14. \[ V = lwh \quad \Rightarrow \quad 10 = (2w)(w)h = 2w^2h, \text{ so } h = \frac{5}{w^2}. \]

The cost is \[ 10(2w^2) + 6[2(2wh) + 2(hw)] = 20w^2 + 36wh, \text{ so} \]
\[ C(w) = 20w^2 + 36w\left(\frac{5}{w^3}\right) = 20w^2 + 180/w. \]

\[ C'(w) = 40w - 180/w^2 = 40\left(\frac{w - \frac{9}{2}}{w^2}\right)/w^2 \quad \Rightarrow \quad w = \frac{\sqrt{92}}{2} \text{ is the critical number. There is an absolute minimum} \]
for \( C \) when \( w = \frac{\sqrt{92}}{2} \) since \( C'(w) < 0 \) for \( 0 < w < \frac{\sqrt{92}}{2} \) and \( C'(w) > 0 \) for \( w > \frac{\sqrt{92}}{2} \).

\[ C\left(\frac{\sqrt{92}}{2}\right) = 20\left(\frac{\sqrt{92}}{2}\right)^2 + \frac{180}{\frac{\sqrt{92}}{2}/2} \approx 163.54. \]

19. From the figure, we see that there are two points that are farthest away from \( A(1, 0) \). The distance \( d \) from \( A \) to an arbitrary point \( P(x, y) \) on the ellipse is \[ d = \sqrt{(x - 1)^2 + (y - 0)^2} \text{ and the square of the distance is} \]
\[ S = d^2 = x^2 - 2x + 1 + y^2 = x^2 - 2x + 1 + (4 - 4x^2) = -3x^2 - 2x + 5. \]
\[ S' = -6x - 2 \quad \text{and} \quad S'' = 0 \quad \Rightarrow \quad x = -\frac{1}{3}. \text{ Now } S'' = -6 < 0, \text{ so we know} \]
that \( S \) has a maximum at \( x = -\frac{1}{3} \). Since \(-1 \leq x \leq 1, \)
\[ S(-\frac{1}{3}) = \frac{18}{3}, \text{ and } S(1) = 0, \text{ we see that the maximum distance is } \sqrt{\frac{18}{3}}. \text{ The corresponding } y\text{-values are} \]
\[ y = \pm\sqrt{4 - 4\left(-\frac{1}{3}\right)^2} = \pm\sqrt{\frac{32}{9}} = \pm\frac{4}{3} \sqrt{2} \approx \pm1.89. \text{ The points are } (-\frac{1}{3}, \pm\frac{4}{3} \sqrt{2}). \]

24. The rectangle has area \( A(x) = 2xy = 2x(8 - x^2) = 16x - 2x^3, \text{ where} \)
\[ 0 \leq x \leq 2 \sqrt{2}. \text{ Now } A'(x) = 16 - 6x^2 = 0 \quad \Rightarrow \quad x = 2 \sqrt{\frac{2}{3}}. \text{ Since} \]
\[ A(0) = A(2 \sqrt{2}) = 0, \text{ there is a maximum when } x = 2 \sqrt{\frac{2}{3}} \text{ Then } y = \frac{18}{3}, \]
so the rectangle has dimensions \( 4 \sqrt{\frac{2}{3}} \) and \( \frac{18}{3} \).

32. \( xy = 180, \text{ so } y = 180/x. \text{ The printed area is} \)
\[ (x - 2)(y - 3) = (x - 2)(180/x - 3) = 186 - 3x - 360/x = A(x). \]
\[ A'(x) = -3 + 360/x^2 = 0 \text{ when } x^2 = 120 \quad \Rightarrow \quad x = 2 \sqrt{30}. \text{ This gives an absolute maximum since } A'(x) > 0 \text{ for } 0 < x < 2 \sqrt{30} \text{ and } A'(x) < 0 \text{ for } x > 2 \sqrt{30}. \text{ When} \]
\[ x = 2 \sqrt{30}, \quad y = 180/(2 \sqrt{30}), \text{ so the dimensions are } 2 \sqrt{30} \text{ in. and } 90/\sqrt{30} \text{ in.} \]
44. Let \( t \) be the time, in hours, after 2:00 PM. The position of the boat heading south at time \( t \) is \((0, -20t)\). The position of the boat heading east at time \( t \) is \((-15 + 15t, 0)\). If \( D(t) \) is the distance between the boats at time \( t \), we minimize \( f(t) = [D(t)]^2 = 20^2 t^2 + 15^2 (t - 1)^2 \).

\[
f'(t) = 800t + 450(t - 1) = 1250t - 450 = 0 \quad \text{when} \quad t = \frac{450}{1250} = 0.36 \text{ h}.
\]

\(0.36 \text{ h} \times \frac{60 \text{ min}}{1 \text{ h}} = 21.6 \text{ min} = 21 \text{ min } 36 \text{ s}.\) Since \( f''(t) > 0 \), this gives a minimum, so the boats are closest together at 2:21:36 PM.

46. In isosceles triangle \( AOB \), \( \angle O = 180^\circ - \theta - \theta \), so \( \angle BOC = 2\theta \). The distance rowed is \( 4 \cos \theta \) while the distance walked is the length of arc \( BC = 2(2\theta) = 4\theta \). The time taken is given by \( T(\theta) = \frac{4 \cos \theta}{2} + \frac{4\theta}{4} = 2 \cos \theta + \theta \), \( 0 \leq \theta \leq \frac{\pi}{2} \).

\[
T'(\theta) = -2 \sin \theta + 1 = 0 \quad \Leftrightarrow \quad \sin \theta = \frac{1}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{6}.
\]

Check the value of \( T \) at \( \theta = \frac{\pi}{6} \) and at the endpoints of the domain of \( T \); that is, \( \theta = 0 \) and \( \theta = \frac{\pi}{2} \).

\( T(0) = 2, T\left(\frac{\pi}{6}\right) = \sqrt{3} + \frac{\pi}{6} \approx 2.26 \), and \( T\left(\frac{\pi}{2}\right) = \frac{\pi}{2} \approx 1.57 \). Therefore, the minimum value of \( T \) is \( \frac{\pi}{2} \) when \( \theta = \frac{\pi}{2} \); that is, the woman should walk all the way. Note that \( T''(\theta) = -2 \cos \theta < 0 \) for \( 0 \leq \theta < \frac{\pi}{2} \), so \( \theta = \frac{\pi}{6} \) gives a maximum time.

72. (a) Let \( D \) be the point such that \( a = |AD| \). From the figure, \( \sin \theta = \frac{b}{|BC|} \Rightarrow |BC| = b \csc \theta \) and

\[
\cos \theta = \frac{|BD|}{|BC|} = \frac{a - |AB|}{|BC|} \quad \Rightarrow \quad |BC| = (a - |AB|) \sec \theta. \quad \text{Eliminating} \quad |BC| \quad \text{gives}
\]

\[
(a - |AB|) \sec \theta = b \csc \theta \quad \Rightarrow \quad b \cot \theta = a - |AB| \quad \Rightarrow \quad |AB| = a - b \cot \theta. \quad \text{The total resistance is}
\]

\[
R(\theta) = C \frac{|AB|}{r_1^4} + C \frac{|BC|}{r_2^4} = C \left( \frac{a - b \cot \theta}{r_1^4} + \frac{b \csc \theta}{r_2^4} \right).
\]

(b) \( R'(\theta) = C \left( \frac{b \csc^2 \theta}{r_1^4} - \frac{b \csc \theta \cot \theta}{r_2^4} \right) = bC \csc \theta \left( \frac{\csc \theta}{r_1^4} - \frac{\cot \theta}{r_2^4} \right) \)

\[
R'(\theta) = 0 \quad \Leftrightarrow \quad \frac{\csc \theta}{r_1^4} = \frac{\cot \theta}{r_2^4} \quad \Leftrightarrow \quad \frac{r_2^4}{r_1^4} = \frac{\cot \theta}{\csc \theta} = \cos \theta.
\]

\[
R'(\theta) > 0 \quad \Leftrightarrow \quad \frac{\csc \theta}{r_1^4} > \frac{\cot \theta}{r_2^4} \quad \Rightarrow \quad \cos \theta < \frac{r_2^4}{r_1^4} \quad \text{and} \quad R'(\theta) < 0 \quad \text{when} \quad \cos \theta > \frac{r_2^4}{r_1^4}, \quad \text{so there is an absolute minimum}
\]

when \( \cos \theta = \frac{r_2^4}{r_1^4} \).

(c) When \( r_2 = \frac{3}{2} r_1 \), we have \( \cos \theta = \left( \frac{3}{2} \right)^4 \), so \( \theta = \cos^{-1}\left( \frac{3}{2} \right)^4 \approx 79^\circ \).