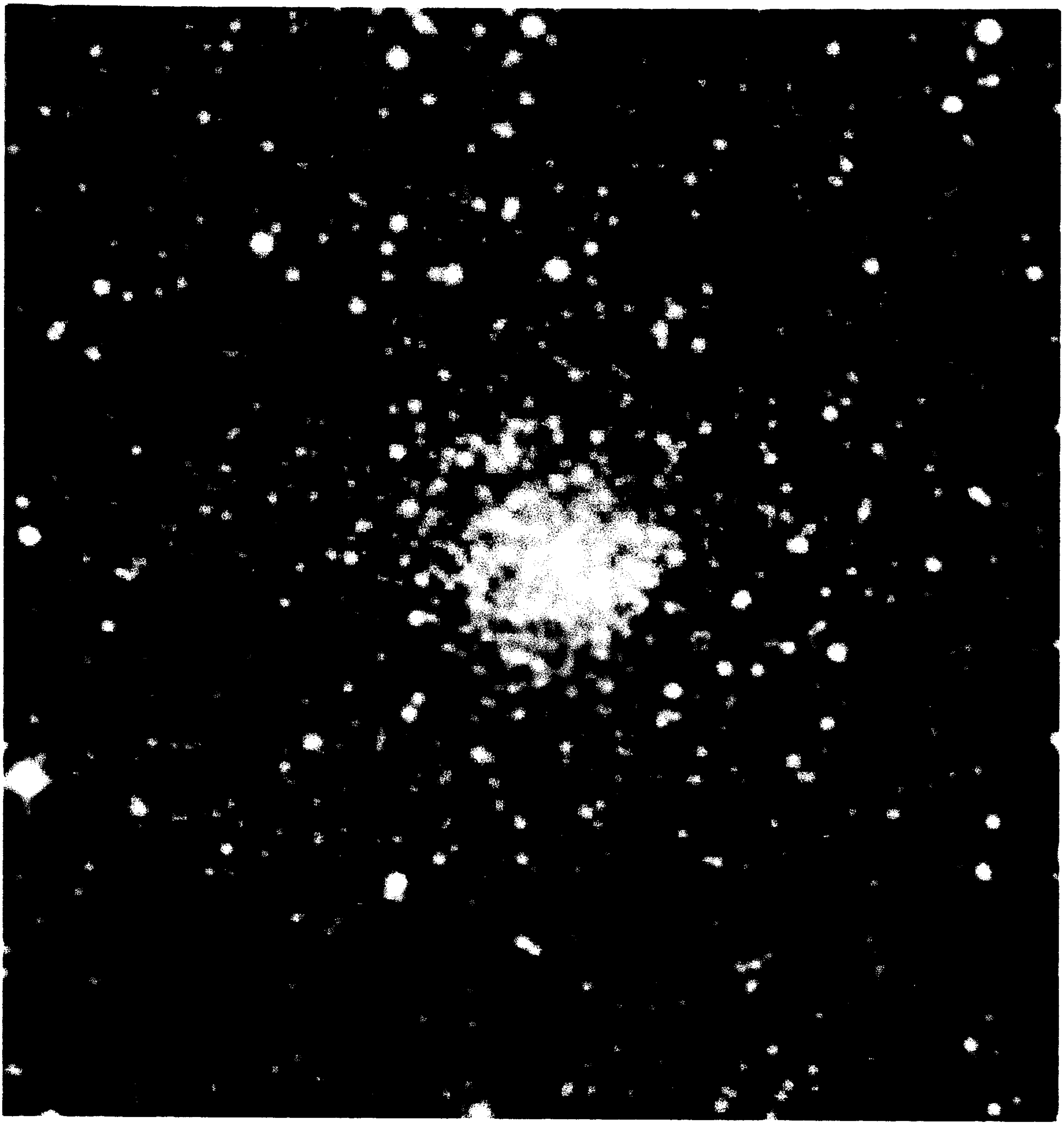
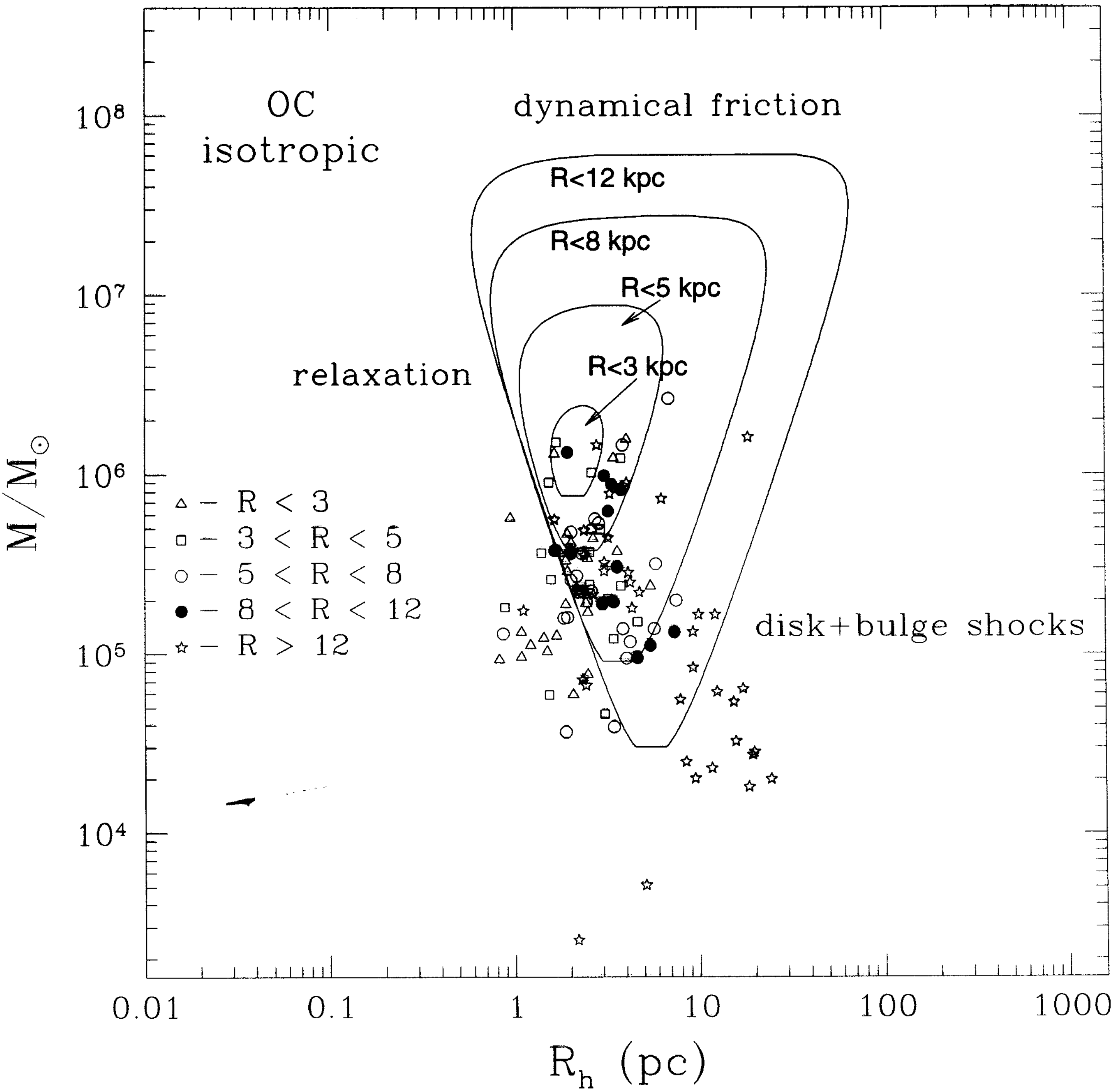


NGC 6712

$M \sim 10^5 M_\odot$

Gnedin & Ostriker 1997
cf. Fall & Rees '77

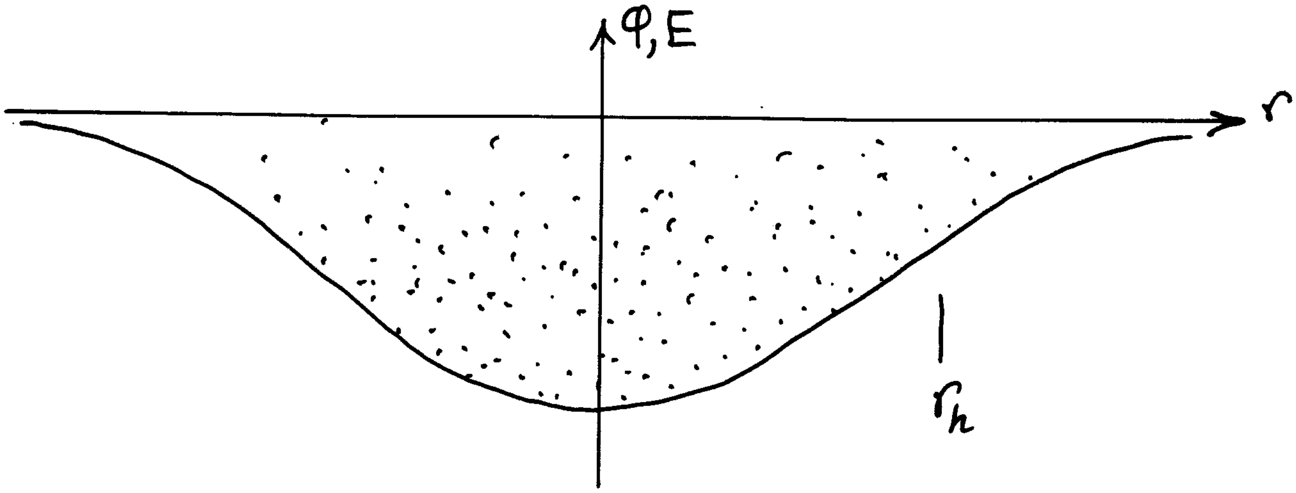


THEORY

- Isolated systems
- Systems on circular galactic orbits
- Systems on oval galactic orbits

Dynamics only

1. ISOLATED SYSTEMS

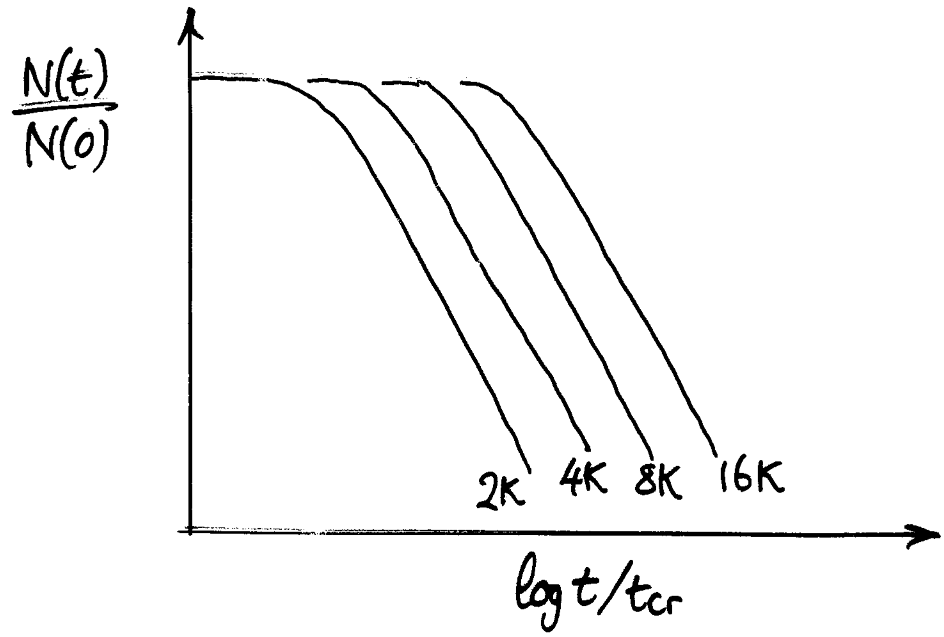


2-body relaxation $\langle(\delta E)^2\rangle \sim \dfrac{t}{t_r}\langle E^2\rangle$

relaxation time $t_r \sim \dfrac{N}{\ln \gamma N}\, t_{cr}$

crossing time $t_{cr} \simeq \dfrac{2r_h}{\langle v^2\rangle^{1/2}}$

orthodox theory $\dfrac{dN}{dt} \simeq -\dfrac{\mu N}{t_r}$



Formulae for the escape rate (Plummer)

Orthodox formula (Spitzer, King, Chandrasekhar, Johnstone, ...)

$$\frac{dN}{dt} \simeq -.0324 \left(\log N - 0.45\right)\sqrt{\frac{Gmn}{r_0^3}}$$

n = number density

m = stellar mass

r_0 = size of cluster

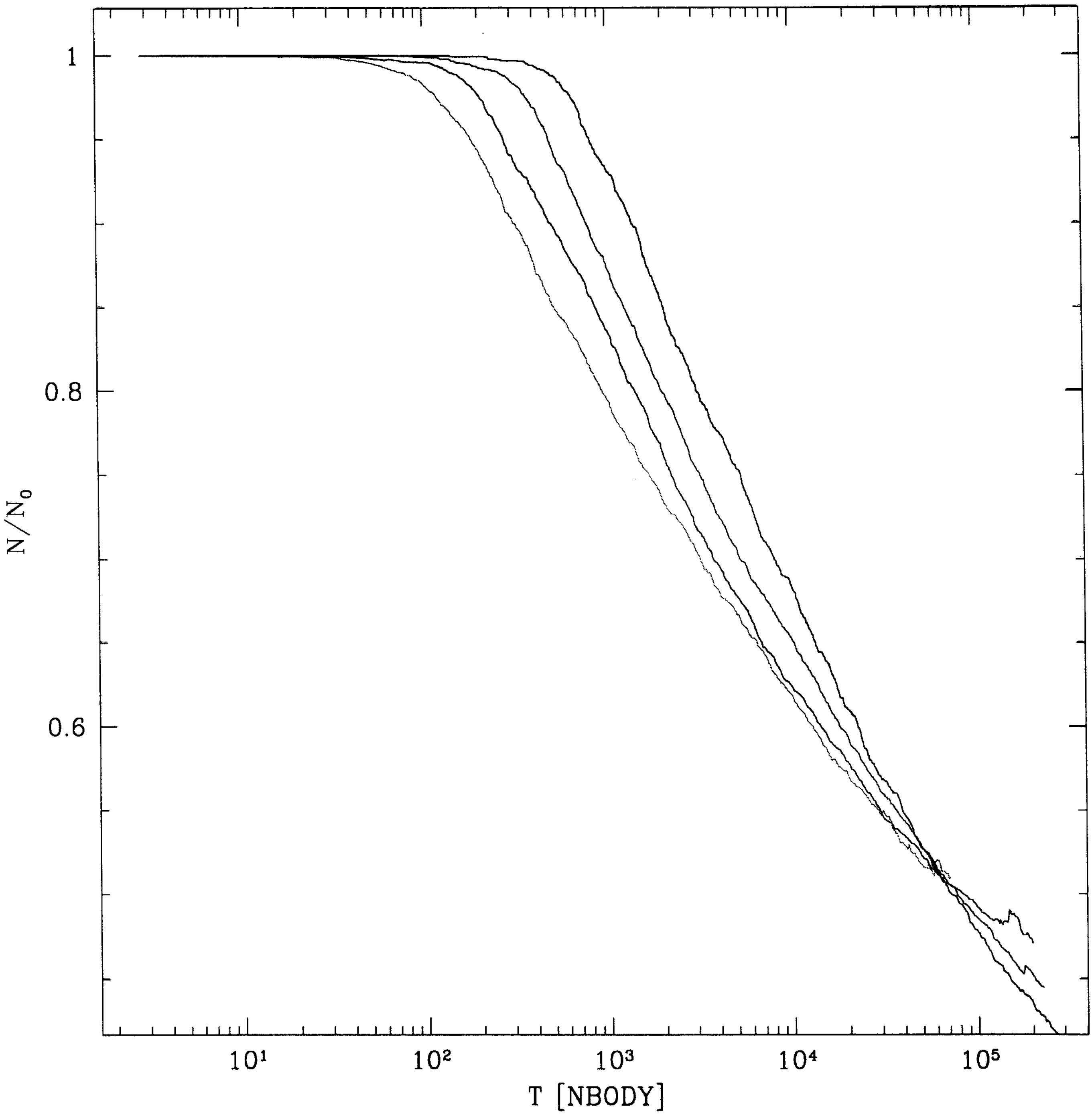
Hénon's formula (1960)

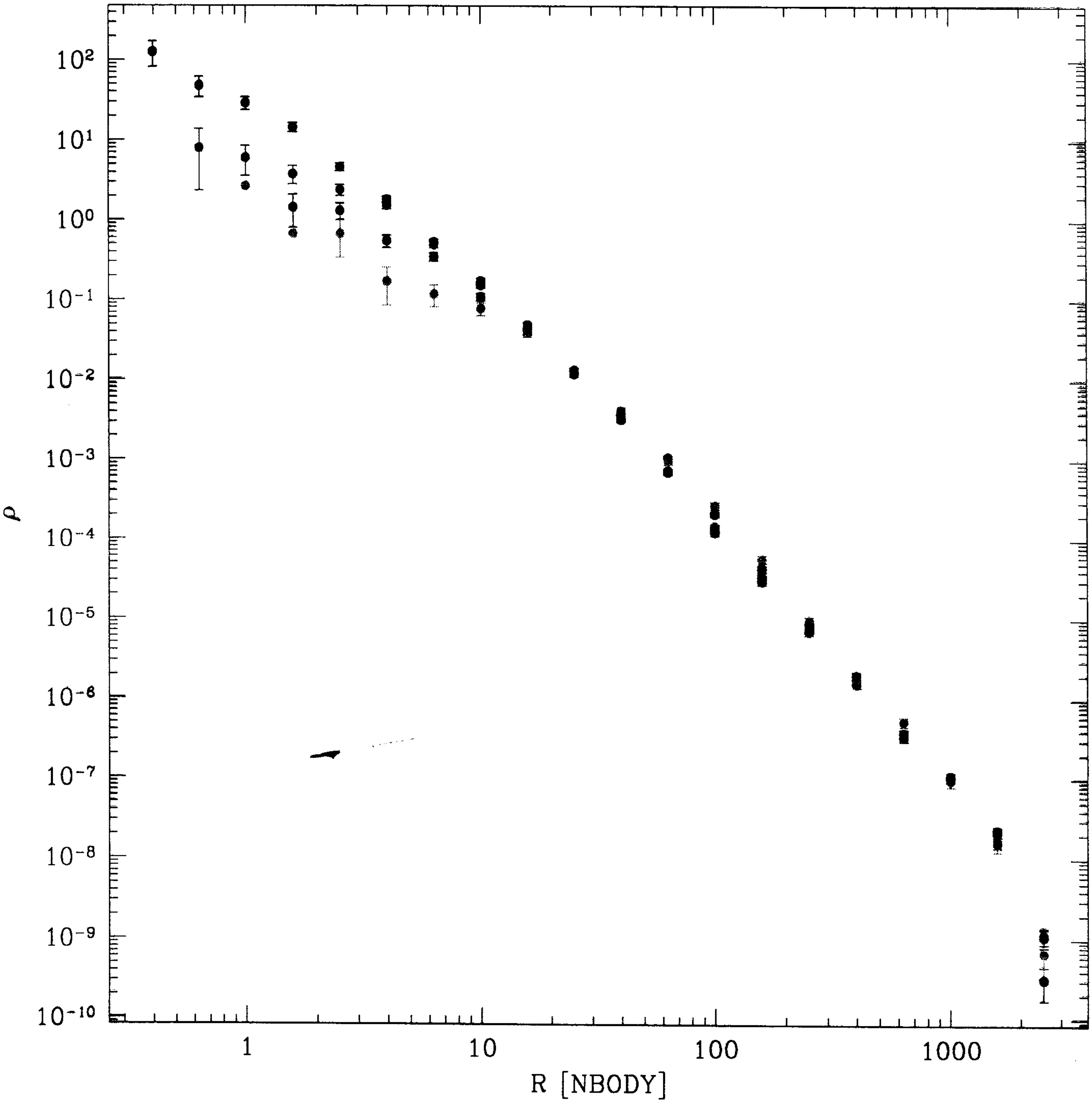
$$\frac{dN}{dt} = -.00868\sqrt{\frac{Gmn}{r_0^3}}$$

or generally $\dot{N} = -\frac{256}{3}\sqrt{2}\,\pi^4 G^2 m^2 \int_0^{r_{max}} r^2 dr \iint \frac{f(E)f(E')(E+E'-\varphi)^{3/2}}{E^2}\, dE\, dE'$

cf. Wielen '88 IAU 126

Baumgardt





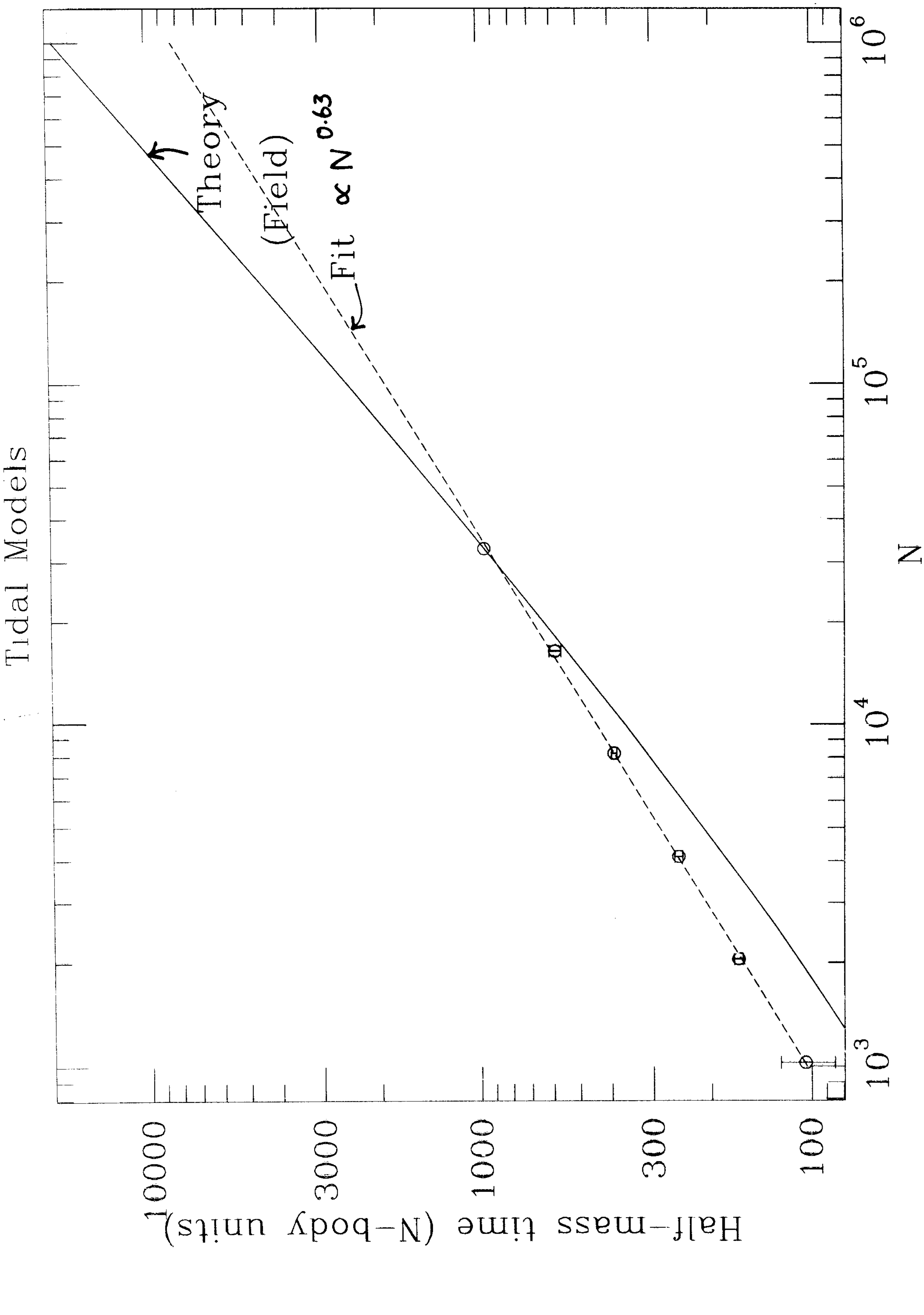
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
ρ
1
10
100
1000
R [NBODY]

CLUSTER ON CIRCULAR GALACTIC ORBIT

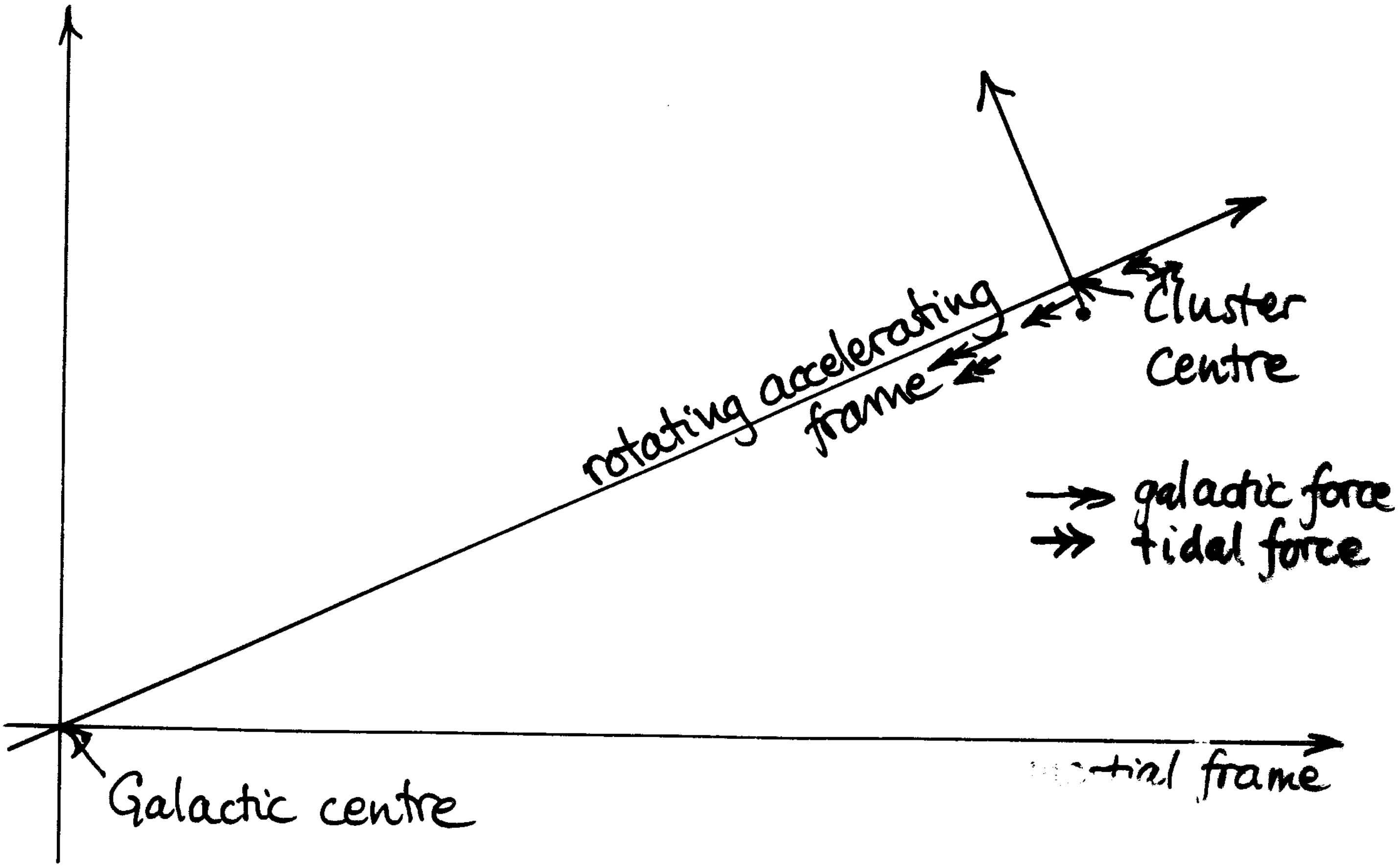
Theory:

time scale of mass loss $\propto t_r \propto \frac{N}{\ln \gamma N} t_{cr}$

Example of difficulty of scaling

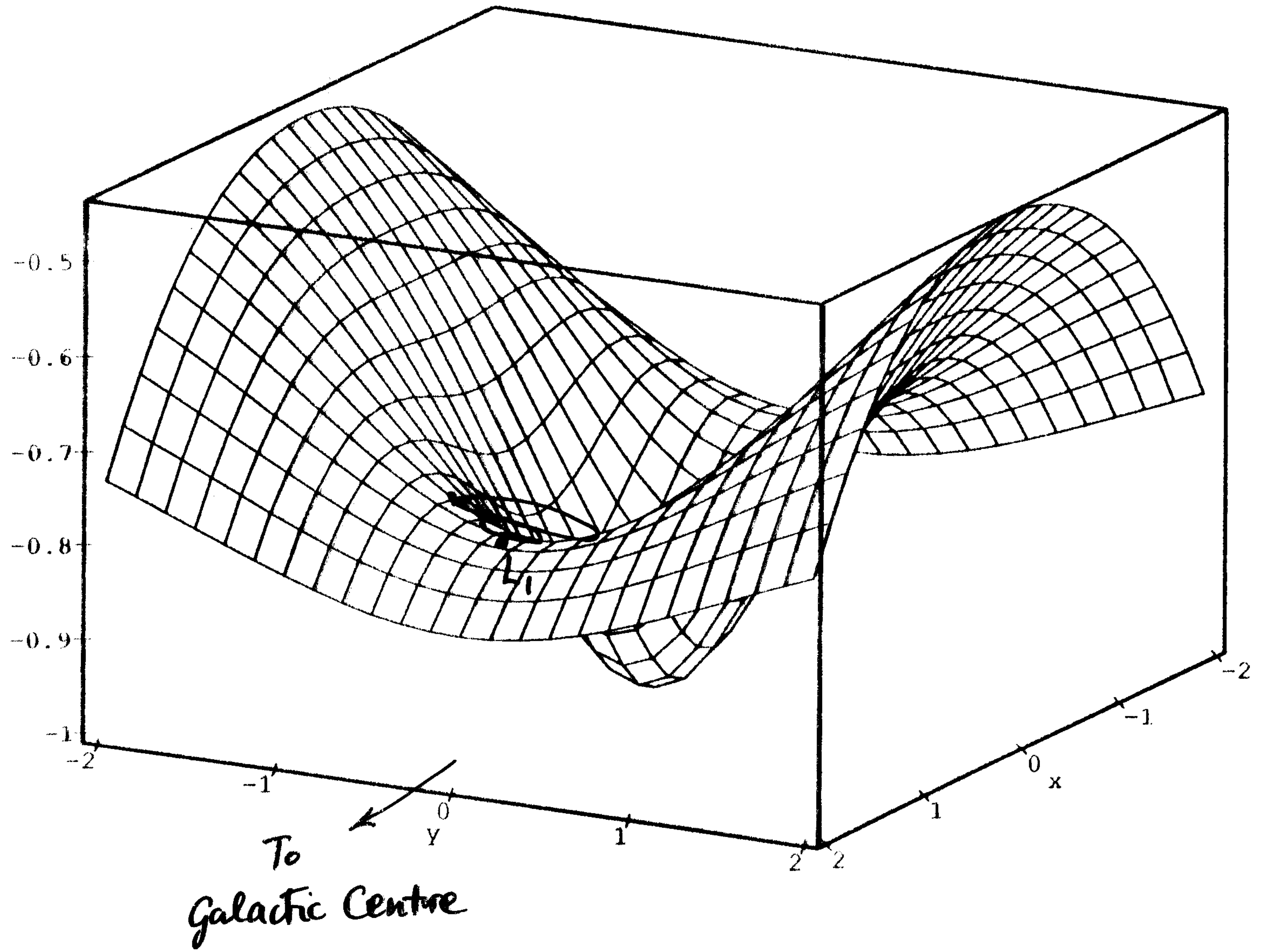


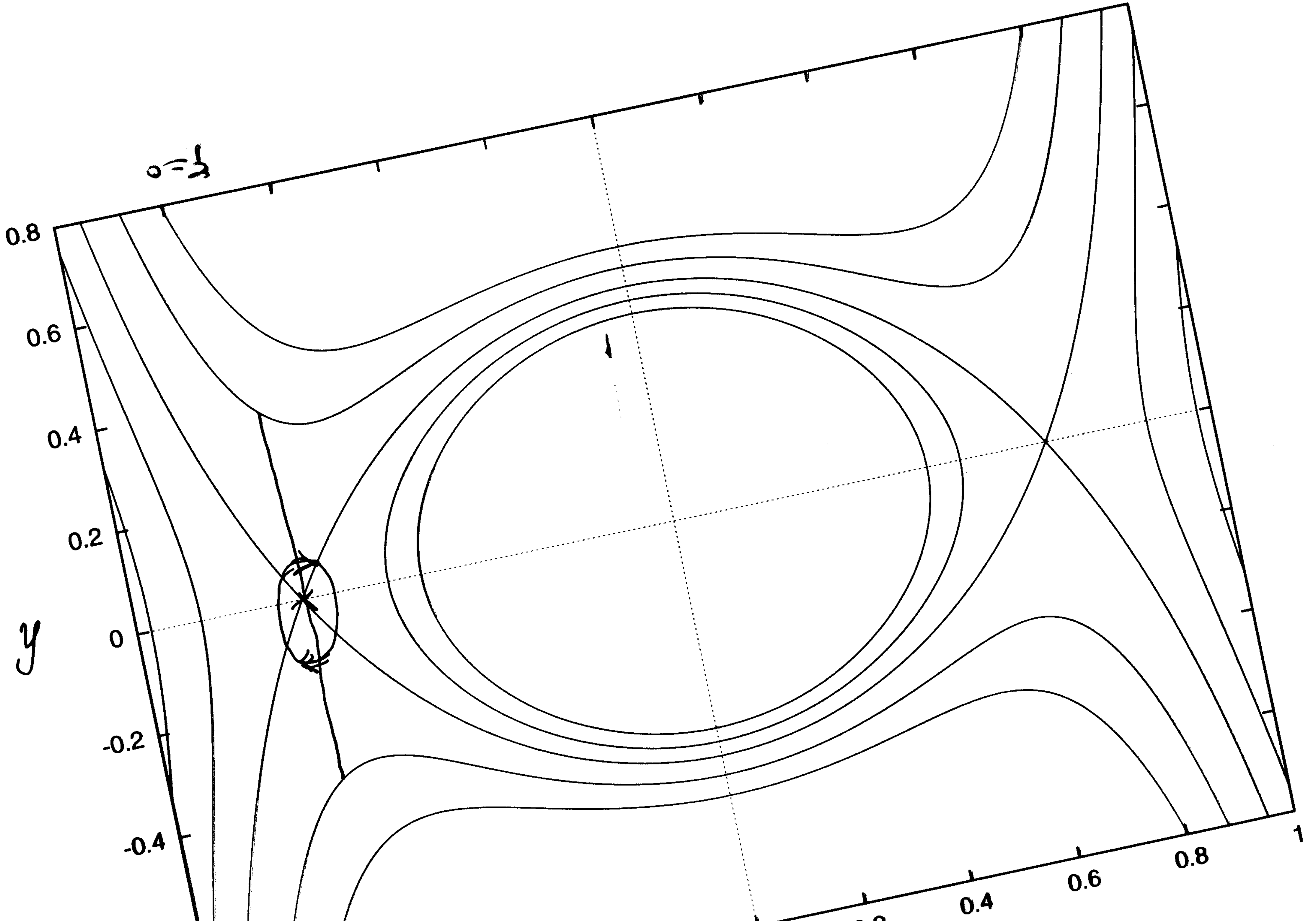
2. CLUSTER ON CIRCULAR ORBIT

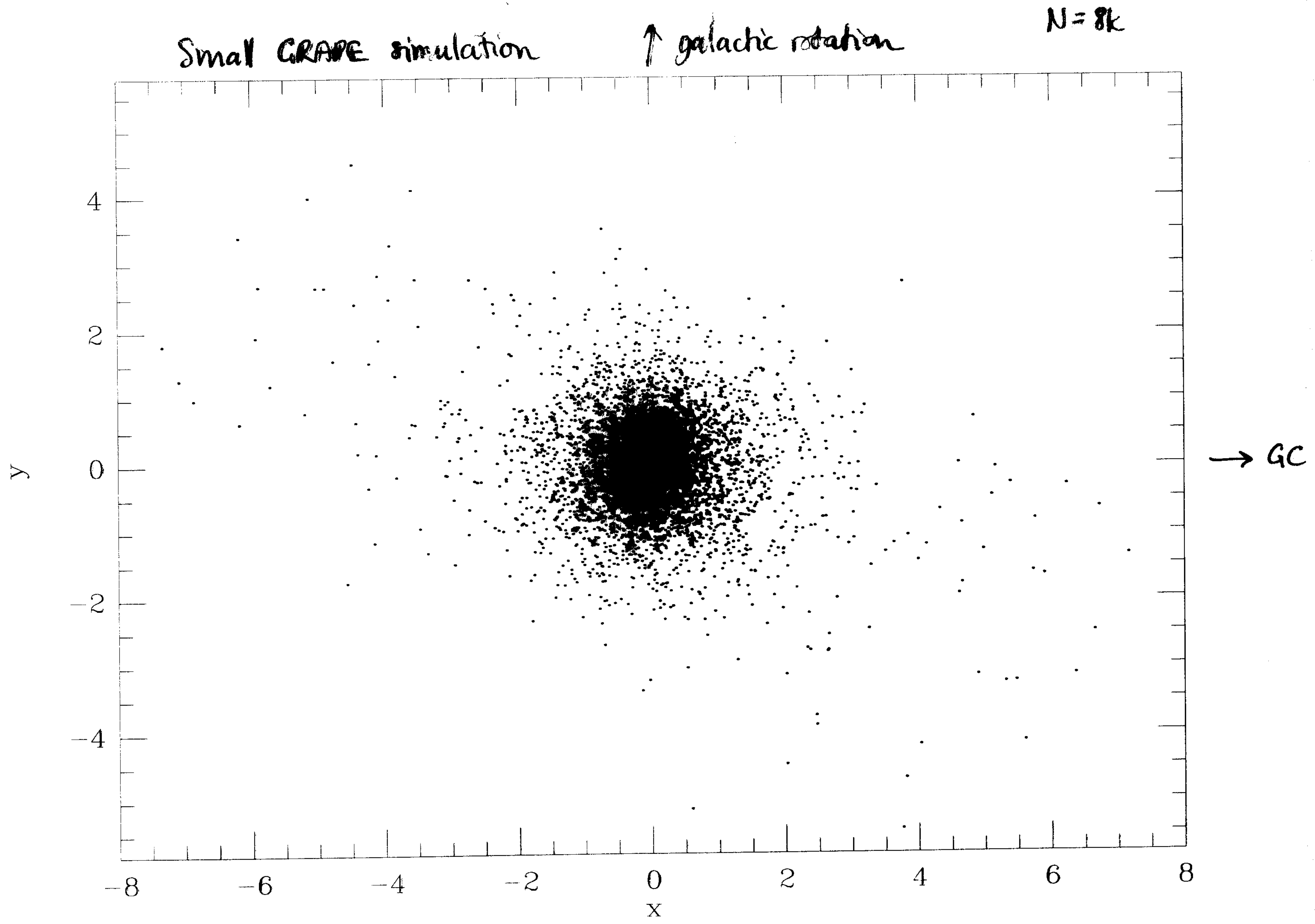


- tidal field
- coriolis force

Potential Well of Cluster in Galactic tide




y
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0.4
0.6
0.8
1



Small GRAPE simulation
↑ galactic rotation
N = 8k
→ GC
y
x
4
2
0
−2
−4
−8
−6
−4
−2
0
2
4
6
8

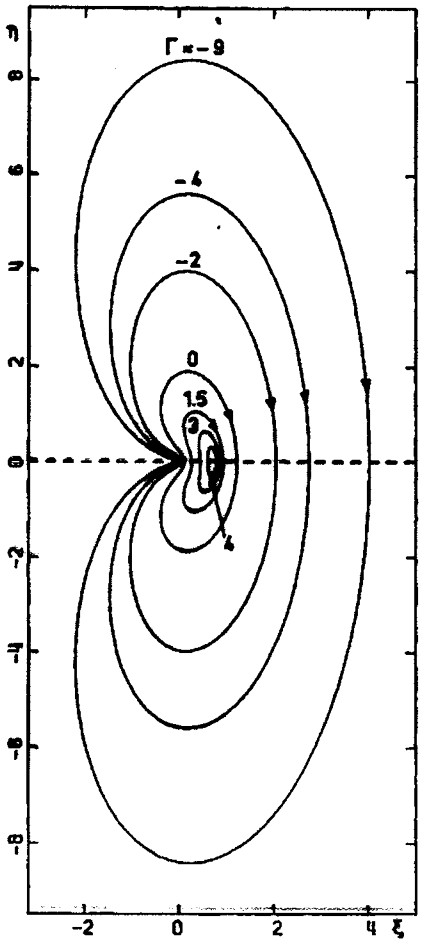


Fig. 2. Family *a* of periodic orbits. All orbits are unstable

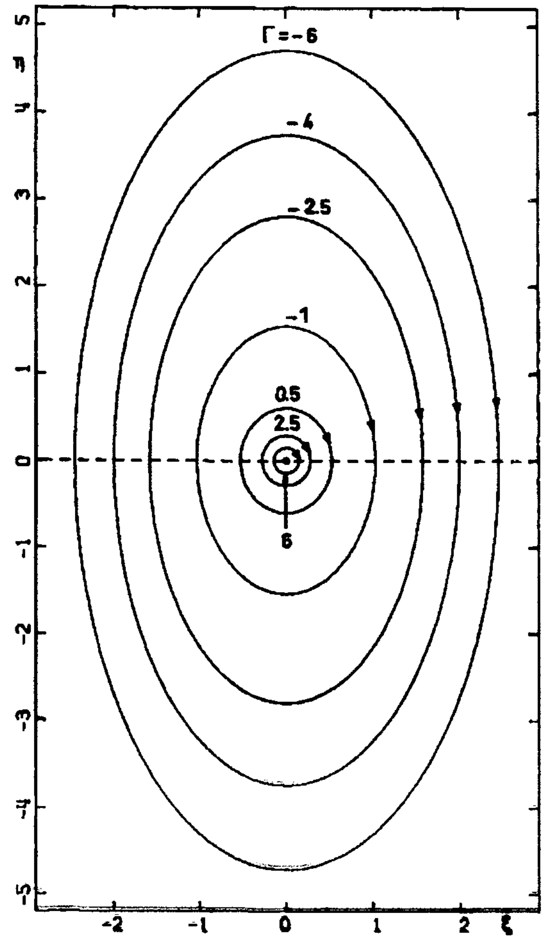


Fig. 3. Family *f* of periodic orbits. All orbits are stable

The simplest case is provided by family *f*. As Fig. 3 shows, for $\Gamma \to -\infty$, all points of the orbit go more and more away from M_2. Therefore the attraction of M_2 can be neglected in a first approximation, and the equations of motion (2) reduce to:

$$\ddot{\xi} = 2\dot{\eta} + 3\xi, \quad \ddot{\eta} = -2\dot{\xi}. \tag{7}$$

This system is easily integrated, and the general solution is:

$$\begin{aligned} \xi &= K_1 \cos t + K_2 \sin t + 2K_3, \\ \eta &= -2K_1 \sin t + 2K_2 \cos t - 3K_3 t + K_4, \end{aligned} \tag{8}$$

where K_1, K_2, K_3, K_4 are the four constants of integration. The corresponding value of Jacobi's constant is found by substitution into (4):

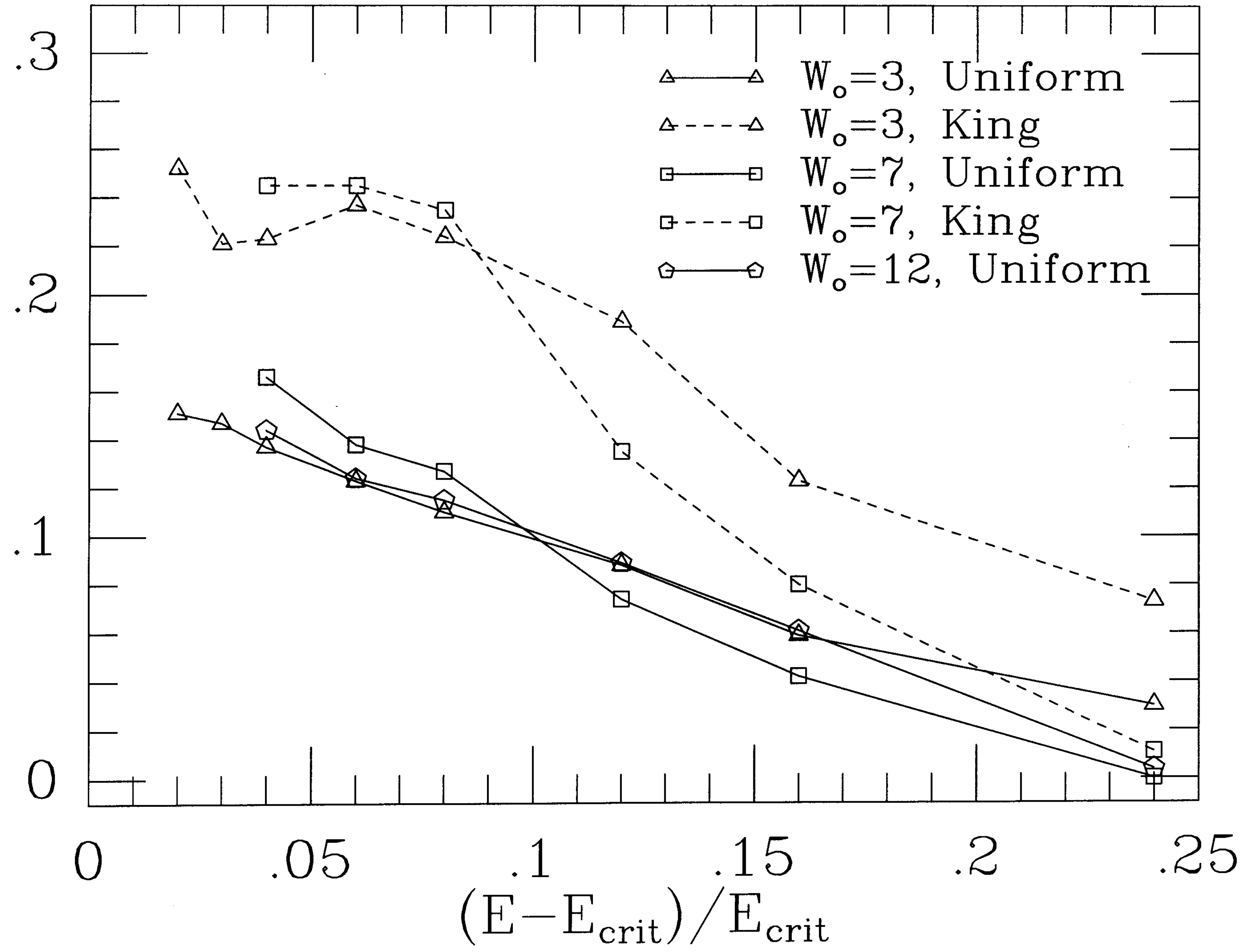
$$\Gamma = 3K_3^2 - K_1^2 - K_2^2. \tag{9}$$

Eqs. (8) represent in general an epicycloidal motion. Since we are interested in periodic orbits only, we must set: $K_3 = 0$. Also, since we restrict our attention to orbits symmetrical with respect to the ξ axis, we must set: $K_4 = 0$. By an appropriate choice of the origin of time, we can also have $K_2 = 0$, and the equations reduce to (cf. Matukuma, 1932):

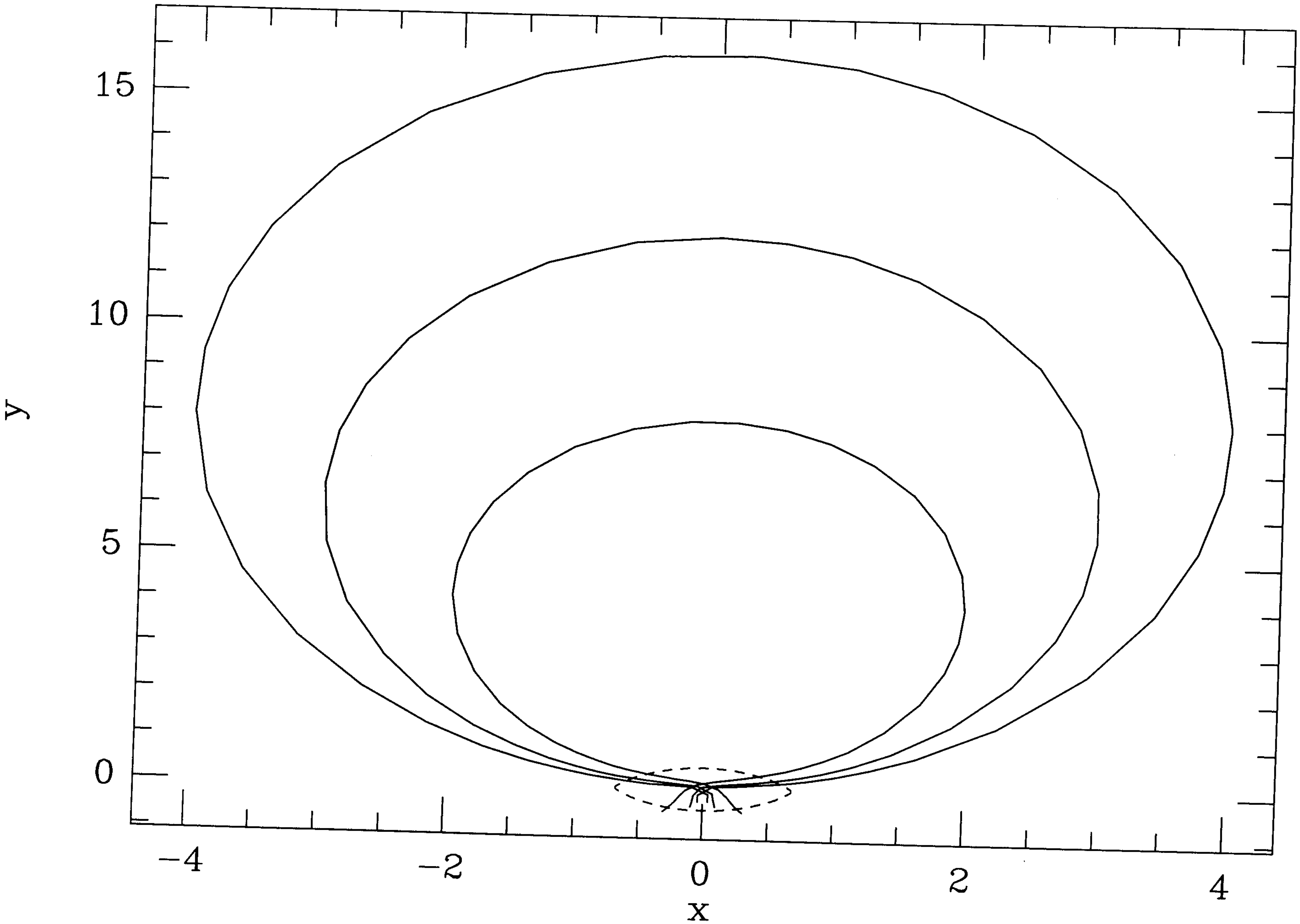
$$\xi = K_1 \cos t, \quad \eta = -2K_1 \sin t, \quad \Gamma = -K_1^2. \tag{10}$$

The orbit is an ellipse, having its center in M_2, its major axis in the vertical direction, an axis ratio equal to 2, a period equal to 2Π and a retrograde direction of rotation. The relation between Γ and ξ_0, abscissa of the intersection of the ξ axis in the positive direction, is found by elimination of K_1:

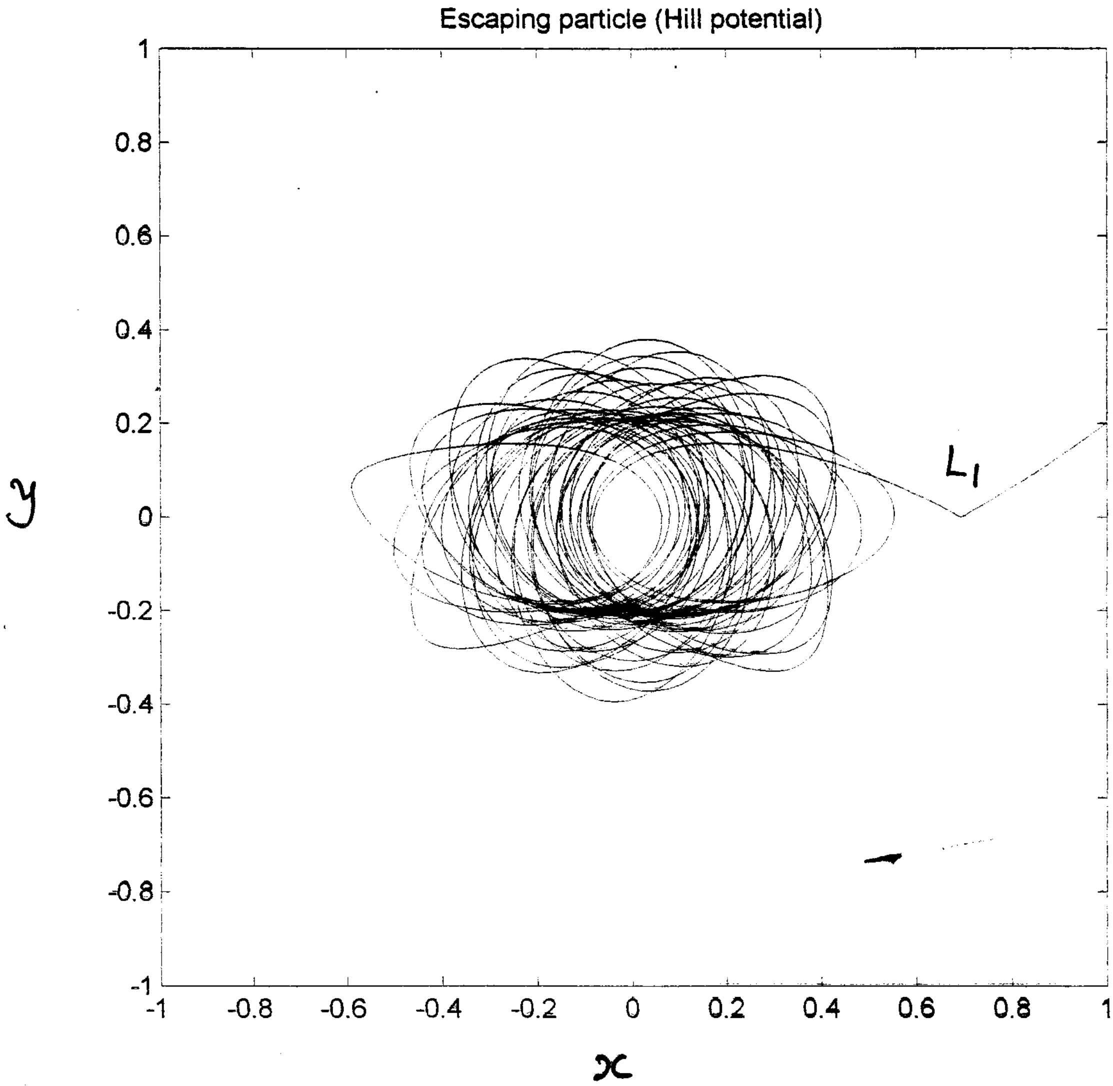
$$\xi_0 = -(-\Gamma)^{1/2}. \tag{11}$$



W$_o$=3, Uniform
W$_o$=3, King
W$_o$=7, Uniform
W$_o$=7, King
W$_o$=12, Uniform
.3
.2
.1
0
0
.05
.1
.15
.2
.25
$(E-E_{crit})/E_{crit}$



y
0
5
10
15
x
−4
−2
0
2
4



cf. King ~ 1961 (pers. comm.)