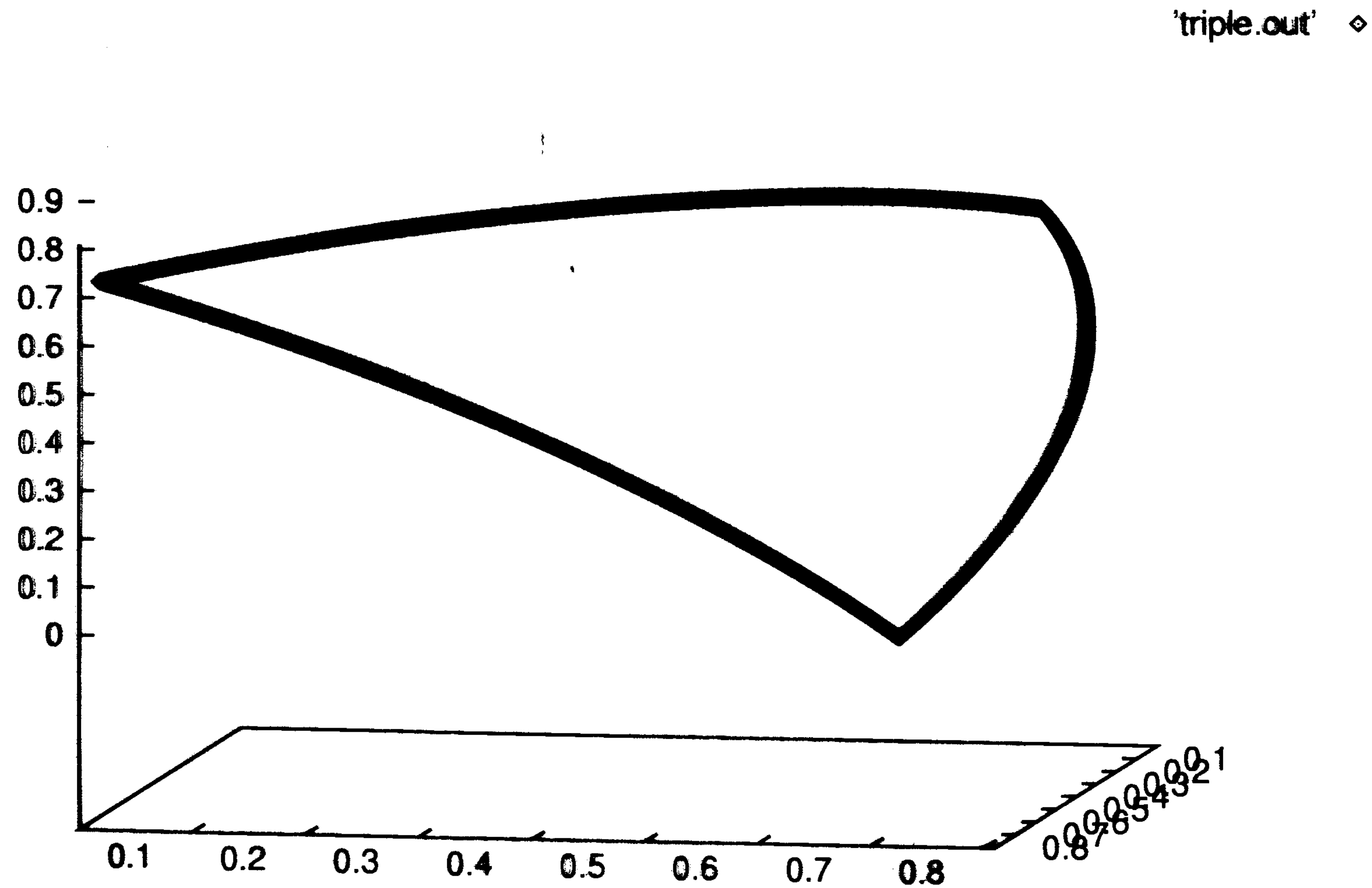
↑ k.e. of rotation $= \frac{c^2}{I}$ ← angular momentum, ← moment of inertia

attempt to minimise $\int \mathcal{L}\, dt$ by assuming $c = 0$,

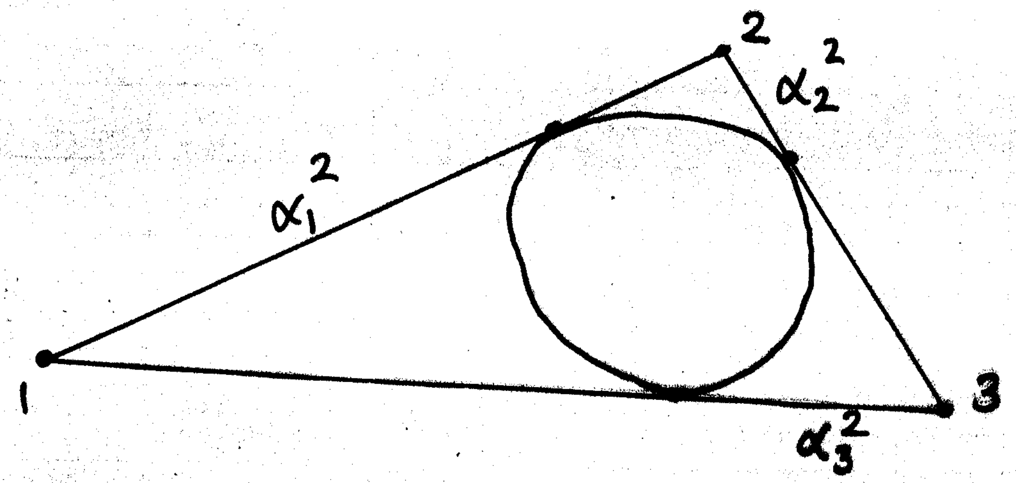
i.e. $\delta \int (T_{def} + U)\, dt = 0$.

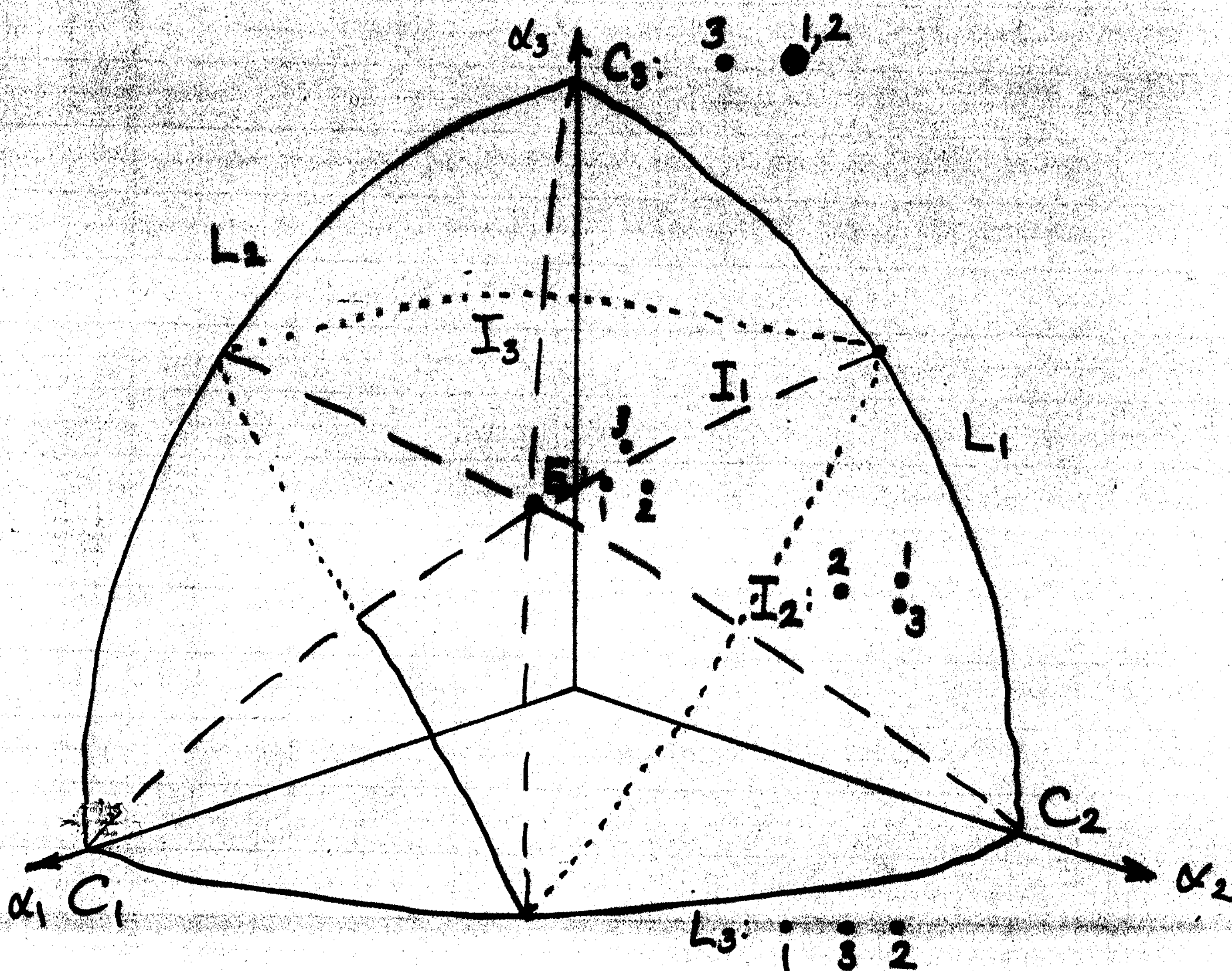
Existence of minimiser "standard".

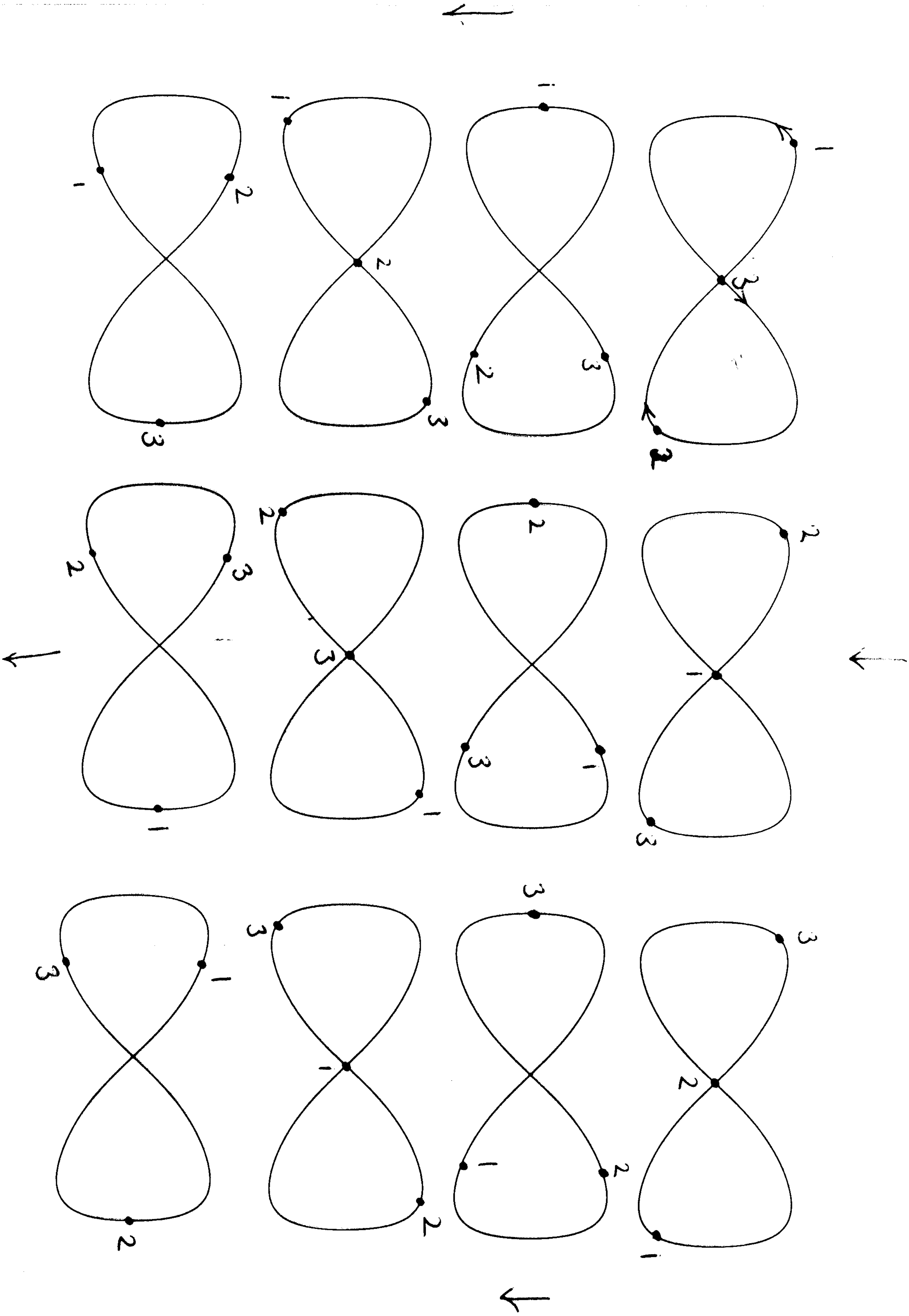
Figure 8



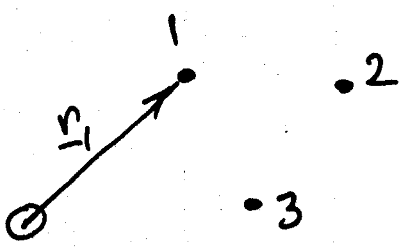
Lemaitre's Parametrisation







Classical Gravitational 3-Body Problem



$$\ddot{\underline{r}}_i = -G \sum_{\substack{j=1 \\ j \neq i}}^{3}{}' m_j \frac{\underline{r}_i - \underline{r}_j}{|\underline{r}_i - \underline{r}_j|^3}, \quad i=1,2,3$$

Take $G m_i = 1$ (scaled, equal masses)

$\underline{r}_i \in \mathbb{R}^2$ (planar)

At $t=0$

$\underline{r}_1 = (a, b)$, $a = -.97000436$
$b = -.24308753$

$\underline{r}_2 = -\underline{r}_1$

$\underline{r}_3 = \underline{0}$

$\dot{r}_3 = (c, d)$, $c = 0.93240737$
$d = 0.86473146$

Source http://orca.ucsc.edu/~rmont/index.html