Show that $\infty^0$ is an indeterminate form

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To show that $+\infty^0$ is an indeterminate form, you have to show that knowing that $\lim_{x \to a} f(x) = +\infty$ and $\lim_{x \to a} g(x) = 0$ is not enough to compute the limit of $f(x)g(x)$! This means you have to find two sets of functions $f, g$ and $f_1, g_1$ such that:

- $\lim_{x \to a} f(x) = +\infty$, $\lim_{x \to a} f_1(x) = +\infty$
- $\lim_{x \to a} g(x) = 0$, $\lim_{x \to a} g_1(x) = 0$
- $\lim_{x \to a} f(x)^{g(x)} \neq \lim_{x \to a} f_1(x)^{g_1(x)}$

Hint: this will be very similar to the examples we found in class for the case of the indeterminate form $1^\infty$.

Possible solution: choose any two of the following:

- $\lim_{x \to +\infty} x^0 = 1$ (this is an indeterminate form of type $\infty^0$ because $\lim_{x \to +\infty} x = +\infty$, $\lim_{x \to +\infty} 0 = 0$)
- $\lim_{x \to +\infty} (e^x)^{\frac{1}{x}} = \lim_{x \to +\infty} e = e$ (this is an indeterminate form of type $\infty^0$ because $\lim_{x \to +\infty} e^x = +\infty$, $\lim_{x \to +\infty} \frac{1}{x} = 0$)
- $\lim_{x \to +\infty} \left(e^{x^2}\right)^{\frac{1}{2}} = \lim_{x \to +\infty} e^x = +\infty$

(this is an indeterminate form of type $\infty^0$ because $\lim_{x \to +\infty} e^x = +\infty$, $\lim_{x \to +\infty} \frac{1}{x} = 0$)