Study suggestions for Midterm 2

October 29, 2009

All exam questions will be free response (no true/false, no multiple choice, etc.). Some of the questions will just ask you to compute things, others might ask you to give an example, to apply a theorem, etc.

The second midterm and the final will be cumulative. This means that in addition to what is listed below, you should go back to the study guide for the first midterm and make sure you still remember those things.

This is what I expect you to be able to do:

Know the definitions: To “know a definition” means that you can state it precisely, give an example of something that satisfies it, give an example of something that fails to satisfy it, and justify that your examples are correct.

You should remember the following definitions:

- $f$ is differentiable at a point
- $f$ is differentiable on an interval
- The derivative of a function at a point
- Linearization
- Absolute maximum and minimum
- Local maximum and minimum
- Critical number
- Increasing and decreasing function
- Concave up/down
- Inflection point

Know the theorems: To “know a theorem” means that you can state it precisely, that you understand which parts of the theorem are the premises and which are the conclusion, that you can show an example in which the theorem applies, and that you can give examples to show why each of the premises is necessary to make the theorem true. I will not ask you to prove these theorems in the midterm.

- Intermediate Value theorem
- $f$ is differentiable at a point $\Rightarrow f$ is continuous at a point
- L’Hospital’s rule
- The Extreme Value Theorem
- Fermat’s theorem
- Rolle’s theorem
- Mean value theorem and its applications (e.g. what can you say when $f'(x) = 0$ / $f'(x) > 0$ / $f'(x) < 0$ on an interval)

Limits:
- Be able to apply l’Hospital’s rule to find the value of a limit;
- Know how to deal with indeterminate products/differences/powers
Differentiation:

- Implicit differentiation and how to find the tangent to a graph defined implicitly
- Logarithmic differentiation
- Related rates: apply the strategy we learned in class to solve problems involving functions and their derivatives
- Find the linear approximation of a function at a point
- Find the global max/min of a function on a closed interval

Derivatives: Remember all the derivative laws we found in class (see below) and be able to apply them, i.e. find derivatives that require a combination of chain, product, quotient rule, and others. Give an equation for the tangent line to a graph at a given point.

<table>
<thead>
<tr>
<th>Function</th>
<th>Derivative</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0</td>
</tr>
<tr>
<td>$x^n$</td>
<td>$nx^{n-1}$</td>
</tr>
<tr>
<td>$e^x$</td>
<td>$e^x$</td>
</tr>
<tr>
<td>$f(x) + g(x)$</td>
<td>$f'(x) + g'(x)$</td>
</tr>
<tr>
<td>$cf(x)$</td>
<td>$cf'(x)$</td>
</tr>
<tr>
<td>$f(x)g(x)$</td>
<td>$f'(x)g(x) + f(x)g'(x)$</td>
</tr>
<tr>
<td>$f(x)/g(x)$</td>
<td>$f(x)g(x) - f(x)g'(x)$</td>
</tr>
<tr>
<td>$\frac{1}{g(x)}$</td>
<td>$-\frac{g'(x)}{(g(x))^2}$</td>
</tr>
<tr>
<td>$\sin(x)$</td>
<td>$\cos(x)$</td>
</tr>
<tr>
<td>$\cos(x)$</td>
<td>$-\sin(x)$</td>
</tr>
<tr>
<td>$\tan(x)$</td>
<td>$\sec^2(x) = 1 + \tan^2(x)$</td>
</tr>
<tr>
<td>$a^x$</td>
<td>$a^x \ln a$</td>
</tr>
<tr>
<td>$f(g(x))$</td>
<td>$f'(g(x))g'(x)$</td>
</tr>
<tr>
<td>$\ln(x)$</td>
<td>$\frac{1}{x}$</td>
</tr>
<tr>
<td>$\arcsin x$</td>
<td>$\sqrt{1-x^2}$</td>
</tr>
<tr>
<td>$\arctan x$</td>
<td>$\frac{1}{1+x^2}$</td>
</tr>
</tbody>
</table>

Curve sketching: The 8 steps to sketch a curve:

1. Find the domain of $f$
2. Find the x and y intercepts
3. Is $f$ symmetric?
4. Find horizontal and vertical asymptotes
5. Find the intervals on which $f$ is increasing and decreasing
6. Find the local maximum and minimum values
7. Compute the concavity and the points of inflection
8. Use the above information to sketch the curve