Calculus 2 Quiz 5

Solution

Show details in the space next to each problem.
You must show your work to receive full credit.

Is the following series convergent? Is it absolutely convergent?

\[ \sum_{n=1}^{\infty} (-1)^n \frac{n}{5+n^2} \]

This is an alternating series so we can use the alternating series test. In this case

\[ b_n = \frac{n}{5+n^2} \]

First of all,

\[ \lim_{n \to \infty} b_n = \lim_{n \to \infty} \frac{1}{\frac{5}{n} + n} = 0 \]

. So we just have to check that the sequence of the \( b_n \)s is decreasing. Let \( f(x) = \frac{x}{5+x^2} \). Then

\[ f'(x) = \frac{5+x^2-2x^2}{(5+x^2)^2} = \frac{5-x^2}{(5+x^2)^2} \]

which is negative for \( x \geq 3 \). Hence \( \{b_n\} \) is a decreasing sequence for \( n \geq 3 \) and the series is convergent.

To test for absolute convergence we can use the limit comparison test on the series \( \sum_{n=1}^{\infty} \frac{n}{5+n^2} \). Let’s compare with \( \sum_{n=1}^{\infty} \frac{1}{n} \).

\[ \lim_{n \to \infty} \frac{\frac{n^2}{5+n^2}}{\frac{1}{n}} = \lim_{n \to \infty} \frac{n^2}{5+n^2} = 1 \]

hence since the series \( \sum_{n=1}^{\infty} \frac{1}{n} \) is divergent, the series \( \sum_{n=1}^{\infty} \frac{n}{5+n^2} \) is also divergent therefore \( \sum_{n=1}^{\infty} (-1)^n \frac{n}{5+n^2} \) is not absolutely convergent.