School of Mathematics

A Decomposition-Based Warmstart Method for Stochastic Programming

Marco Colombo, Andreas Grothey

Decomposition Based IPM-Warmstarts: Overview

- (Anotated) whistle-stop tour of
  - Stochastic Programming
  - Interior Point Methods
  - Warmstarting Interior Point Methods
- Reduced tree based SP warmstarts
- Decomposition based SP warmstarts
- Numerical Results

Stochastic Programming:

A Stochastic Programming Problem is given by

$$\begin{array}{rl} \min_{x,y} & c_0^T x + \mathbb{E}_\xi[c(\xi)^T y(\xi)] \\ \text{s.t.} & Ax = b, \\ & T(\xi)x + W(\xi)y(\xi) = h(\xi) \quad \text{a.s.} \\ & x \geq 0, y(\xi) \geq 0 \end{array}$$

This models a decision process $x \to \xi \to y(\xi)$

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This models a decision process $x \rightarrow \xi \rightarrow y(\xi)$

Deterministic equivalent:

For a discrete distribution $\xi = \{\xi_i : P(\xi = \xi_i) = p_i\}_i$, $(\mathcal{T} = \{p_i, \xi_i\}_i)$:

$$\begin{array}{rl} \min & c_0^T x + \sum_i p_i c_i^T y_i \\ \text{s.t.} & Ax = b, \\ & T_i x + W_i y_i = h_i \quad \forall i \\ & x \geq 0, y_i \geq 0 \end{array} \qquad (P(\mathcal{T}))$$

Structure of Deterministic equivalent

The constraint matrix of the deterministic equivalent

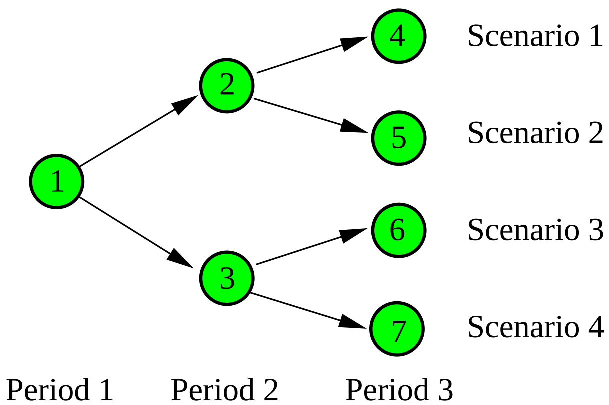
$$\begin{array}{ll} \min & c_0^T x + \sum_i p_i c_i^T y_i \\ \text{s.t.} & \begin{bmatrix} W_1 & & & & T_1 \\ & W_2 & & & T_2 \\ & & \ddots & & \vdots \\ & & & W_n & T_n \\ & & & & A \end{bmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \\ x \end{pmatrix} = \begin{pmatrix} h_1 \\ h_2 \\ \vdots \\ h_n \\ b \end{pmatrix} \end{array}$$

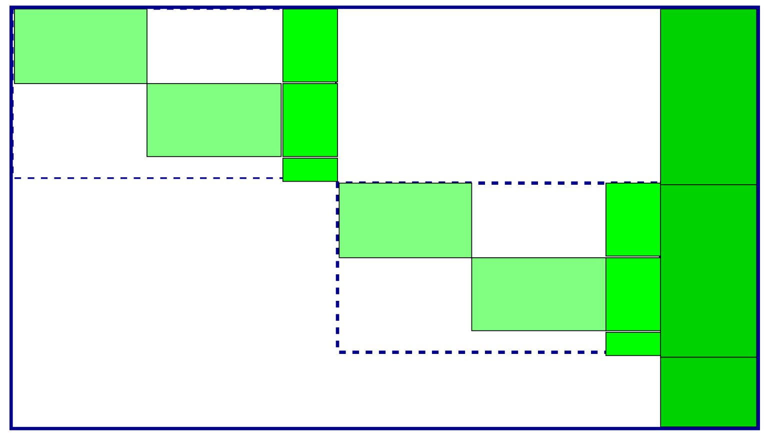
is a **column bordered block-diagonal** matrix.

Structure can be exploited

Multistage Stochastic Programming

(models decision process $x_1 \longrightarrow \xi_2 \longrightarrow x_2(\xi_2) \longrightarrow \ldots \longrightarrow \xi_T \longrightarrow x_T(\xi_T)$)





Scenario Tree Constraint Matrix

$\Rightarrow$ **nested** column bordered block-diagonal constraint matrix

Symmetrical event tree with K realizations/node and T periods corresponds to

$$K^{T-1} \text{ scenarios} \qquad \frac{K^T - 1}{K - 1} \text{ nodes (blocks)}$$

Applications

(Multistage) Stochastic Programming has many applications

- Portfolio Optimization
- Planning under uncertainty
- Electricity Generation Planning (involving hydro or wind)
- etc

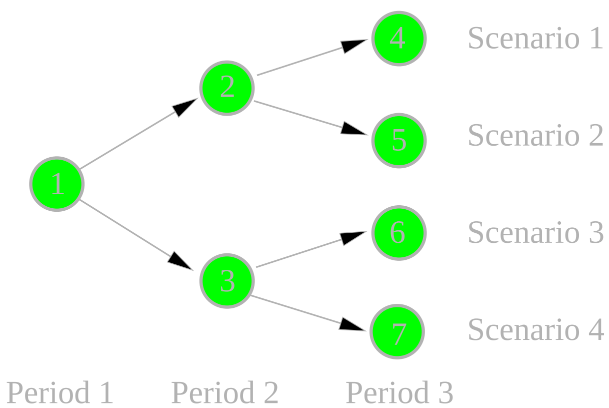
Solving Stochastic Programming Problems:

Problem: size of deterministic equivalent quickly becomes very large
⇒Difficult for standard solvers. Suitable approaches are:

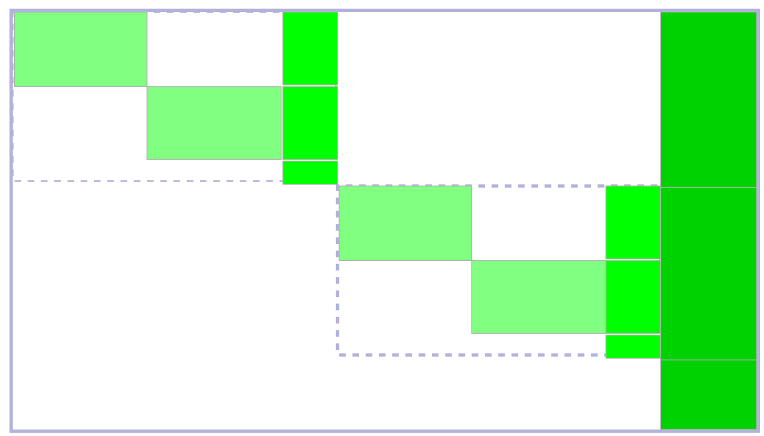
- Decomposition (Benders, L-shaped method)
- Interior Point Method
  (*Gondzio, G. (2005): solved multistage SP problem with* 10^9 *variables on 1280 processors in under 2h*)

Stochastic Programming Warmstarts

- **Idea:** speed up solution process by crash-starting from a smaller tree
- **But:** IPMs are notoriously bad at exploiting a known starting point



Scenario Tree



Constraint Matrix

Interior Point Methods (for LP)

$$\min c^\top x \qquad \text{s.t.} \quad \begin{aligned} Ax &= b \\ x &\geq 0 \end{aligned} \tag{LP}$$

Optimality conditions:

$$\begin{aligned} c - A^\top \lambda - s &= 0 \\ Ax &= b \\ XSe &= 0 \\ x, s &\geq 0 \end{aligned} \tag{KKT}$$

Interior Point Methods (for LP)

$$\min c^\top x - \mu \sum \ln x_i \quad \text{s.t.} \quad \begin{aligned} Ax &= b \\ x &\geq 0 \end{aligned} \tag{LP}$$

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Central Path:

The set of all solutions to the optimality conditions for $\mu > 0$.
The central path joins the analytic center (for $\mu = \infty$) with the LP solution (for $\mu = 0$).

Neighbourhoods (of the central path)

$$\begin{aligned} \mathcal{N}_2(\theta) &:= \{(x, \lambda, s) \in \mathcal{F}^0 : \|XSe - \mu e\|_2 \leq \theta\mu\} \\ \mathcal{N}_{-\infty}(\gamma) &:= \{(x, \lambda, s) \in \mathcal{F}^0 : x_i s_i \geq \gamma\mu\} \end{aligned}$$

where $\mathcal{F}^0 := \{(x, \lambda, s) : c - A^\top \lambda - s = 0, Ax = b, x, s > 0\}$.

Path Following Methods

- choose $x_0, \lambda_0, s_0 > 0, \mu = x_0^\top s_0/n$
- compute Newton step $(\Delta x, \Delta s, \Delta\lambda)$ for (KKT) and given $\mu^+ < \mu$.
- compute stepsizes

$$\alpha = \max_{\alpha>0}\{\alpha : x + \alpha\Delta x \geq 0, s + \Delta s \geq 0, (x, s) \in \mathcal{N}_*(\tau)\}$$

- take step

$$\begin{aligned} x_+ &= x + 0.995\alpha\Delta x \\ \lambda_+ &= \lambda + 0.995\alpha\Delta\lambda \\ s_+ &= z + 0.995\alpha\Delta s \end{aligned}$$

- update μ:

$$\mu_+ = \sigma\frac{x_+^\top s_+}{n}, \quad 0 < \sigma < 1$$

Warmstarting Interior Point Methods

- Many applications require the solution of a series of optimization problems (SQP, B&B, MPC, etc)
- Simplex/Active Set Methods are easy and efficient to warmstart
- IPM Warmstart is seen as difficult to impossible
- One of the main arguments against IPM

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Some research (among others)

- Mitchell, Todd '92
- Hippolito '93
- Lustig, Marsten, Shanno '94
- Gondzio '98
- Gondzio, Vial '99
- Yıldırım, Wright '02
- Gondzio, G. '03/'08
- John,Yıldırım '06
- Benson,Shanno '06

⇒ Message:

- Warmstarting IPMs is possible! (Can save around 50%-60% of iterations)

Warmstarting Interior Point Methods

Aim: Use information from solution process of

$$\min c^\top x \ \text{ s.t. } \begin{aligned} Ax &= b \\ x &\geq 0 \end{aligned} \tag{LP}$$

to construct a starting point for (nearby problem)

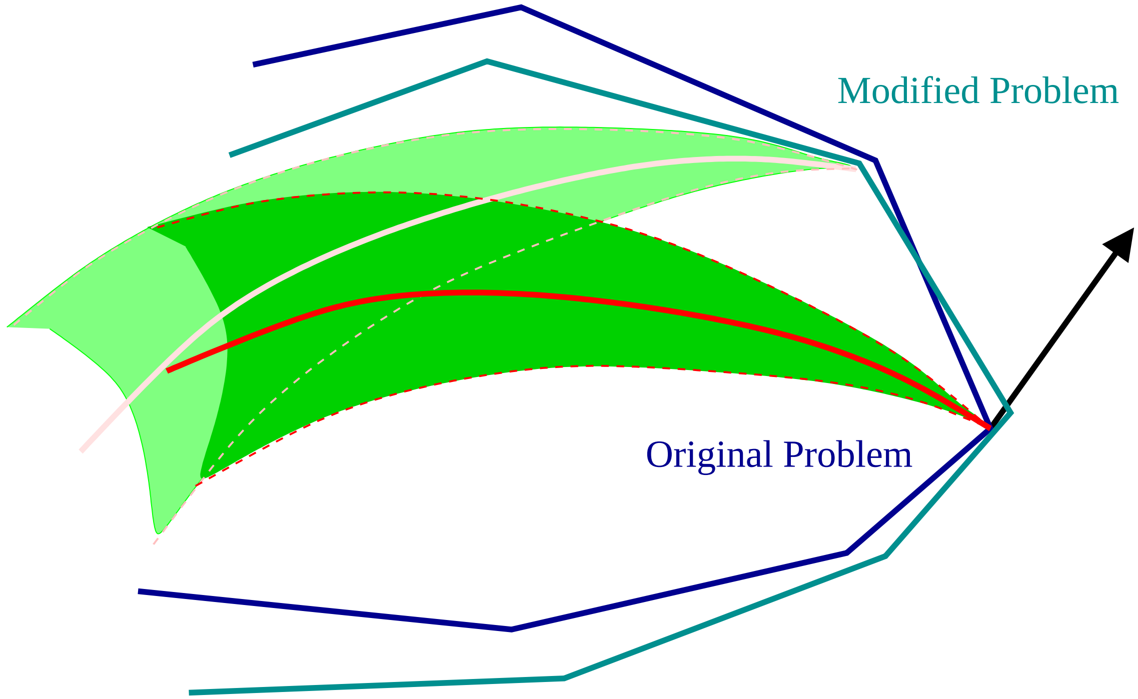
$$\min \tilde{c}^\top x \ \text{ s.t. } \begin{aligned} \tilde{A}x &= \tilde{b} \\ x &\geq 0 \end{aligned} \tag{$\widetilde{\text{LP}}$}$$

where $\tilde{A} \approx A, \tilde{b} \approx b, \tilde{c} \approx c$

- It is **not** a good idea to use the solution of (LP) to start ($\widetilde{\text{LP}}$).
- *Unlike for the Simplex/Active Set Method!*

Why?

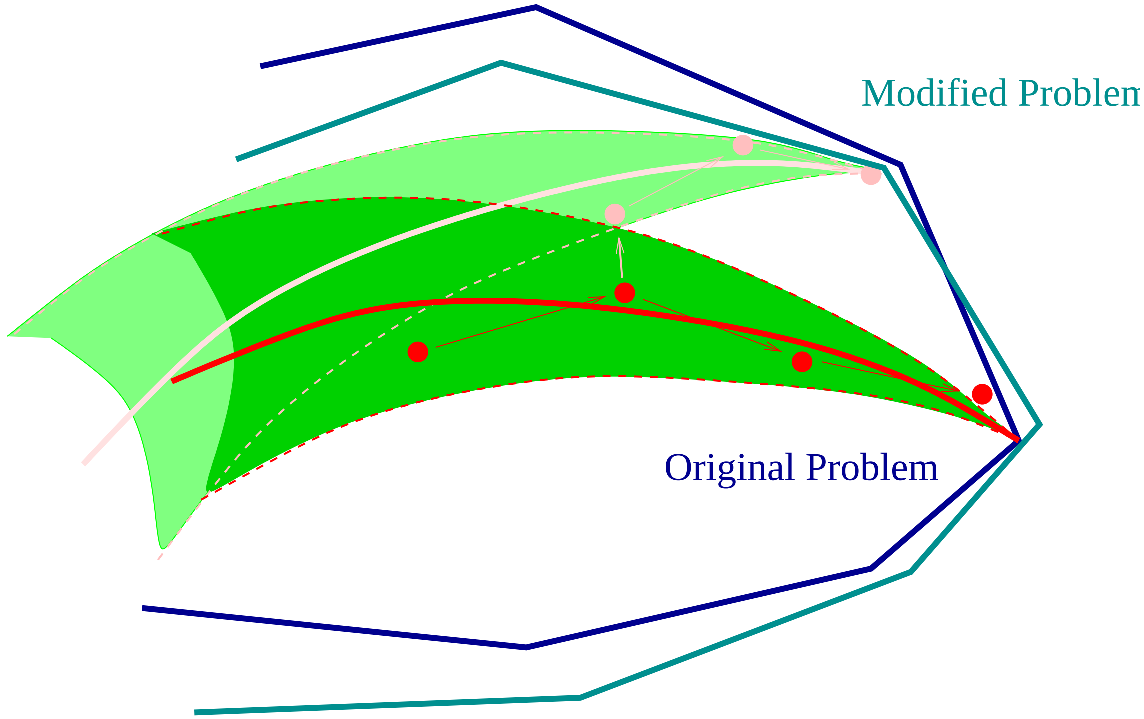
Hippolito (1993): Search direction is parallel to nearby constraints



⇒ only small step in search direction can be taken

Warmstarting Heuristics

Idea: Start close to the (new) central path, not close to the (old) solution



⇒ Start from a previous iterate and do additional *modification* step.

- Yıldırım/Wright ('02): Weighted Least Squares (WLS), Newton correction
- Gondzio/G. ('03/'08): Splitting Directions, Unblocking

Interior Point Warmstarts: Theoretical Results

A typical warmstart results is (Assume $\tilde{A} = A$):

Lemma (based on Yıldırım/Wright '02). Let $(x, \lambda, s) \in \mathcal{N}_2(\theta_0)$ for problem (LP) then the full WLS step $(\Delta x, \Delta\lambda, \Delta s)$ in the perturbed problem ($\tilde{\text{LP}}$) is feasible and

$$(x + \Delta x, \lambda + \Delta\lambda, s + \Delta s) \in \tilde{\mathcal{N}}_2(\theta)$$

provided that

$$\delta_{bc} \leq \frac{\theta - \theta_0}{2C(d)} \min\left\{\frac{1}{2n+1}, \frac{\mu}{4C(d)\|d\|}\right\}$$

$\Rightarrow$ **"small δ_{bc}, large μ"**

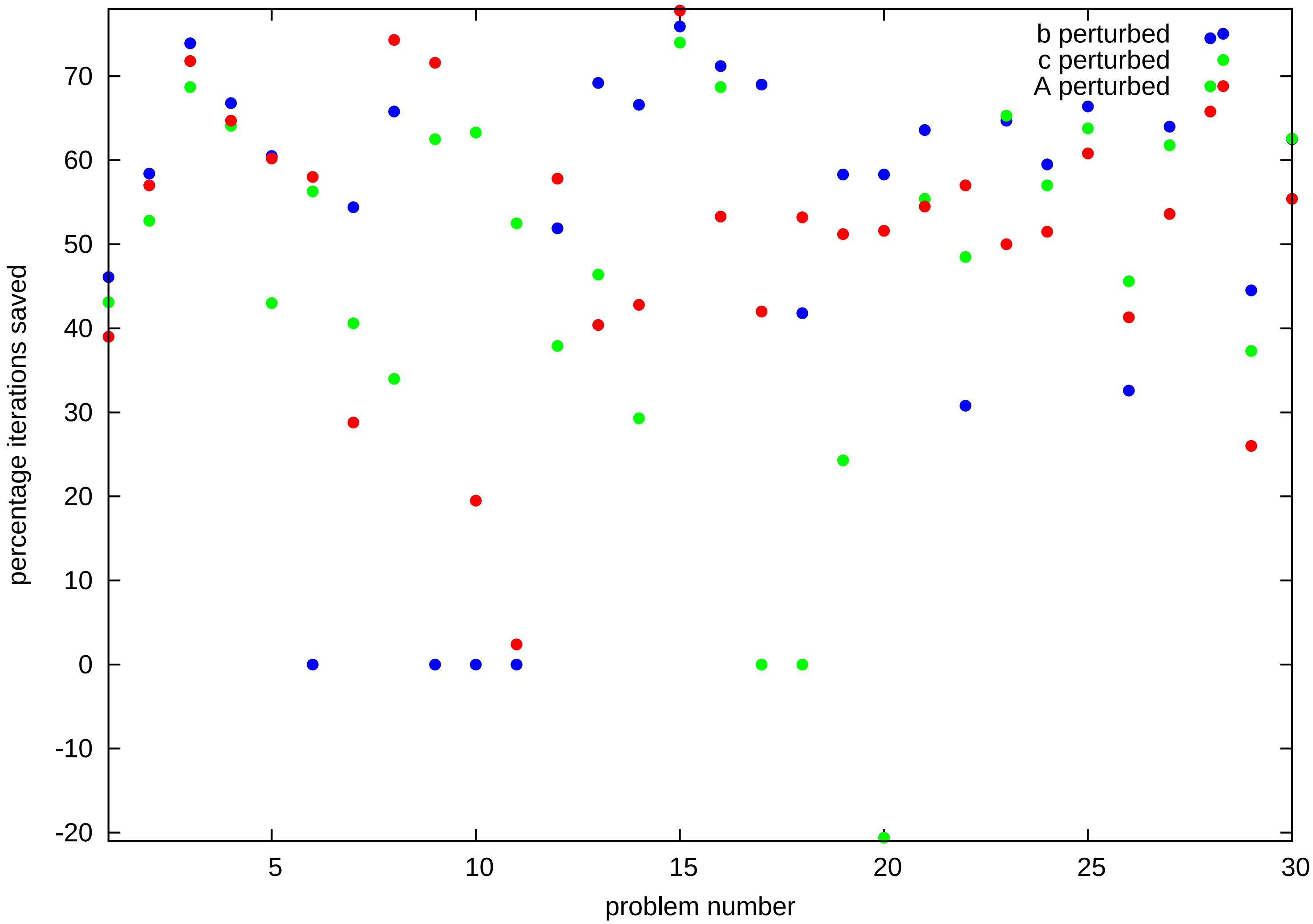
$C(d)$ is the Renegar condition number of the problem $d = (A, b, c)$:

$$C(d) = \frac{\|d\|}{\rho(d)}, \quad \rho(d) = \text{“\textit{distance to infeasibility}”}$$

and

$$\delta_{bc} := \frac{\Delta c}{\|d\|} + 2C(d)\frac{\Delta b}{\|d\|}$$

Results for LP problems (NETLIB) (Gondzio, G. '08)



Stochastic Programming Warmstarts

- **Idea:** speed up solution process by crash-starting from a smaller tree
- **But:** IPMs are notoriously bad at exploiting a known starting point

The aim of a crash-start is to construct (cheaply) a point on (or near) the central path.

primal feasibility	dual feasibility	centrality
$Ax = b$	$\sum_i T^T \lambda_i + s = c_0$	$XSe = \mu e$
$Tx + W_i y_i = h_i$	$W_i^T \lambda_i + z_i = p_i c_i$	$Y_i Z_i e = \mu e$
		$x, s, y, z \geq 0$

The (unique) solution is denoted by

$$(x_\mu(\mathcal{T}), y_\mu(\mathcal{T}), \lambda_\mu(\mathcal{T}), s_\mu(\mathcal{T}), z_\mu(\mathcal{T}))$$

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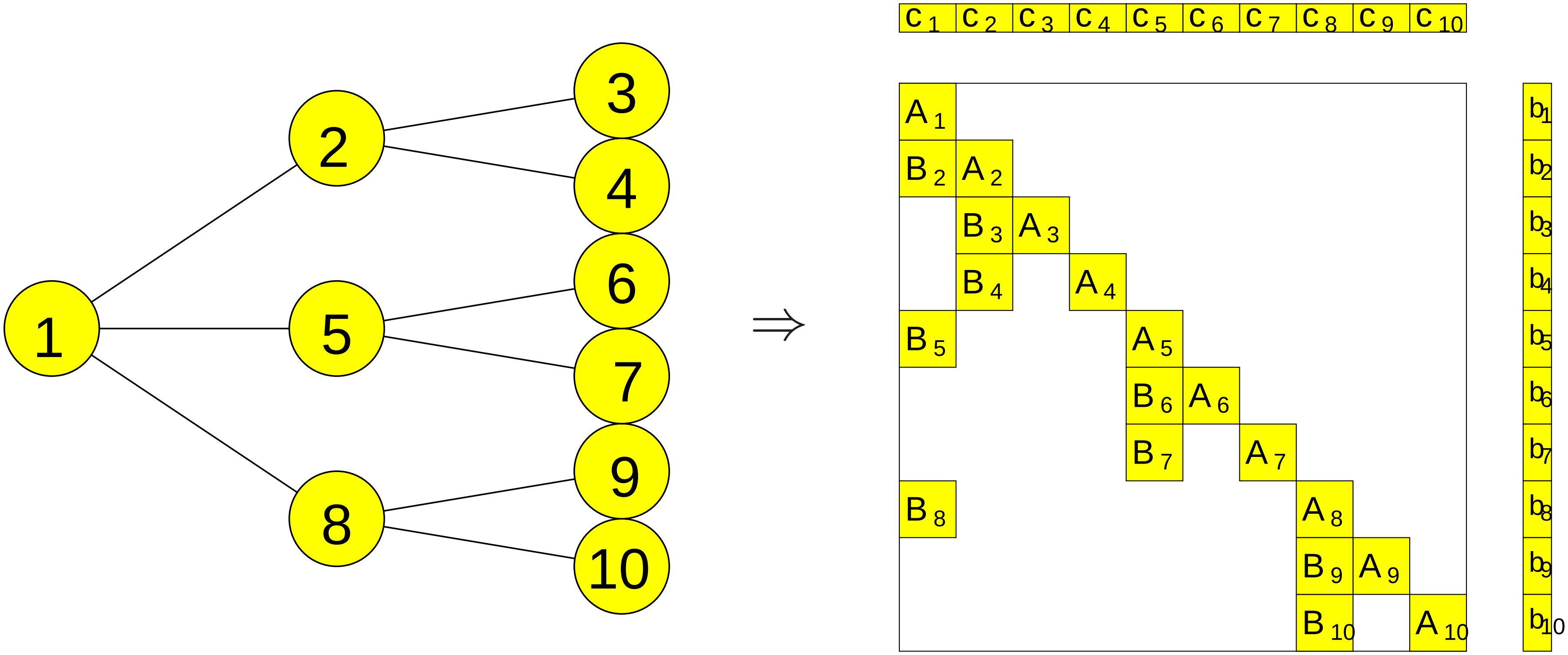
The (unique) solution is denoted by

$$(x_\mu(\mathcal{T}), y_\mu(\mathcal{T}), \lambda_\mu(\mathcal{T}), s_\mu(\mathcal{T}), z_\mu(\mathcal{T}))$$

⇒ The stochastic programming warmstart schemes attempt to (cheaply) find an approximation to $(x_\mu(\mathcal{T}), y_\mu(\mathcal{T}), \lambda_\mu(\mathcal{T}), s_\mu(\mathcal{T}), z_\mu(\mathcal{T}))$

⇒ **Reduced tree based warmstart** (Colombo, Gondzio, G. '09)

Stochastic Programming Warmstarts



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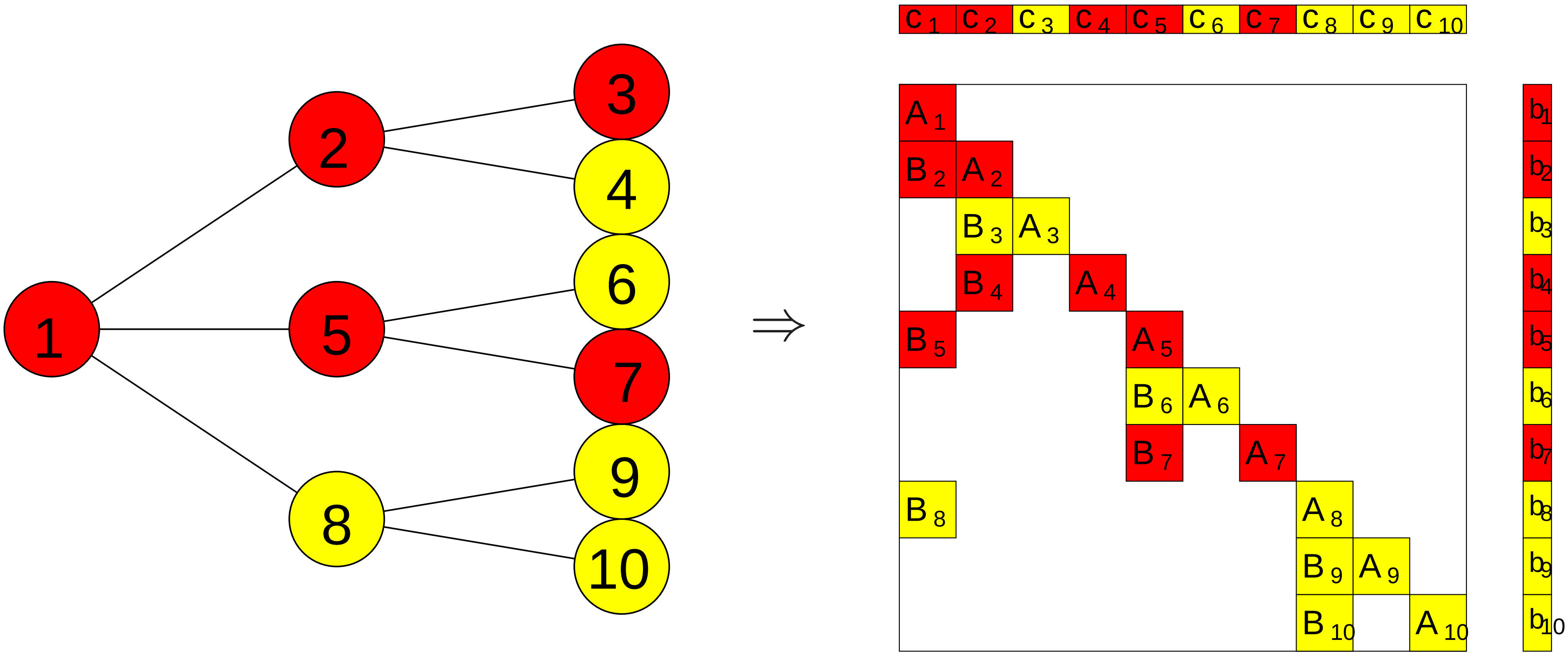
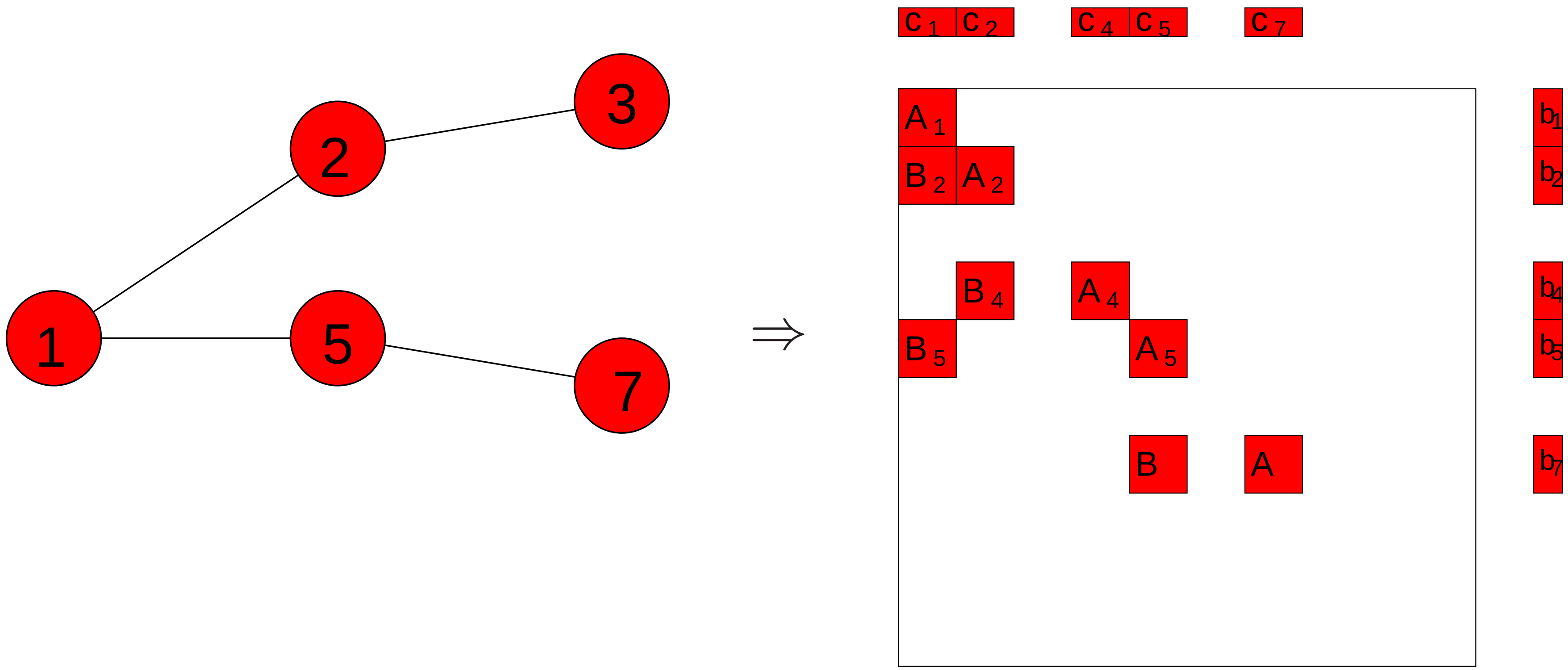


- Select sample scenarios

Stochastic Programming Warmstarts



- Select sample scenarios
- Aggregate Scenarios/Reduce Problem

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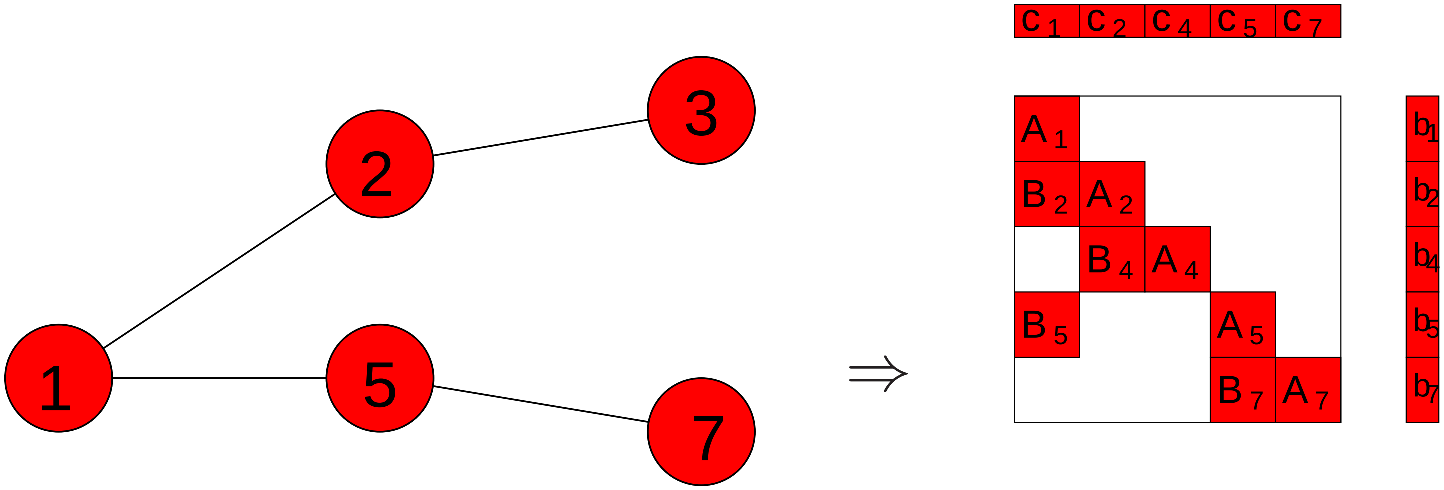

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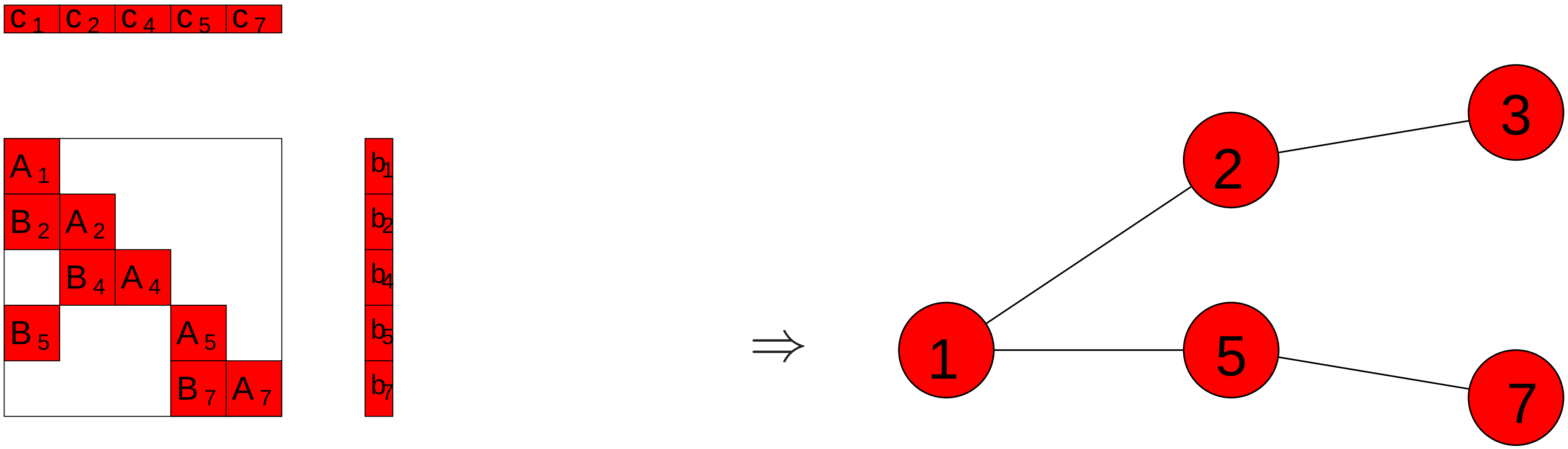
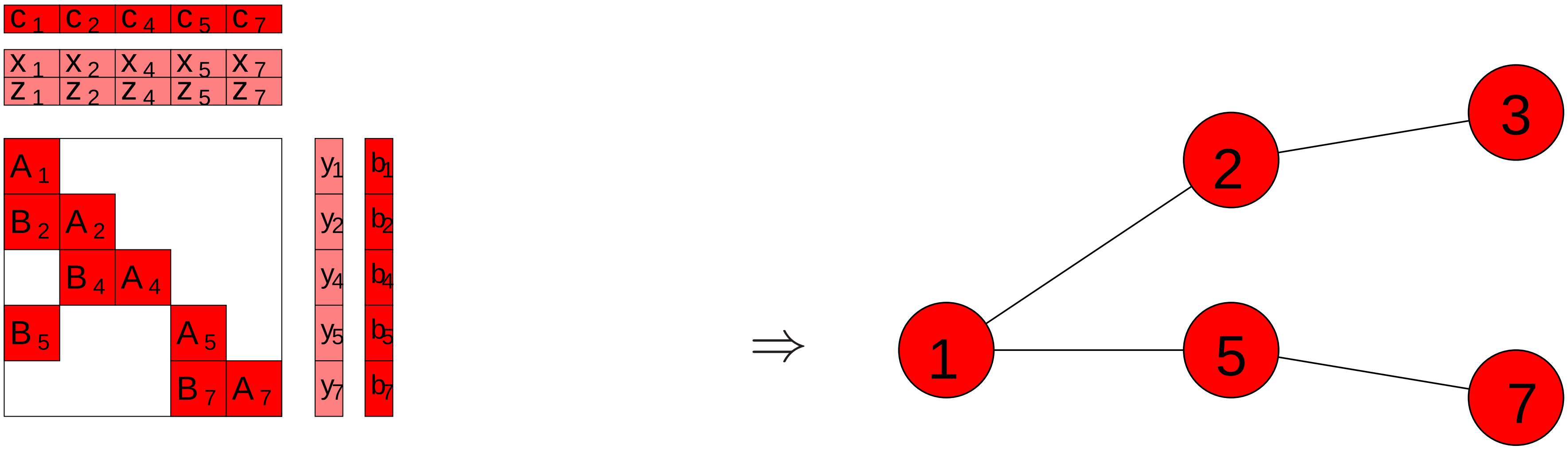


To solve the problem by warmstarting, reverse the process

Stochastic Programming Warmstarts



To solve the problem by warmstarting, reverse the process

- Find central point for reduced problem

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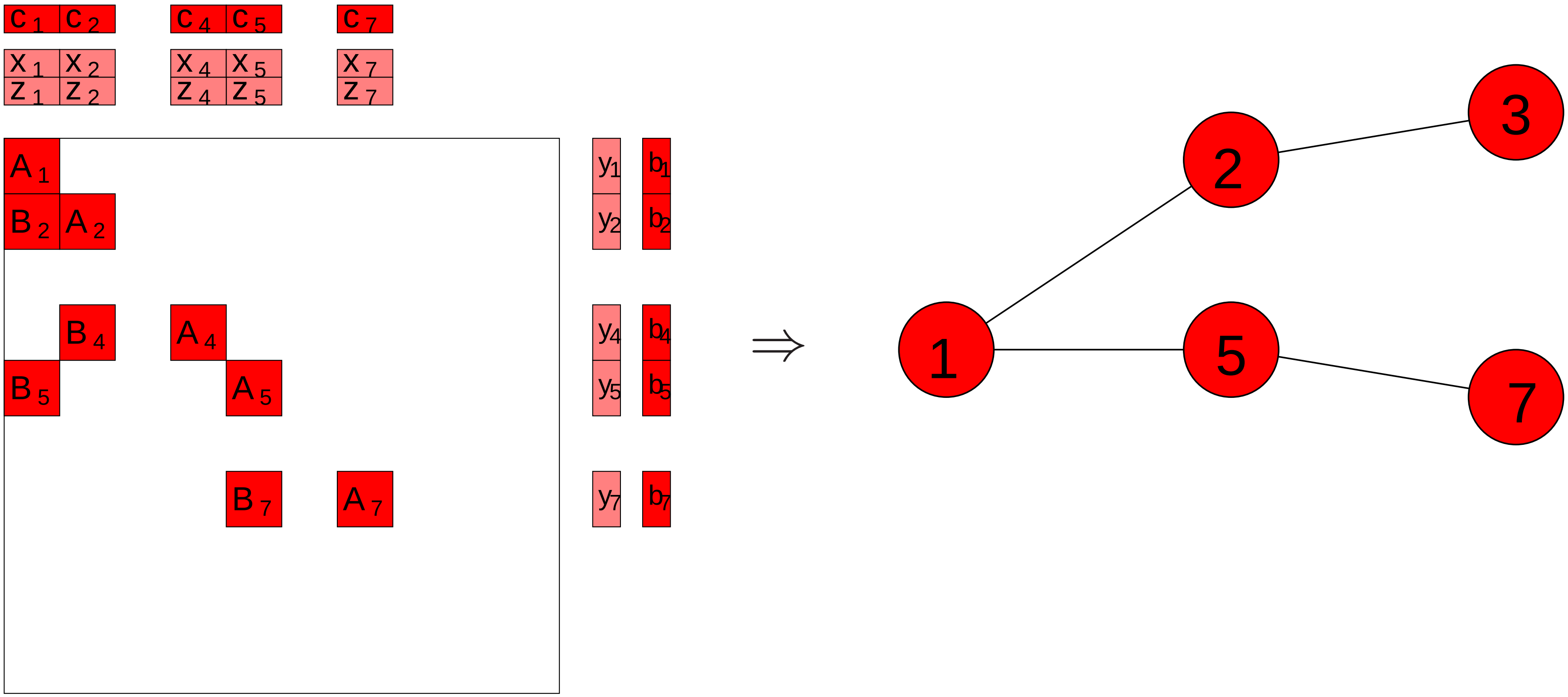
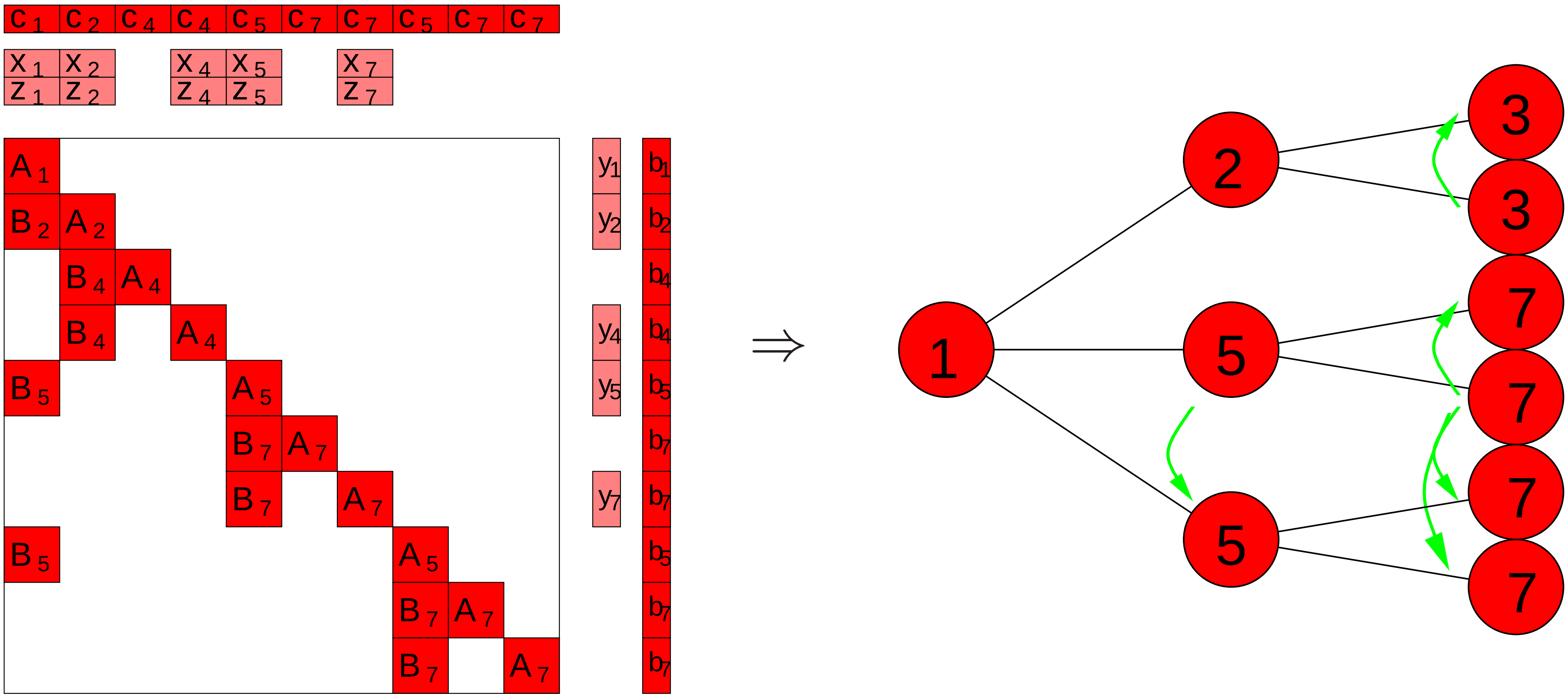


To solve the problem by warmstarting, reverse the process

- Find central point for reduced problem
- Expand the problem to original size

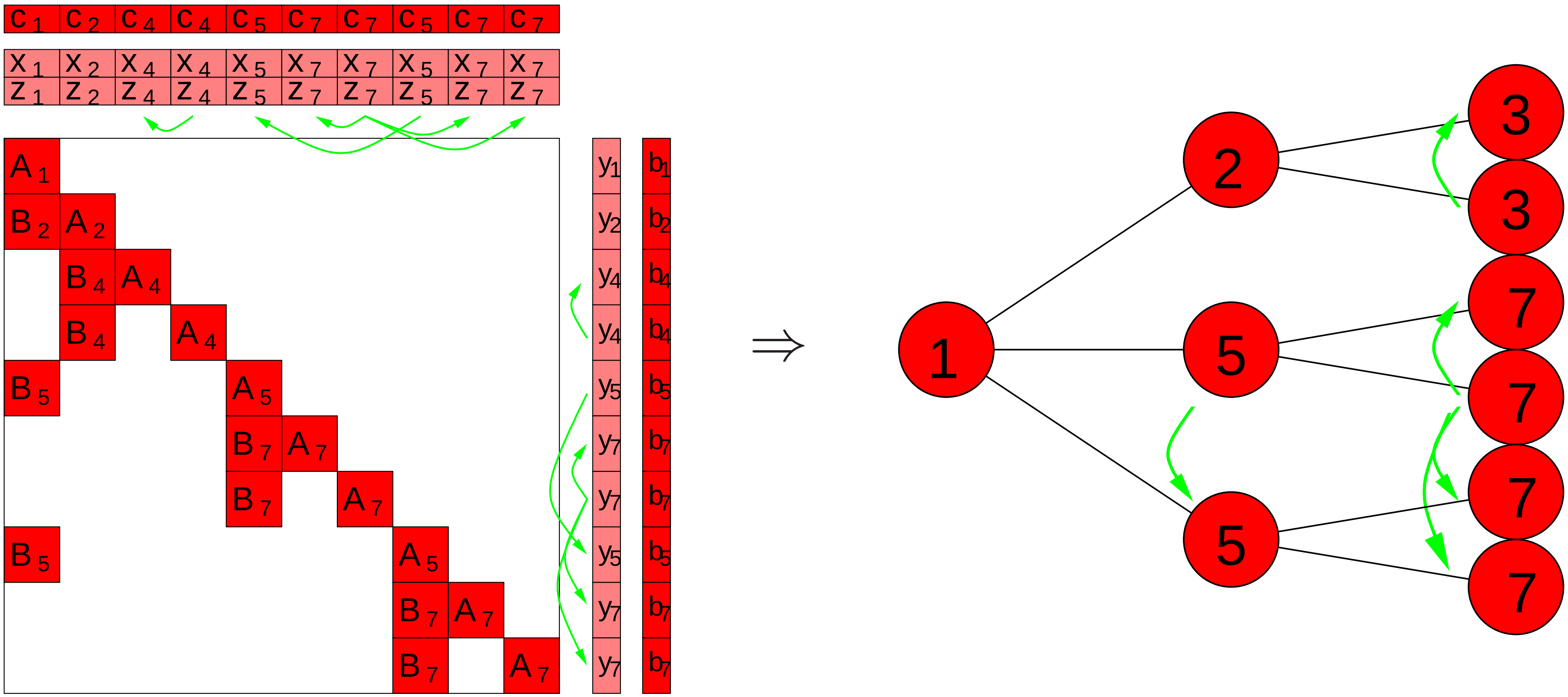
Stochastic Programming Warmstarts



To solve the problem by warmstarting, reverse the process

- Find central point for reduced problem
- Expand the problem to original size (by duplicating scenarios)

Stochastic Programming Warmstarts



To solve the problem by warmstarting, reverse the process

- Find central point for reduced problem
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- Expand solution to primal/dual feasible point for expanded problem

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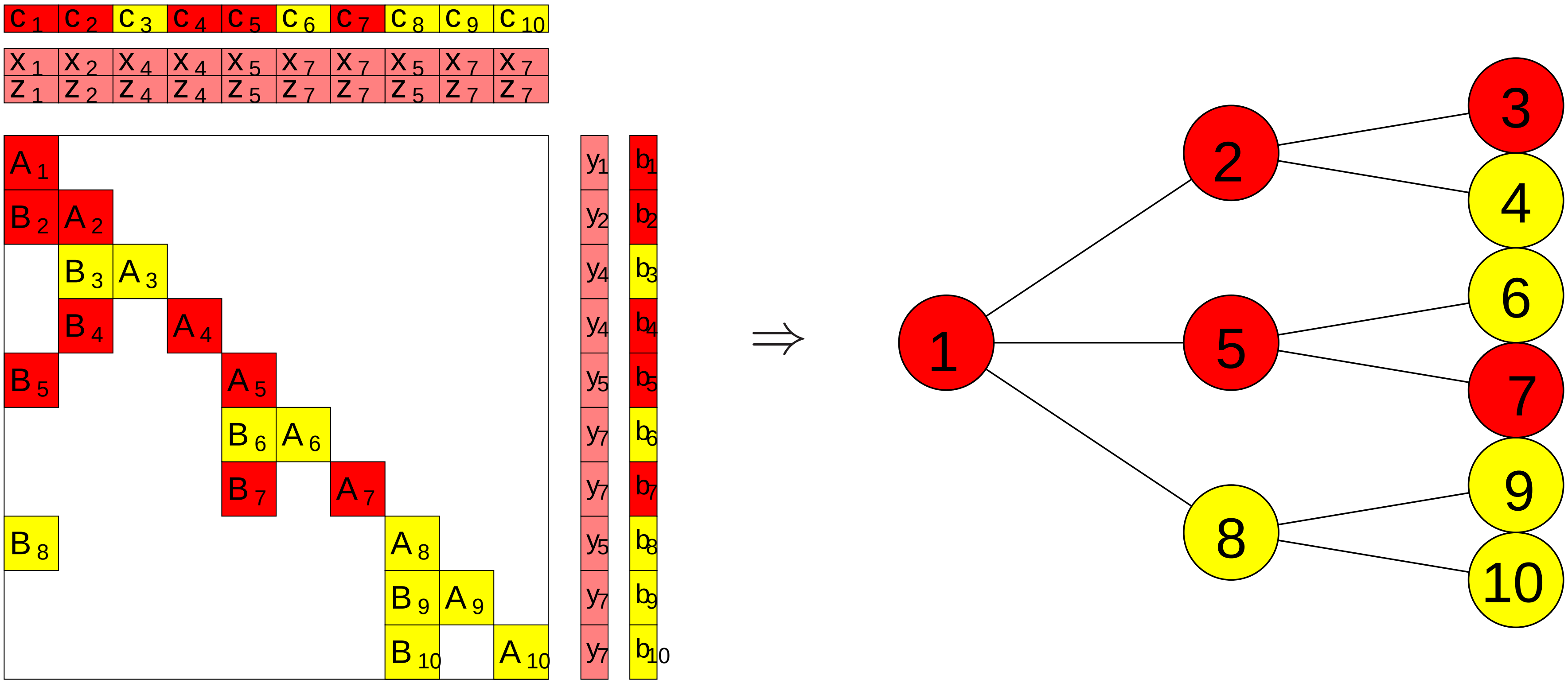
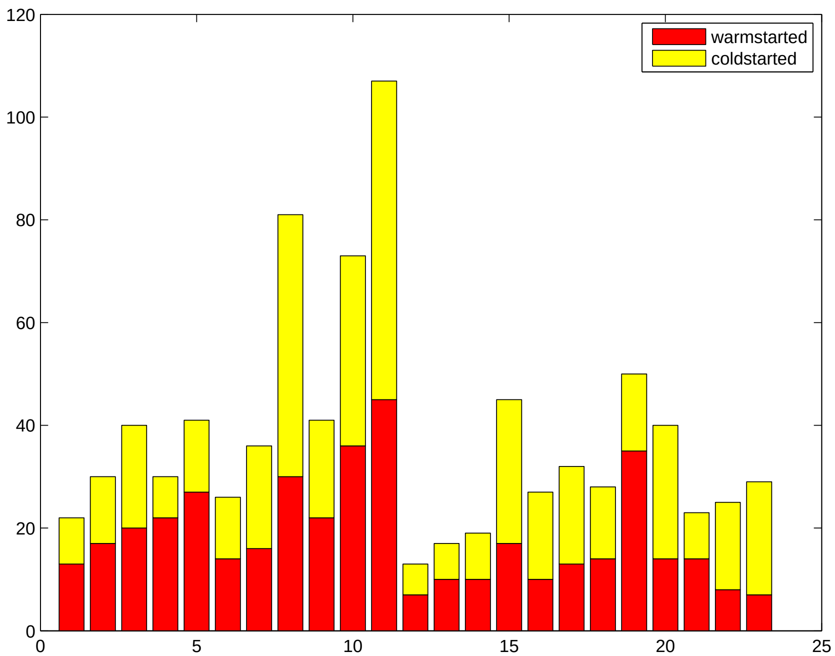


To solve the problem by warmstarting, reverse the process

- Find central point for reduced problem
- Expand the problem to original size (by duplicating scenarios)
- Expand solution to primal/dual feasible point for expanded problem
- Use this to warmstart full problem

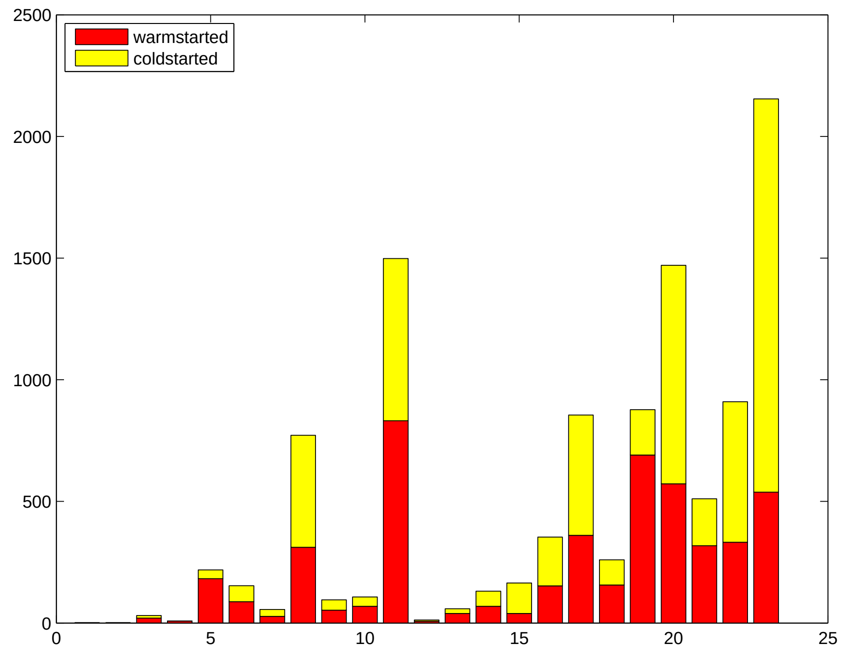
Reduced Tree Warmstart: Results

- SP test problems & Capacity assignment problems
- Ranging from 1,000 - 100,000 variables



Number of IPM iterations

-50.5%



Total solution time (s)

-42.1%

Crash starting Interior Point methods

The aim of a crash-start is to construct (cheaply) a point on (or near) the central path.

$$
\begin{array}{ccc}
\text{primal feasibility} & \text{dual feasibility} & \text{centrality} \\
\begin{aligned} Ax &= b \\ Tx + W_i y_i &= h_i \end{aligned} &
\begin{aligned} \sum_i T^T \lambda_i + s &= c_0 \\ W_i^T \lambda_i + z_i &= p_i c_i \end{aligned} &
\begin{aligned} XSe &= \mu e \\ Y_i Z_i e &= \mu e \\ x, s, y, z &\geq 0 \end{aligned}
\end{array}
$$

Analysis of reduced tree warmstart scheme:

- The tree expansion constructs a primal/dual feasible (not central) point
- Scenario changes destroy primal/dual feasibility.

Is it possible to regain primal-/dual- feasibility & centrality?

Decomposition based Warmstarting Scheme

Idea:

- Use estimate of first stage decisions x, s from reduced tree $\mathcal{T}_R$.
- Given these solve scenario subproblems to get $(y_i, z_i, \lambda_i) \forall i \in \mathcal{T}$.

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Given x, s the conditions for a point on the central path (almost) decompose into

$$\begin{aligned} W_i y_i &= h_i - Tx \\ W_i^T \lambda_i + z_i &= p_i c_i \\ Y_i Z_i e &= \mu e \end{aligned}$$

which are the optimality conditions for the scenario subproblem

$$\begin{array}{ll} \min_{y_i} & p_i c_i^T y_i - \mu \sum_j \ln y_{ij} \\ \text{s.t.} & W_i y_i = h_i - Tx \end{array} \qquad (P_i(x))$$

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Note:

- Disregard the global dual constraint: $\sum_i T^T \lambda_i + s = c_0$.
- Satisfied (approximately) if $\mathcal{T}_R \approx \mathcal{T}$.

Decomposition based Warmstarting Scheme: Algorithm

- Input: Reduced tree $\mathcal{T}_R$, target $\mu : \mu_T$.

1. Solve $P(\mathcal{T}_R)$ to find $(\hat{x}, \hat{s}) := (x_{\mu_T}(\mathcal{T}_R) s_{\mu_T}(\mathcal{T}_R))$.
   as approximation to $(x_{\mu_T}(\mathcal{T}), s_{\mu_T}(\mathcal{T}))$

2. $\forall i$:Solve scenario subproblems $P_i(\hat{x})$
   to get μ_T-centers $(\hat{y}_i, \hat{z}_i, \hat{\lambda}_i) := (y_{\mu,i}, z_{\mu,i}, \lambda_{\mu,i})$.
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- Output: $(\hat{x}, \hat{y}, \hat{\lambda}, \hat{s}, \hat{z})$ as the warmstart point for $P(\mathcal{T})$.

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- Output: $(\hat{x}, \hat{y}, \hat{\lambda}, \hat{s}, \hat{z})$ as the warmstart point for $P(\mathcal{T})$.

Rationale:

- If $\mathcal{T}_R \approx \mathcal{T}$
- then $(\hat{x}, \hat{s}) := (x_{\mu_T}(\mathcal{T}_R)s_{\mu_T}(\mathcal{T}_R)) \approx (x_{\mu_T}(\mathcal{T})s_{\mu_T}(\mathcal{T}))$
- and thus $(\hat{y}, \hat{\lambda}, \hat{z}) \approx (y_{\mu_T}(\mathcal{T})\lambda_{\mu_T}(\mathcal{T}), z_{\mu_T}(\mathcal{T})$

Decomposition based Warmstarting Scheme: Analysis

By construction $(\hat{x}, \hat{y}, \hat{\lambda}, \hat{s}, \hat{z})$ is

- *Central*
- *Primal feasible*

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Dual feasibility depends on residual in $\sum_i T^T \lambda_i + s = c_0$:

$$= \mathcal{O}(\|\hat{\lambda} - \lambda_\mu(\mathcal{T}))\| \stackrel{?}{=} \mathcal{O}(d(\mathcal{T}, \mathcal{T}_R))$$

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Wasserstein distance: a standard measure of distance between trees

$$\mathcal{T} = \{(p_i, \xi_i)\}, \qquad \mathcal{T}_R = \{(\tilde{p}_i, \tilde{\xi}_i\}$$

is the *Wasserstein* or *Transportation* distance:

$$W_1(\mathcal{T}, \mathcal{T}_R) := \min_{\eta \geq 0} \left\{ \sum_{(\pi_i, \xi_i) \in \mathcal{T}} \sum_{(\tilde{\pi}_j, \tilde{\xi}_j) \in \mathcal{T}_R} \|\xi_i - \tilde{\xi}_j\| \eta_{ij} : \sum_i \eta_{ij} = \tilde{\pi}_j, \sum_j \eta_{ij} = \pi_i \right\}.$$

Decomposition based Warmstarting Scheme: Theoretical Results

Given

- $L_h : \|h(\xi) - h(\tilde{\xi})\| \leq L_h \|\xi - \tilde{\xi}\|$
- $\overline{B} : \|x_{\mu_T}(\mathcal{T}_R)\|_\infty \leq B, \qquad \forall \mathcal{T}_R$
- $L_Q = 3\overline{C(d)}(\overline{C(d)\|d\|} + \mu_T n)\chi(W) L_h$
- $C_\lambda = 4\chi(W)\|T\|\overline{C(d)}^2[\overline{C(d)\|d\|} + \mu_T n]^2/\mu_T$

Lemma. We have

$$\|\lambda_\mu(\mathcal{T}) - \hat{\lambda}\|_\infty \leq \frac{2}{\sqrt{\mu_T}} C_\lambda \overline{B} \sqrt{L_Q} \sqrt{W_1(\mathcal{T}, \mathcal{T}_R)}$$

$\Rightarrow$ **"small $W_1(\mathcal{T}, \mathcal{T}_R)$, large μ"**

Decomposition based Warmstarting Scheme: Theoretical Results

Theorem. Let $\hat{w} = (\hat{x}, \hat{y}, \hat{\lambda}, \hat{s}, \hat{z})$ be the warm-start point for problem $P(\mathcal{T})$ obtained by following the above algorithm starting from the reduced tree $\mathcal{T}_R$. If

$$W_1(\mathcal{T}, \mathcal{T}_R) \leq \frac{\theta^2}{\overline{C(d)}^2 C_\lambda^2} \min \left\{ \frac{\overline{\|d\|}^2}{4(2n+1)^2}, \frac{\mu_T^2}{16\overline{C(d)}^2} \right\}$$

then the warmstart is successful, that is the full WLS step is feasible and

$$(\hat{x} + \Delta x, \hat{y} + \Delta y, \hat{\lambda} + \Delta\lambda, \hat{s} + \Delta s, \hat{z} + \Delta z) \in \mathcal{N}_2^{\mathcal{T}}(\theta).$$

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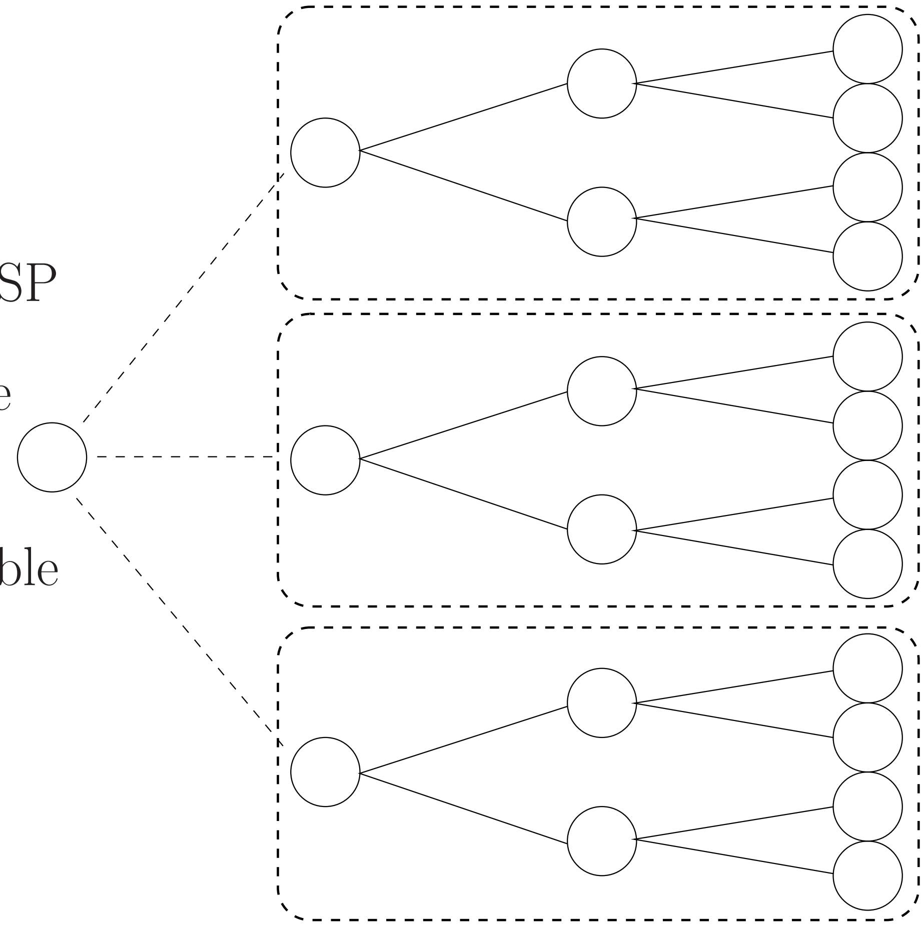
A similar result can be proven if we only require approximate μ_T-centers!

Decomposition based warmstart: Extension to Multistage SP

- Easy to generalize to multistage SP
- Decompose problem at 2nd stage
  $\Rightarrow P_i(x)$ is based on subtree
- Solve each subtree and assemble warmstart point



Test Problem Characteristics

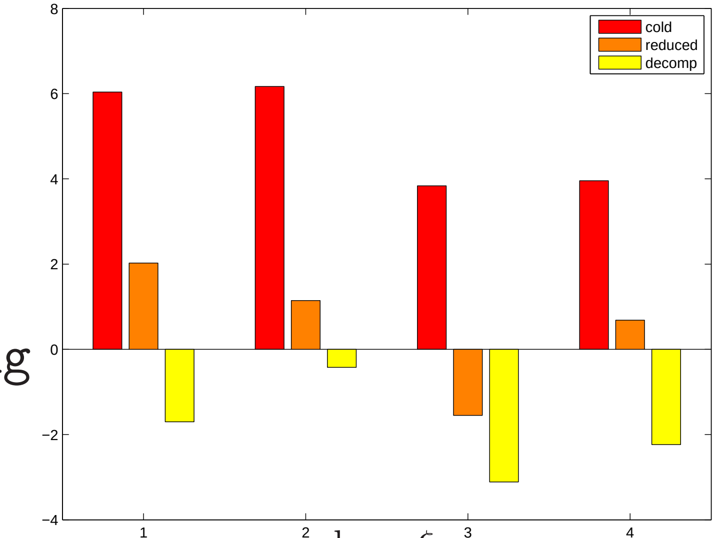
Problem	stages	scenarios	rows	columns	nonzeros
fxm3-6	3	36	6,200	12,628	57,722
fxm3-16	3	256	41,340	85,575	392,252
pltexpA4-6	4	216	26,894	70,364	143,059
swing8-4	8	65,536	262,142	349,522	786,422
mmix-50	2	50	32,951	72,517	366,817
mmix-100	2	100	65,901	145,017	733,617
watson10-64	7	64	15,101	28,097	72,648
watson10-128	8	128	26,237	49,153	128,648
watson10-256	9	256	43,517	82,177	218,888

Numerical Results

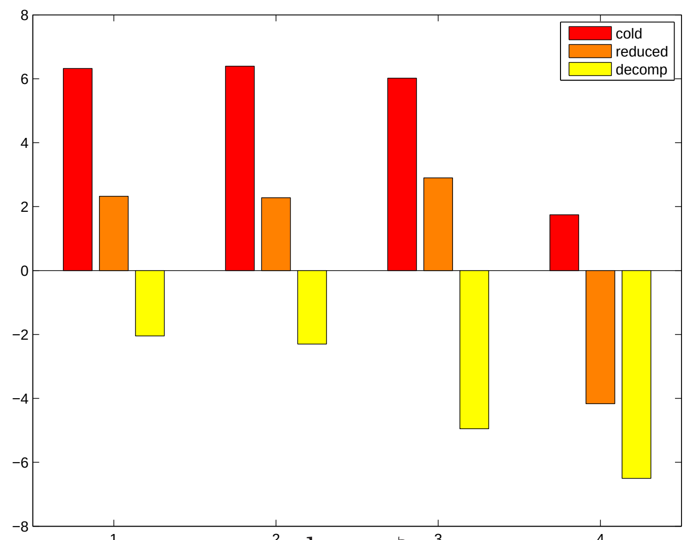
Problem	cold		reduced		decomp	
	iter	time	iter	time	iter	time
fxm3-6	24	3.7	9	1.4	11	3.9
fxm3-16	62	46.2	21	18.7	19	37.0
pltexpA4-6	68	30.3	-	-	40	23.0
swing8-4	35	69.8	66	114.9	14	72.9
mmix-50	31	11.5	25	8.7	10	18.6
mmix-100	35	27.3	48	29.1	13	8.3
watson10-64	58	11.8	89	15.8	6	9.2
watson10-128	74	26.1	106	34.2	15	20.8
watson10-256	85	51.6	-	-	48	56.7

Residuals

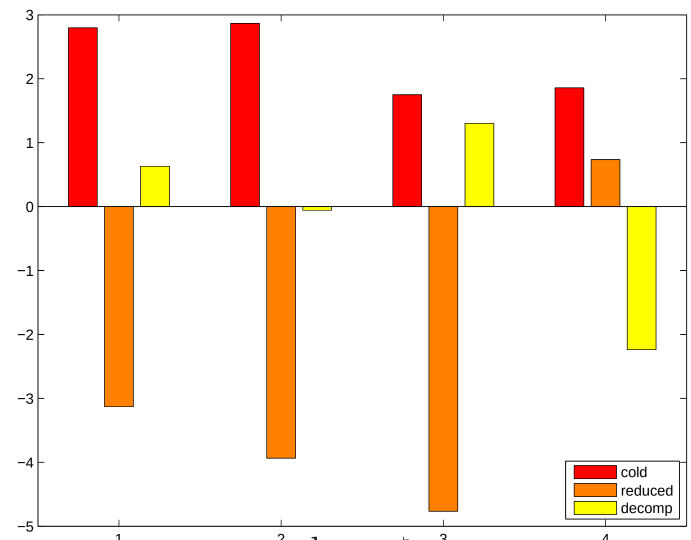
- Plotted are $\log_{10}$ of ξ_μ, ξ_b, ξ_c
- Problems: `fxm6, fxm16, pltexp, swing`



$\log_{10} \xi_u$

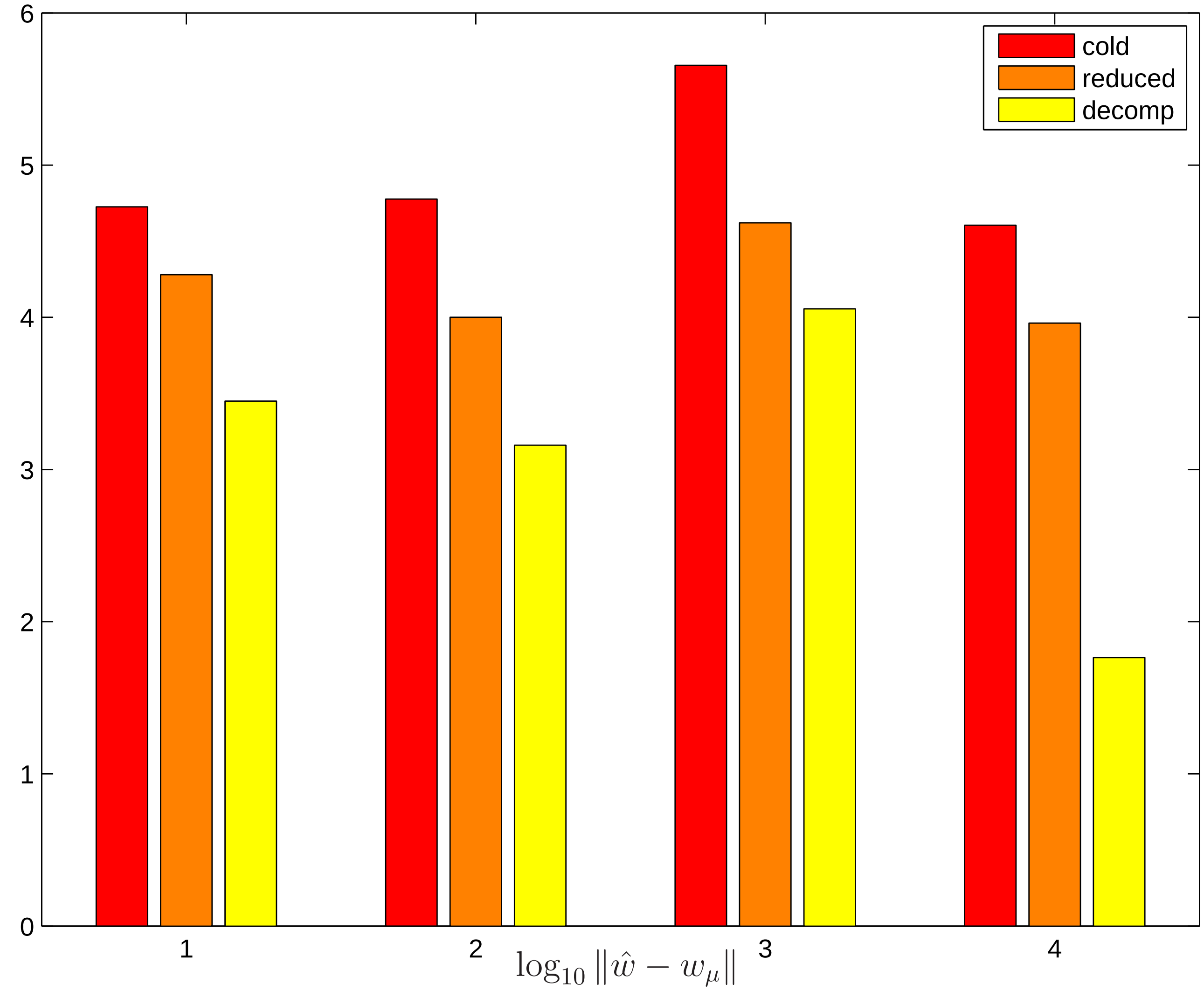


$\log_{10} \xi_b$



$\log_{10} \xi_c$

Distance from central path



Possible Efficiency Gains

- Scenario subproblems can be warmstarted from reduced tree solution
- Only resolve scenario subproblems for which reduced tree solution has large residuals
  $\Rightarrow$ Hybrid Reduced Tree/Decomposition warmstart.

Conclusions

- IPM **can** be warmstarted
- Can exploit stochastic programming structure to significantly speed-up solution.
- Decomposition based warmstart shows potential for further time-savings.

Future Work:

- Explore Hybrid-Scheme
- Warmstart scenario subproblems

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`http://www.maths.ed.ac.uk/ERGO/preprints.html`