Calculus 2 Quiz 2

NAME

Show details in the space next to each problem.
You must show your work to receive full credit.

1. (5 points) Compute the following integral:

\[ \int x \tan x \sec x \, dx \]

**Solution:** Integrate by part with \( u = x \), \( dv = \sec x \tan x \) so that \( du = 1 \) and \( v = \sec x \)
so we get

\[
\int x \sec x \tan x \, dx = x \sec x - \int \sec x \, dx = x \sec x - \ln | \sec x + \tan x | + C
\]

2. (5 points) Is the following integral convergent or divergent? Why?

\[ \int_1^\infty \frac{\ln x}{x^3} \, dx \]

**Solution:** I claim that for \( x > 1 \) we have \( x - \ln x > 0 \). In fact \( 1 - \ln 1 = 1 > 0 \) and
for \( x \geq 1 \) we have \( \frac{d}{dx}(x - \ln x) = 1 - \frac{1}{x} \geq 0 \). Therefore \( x - \ln x \) is positive at 1 and
increasing on \([1, \infty)\) hence always positive on that interval, so that \( x > \ln x \) on that
interval. So we have

\[ \frac{\ln x}{x^3} < \frac{1}{x^2} \]

and the integral is convergent by the comparison theorem since

\[ \int_1^\infty \frac{1}{x^2} \]

is convergent.

**Alternative solution:** You can compute the integral by parts: we can set \( u = \ln x \)
and \( dv = \frac{1}{x^3} \) so that \( du = \frac{1}{x} \, dx \) and \( v = \frac{-1}{2x^2} \), so that

\[
\int_1^\infty \frac{\ln x}{x^3} \, dx = \lim_{t \to \infty} \left[ -\ln x \right]_1^t - \int_1^\infty \frac{-1}{2x^3} \]

which is convergent because

\[ \lim_{t \to \infty} -\frac{\ln x}{2x^2} = \lim_{t \to \infty} -\frac{1}{4x^2} = 0 \]

and

\[ \int_1^\infty \frac{1}{2x^2} \]

is convergent.