CALCULUS 1 QUIZ 3

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Show details in the space next to each problem. 
You must show your work to receive full credit.

1. Find the absolute maximum and absolute minimum of the function

\[ f(t) = t\sqrt{4-t^2} \]

on the interval \([-1,2]\).

**Solution:**

\[ f'(t) = \frac{\sqrt{4-t^2}}{2\sqrt{4-t^2}}(4) + \frac{1}{2\sqrt{4-t^2}}(-2t) = \frac{4-2t^2}{\sqrt{4-t^2}} \]

\[ f'(x) = 0 \text{ when } x = \sqrt{2} \text{ or } x = -\sqrt{2}, \text{ but } -\sqrt{2} \text{ is not in the domain.} \]

\[ f'(x) \text{ is not defined when } x = 2. \text{ (and } x = -2 \text{ which is not in our domain).} \]

Hence the critical points are \(\sqrt{2}\) and 2.

To find the absolute max and min we have to check the critical points and the endpoints:

\[ f(-1) = -\sqrt{3}, \quad f(2) = 0, \quad f(\sqrt{2}) = 2 \]

hence the max value of \(f\) is 2, attained at \(x = \sqrt{2}\) and the min value is \(-\sqrt{3}\), attained at \(x = -1\).

2. Let

\[ f(x) = 2x^3 - 3x^2 - 12x + 1. \]

- Find the intervals on which \(f\) is increasing and decreasing.
- Find the local maximum and minimum values of \(f\)

**Solution:**

\[ f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x - 2)(x + 1) \]

\[ f'(x) \text{ is positive for } x < -1 \text{ and } x > 2 \text{ hence } f \text{ is increasing on } (-\infty, -1) \text{ and } (2, +\infty). \]

\[ f'(x) \text{ is negative for } -1 < x < 2 \text{ hence } f \text{ is decreasing on } (-1, 2). \]

By the first derivative test, \(f\) has a local max at -1 and a local min at 2. \(f(-1) = -2 - 3 + 12 + 1 = 8\) and \(f(2) = 16 - 12 - 24 + 1 = -19.\)