MILP islanding of power networks by bus splitting

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***Abstract*—A mathematical formulation for the islanding of power networks is presented. Given an area of uncertainty in the network, the proposed approach uses mixed integer linear programming to isolate unhealthy components of the network and create islands, while maximizing load supply. Rather than disconnecting transmission lines, the new method splits the network at its nodes, which are modelled as busbars with switches between lines, generators and loads. DC power flow equations and network constraints are explicitly included in the MILP problem, resulting in balanced, steady-state feasible islands. Numerical simulations on the IEEE 14-bus test network demonstrate the effectiveness of the approach.**

I. Introduction

In recent years, there has been a number of occurrences of wide-area blackouts of power networks. For example, 2003 saw separate blackouts in Italy [1], Sweden/Denmark [2] and USA/Canada [3], affecting millions of customers. The wide-area disturbance in 2006 to the UCTE system caused the system to split in an uncontrollable way [4], forming three islands. While the exact causes of wide-area blackouts differ from case to case, some common driving factors emerge. Modern power systems are being operated closer to limits; liberalization of the markets, and the subsequent increased commercial pressures, has led to a reduction in security margins [5]–[7]. A more recently occurring factor is increased penetration of variable distributed generation, notably from wind power, which brings significant challenges to secure system operation [8].

For several large disturbance events, studies have shown that wide-area blackout could have been prevented by intentionally splitting the system into islands [9]. By isolating the faulty part of the network, the total load disconnected in the event of a cascading failure is reduced. *Controlled* islanding or system splitting is therefore attracting an increasing amount of attention. The problem is how to efficiently split the network into 'viable' islands. Motives for splitting range from islands balanced in load and generation to electro-mechanically stable islands. For example, Sun et al. [10] use ordered binary decision diagrams (OBDDs) to determine sets of balanced islands, while several authors propose that islands be formed around coherent [11]–[13] or controlling [14] groups of generators.

Regardless of motive, splitting is a considerable challenge, since the search space of line cutsets grows combinatorially with network size, and is exacerbated by the requirement for strategies that obey non-linear power flow equations and satisfy operating constraints. Approaches include exhaustive search [11], minimal-flow minimal-cutset determination using breadth-/depth-first search [12], heuristic methods [14], graph simplification and partitioning [10], [13], and power flow tracing [15].

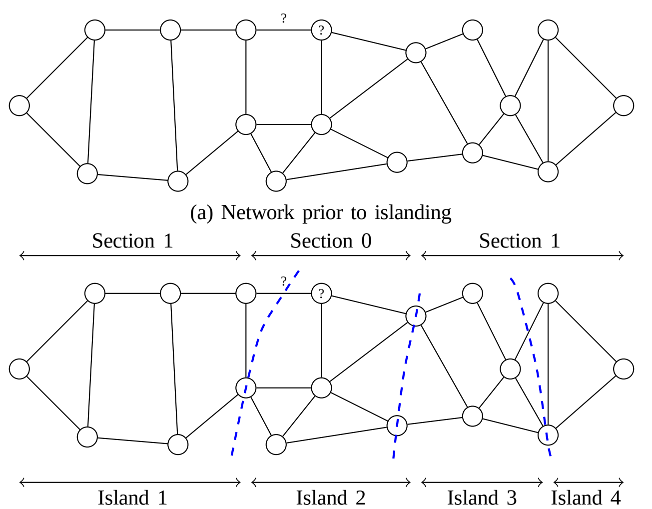
In a recent paper [16], we proposed an optimization-based approach to system islanding and load shedding. Given some uncertain or unhealthy parts of the network, the aim is to isolate—by cutting lines—these parts of the network while minimizing the load shed or at risk. An advantage of this approach is that islanding is in response to specific contingencies, rather than along pre-determined boundaries, so the island containing the impacted area need not be too large. In common with the optimal transmission switching technique of Fisher et el. [17], binary variables represent switches that open or close each line. Solving a MILP optimization determines the optimal set of lines to cut and which loads to shed. Thus, optimal islanding may be viewed as an extension of optimal transmission switching or network topology optimization [18]. Any islands created are balanced, and satisfy DC power flow equations and operating constraints.

In this paper, we propose a new *bus splitting* approach to system islanding. The premise for islanding is the same as that outlined above and in [16]. When partitioning the network, however, we may either disconnect lines or divide the *nodes* of the network by opening switches between busbars. In the latter case, we switch network components—generators, loads and lines—between busbars to obtain an optimal configuration. Busbar switching or splitting as a method of transmission switching has been proposed before, but always in the context of corrective control of flows [19], [20] or voltages [21]. The advantages of bus switching are significant; being an operational action, it can be executed quickly and re-route flows in a short time, with minimal disturbances, while incurring no extra economic cost [21]. In terms of islanding, allowing system splitting via the nodes enlarges the set of feasible islanding solutions. We show that by splitting the network in this way, less load may be required to be shed or lost. Furthermore, although the search space grows combinatorially with the number of extra binary decision variables, we propose cuts and constraints that reduce symmetry, thus shortening computation time.

The organization of this paper is as follows. The next section outlines the motivation and assumptions that underpin the approach. The islanding formulation is developed in Section III, and extra symmetry-breaking constraints and cuts are proposed in Section IV. In Section V, preliminary numerical simulations are presented. Finally, conclusions are drawn in Section VI.

II. Motivation

Following some failure, we assume that limited information is available about the network and its exact state is uncertain; there are parts of the network that are suspected of having a fault and some where we are reasonably sure have no faults. We assume that in such a case, a robust solution to prevent cascading failures is to isolate the uncertain part of the network from the certain part, by forming one or more stable islands. Fig. 1(a) depicts such a situation for a fictional network; uncertain lines and buses are indicated.



(a) Network prior to islanding

(b) Network post islanding

Fig. 1. (a) Fictional network with an uncertain bus, and (b) the islanding of that network by splitting buses and disconnecting lines.

Our aim is to split the network into disconnected sections so that the possible faults are all in one section. It is desirable that this section be small, since it may be prone to failure, and that the other section is able to operate with little load shedding. We would also like the problem section to shed as little load as possible. Fig. 1(b) shows a possible islanding solution for this network, where all uncertain buses have been placed in a section 0 by splitting nearby buses, and uncertain lines with an end in section 1 have been cut. We make the following distinction between *sections* and *islands*.

- The optimized network consists of two sections, an "unhealthy" section 0 and a "healthy" section 1. No lines connect the two sections. On the other hand, neither section is required to be a single, connected component.
- An island is a connected component of the network.

Thus, either section may contain a number of islands, as in fig. 1(b), where section 1 comprises islands 1, 3 and 4, while section 0 is a single island. Islands are formed by a combination of splitting buses and disconnecting lines. The boundaries of sections and the number of islands formed will depend on the optimization.

We will assume that generator outputs and load levels immediately after the initial fault are known. We have central control of generation, load shedding and switches and breakers; we may instantaneously reduce the demand and open or close switches and breakers. Furthermore, we assume that we have a certain degree of control over a generator's output. We require that after the adjustments the system is a feasible equilibrium.

III. MILP Formulation

This section describes the islanding formulation. The arrangement of the busbars at a bus is first described, and constraints are developed to switch between configurations and direct the power flows. Operating constraints and sectioning constraints that split the network to isolate the unhealthy parts are subsequently presented.

Consider a network that comprises a set of buses $\mathcal{B} = \{1, 2, \ldots, n^{\mathrm{B}}\}$ and a set of lines $\mathcal{L} = \{1, 2, \ldots, n^{\mathrm{L}}\}$. The vectors F and T describe the connection topology of the network: a line $l \in \mathcal{L}$ connects bus F_l to T_l. We assume there also exists a set of generators $\mathcal{G} = \{1, 2, \ldots, n^{\mathrm{G}}\}$ and a set of loads $\mathcal{D} = \{1, 2, \ldots, n^{\mathrm{D}}\}$. The sets $\mathcal{BG}$ and $\mathcal{BD}$, indexed by (b, g) and (b, d) respectively, describe the sets of generators and demands connected to each bus.

A. Connection and flow constraints

1) Busbar connections: The bus configuration is shown in fig. 2 and described as follows. Each bus $b \in \mathcal{B}$ is assumed to comprise two busbars. A switch $\eta^{\mathrm{B}}_b \in \{0, 1\}$ connects or disconnects the two busbars; $\eta^{\mathrm{B}}_b = 1$ means that the switch is closed and the busbars are connected. Connected components—lines, generators and loads—are shown also; each may be connected to either of the busbars by means of further switches.

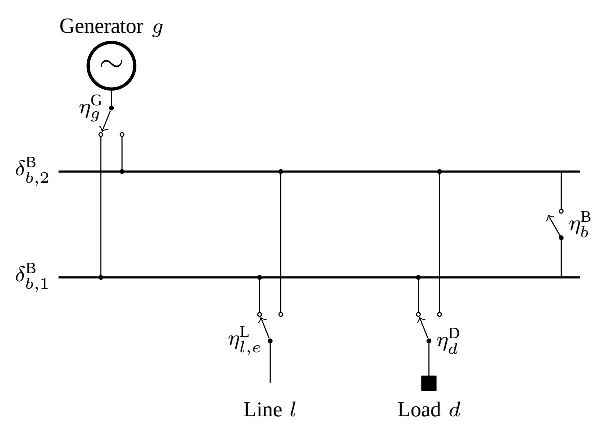


Fig. 2. Busbar configuration for bus b, and notation.

With each busbar is associated a voltage phase angle, thus $\delta^{\mathrm{B}}_{b,1}$ and $\delta^{\mathrm{B}}_{b,2}$. If the interconnecting switch is closed then these must be equal; otherwise they may differ.

$$-\Delta^{+}(1 - \eta^{\mathrm{B}}_b) \leq \delta^{\mathrm{B}}_{b,1} - \delta^{\mathrm{B}}_{b,2} \leq \Delta^{+}(1 - \eta^{\mathrm{B}}_b), \forall b \in \mathcal{B}, \quad (1)$$

where Δ^+ is a sufficiently large number. In addition, real power may flow between the two busbars only if the connecting switch is closed.

$$-P^{\mathrm{B}+}\eta_b^{\mathrm{B}} \le p_b^{\mathrm{B}} \le P^{\mathrm{B}+}\eta_b^{\mathrm{B}}, \forall b \in \mathcal{B}. \tag{2}$$

where $P^{\mathrm{B}+}$ is a sufficiently large number. A positive p_b^{B} represents a real power flow from busbar 1 to busbar 2.

2) Generator and load connections to busbars: We assume that any single generator $g \in \mathcal{G}$ or single load $d \in \mathcal{D}$ is connected to only one bus. Then, for connecting these components to one of the two busbars via the switches shown, we introduce binary variables η_g^{G} and η_d^{D} for each $g \in \mathcal{G}$ and $d \in \mathcal{D}$. If $\eta_g^{\mathrm{G}} = 1$ ($\eta_d^{\mathrm{D}} = 1$) then generator g (load d) is connected to busbar 1 at its bus b, and otherwise it is connected to busbar 2.

Now consider the power flows to and from busbars. The output p_g^{G} of generator $g \in \mathcal{G}$ is the sum of the individual flows onto busbars 1 and 2, of which only one can be non-zero. Suppose the maximum possible real power output of a generator $g \in \mathcal{G}$, after islanding, is $P_g^{\mathrm{G}+}$. Then, for all $g \in \mathcal{G}$,

$$0 \le p_{g,1}^{\mathrm{G}} \le P_g^{\mathrm{G}+}\eta_g^{\mathrm{G}}, \tag{3a}$$
$$0 \le p_{g,2}^{\mathrm{G}} \le P_g^{\mathrm{G}+}(1-\eta_g^{\mathrm{G}}) \tag{3b}$$
$$p_g^{\mathrm{G}} = p_{g,1}^{\mathrm{G}} + p_{g,2}^{\mathrm{G}}. \tag{3c}$$

where $p_{g,1}^{\mathrm{G}}$ is the flow on to busbar 1 of the bus and $p_{g,2}^{\mathrm{G}}$ is the flow on to busbar 2.

Demands are similarly treated. For a load $d \in \mathcal{D}$ with real power demand P_d^{D} supplied with $p_d^{\mathrm{D}} \le P_d^{\mathrm{D}}$,

$$0 \le p_{d,1}^{\mathrm{D}} \le P_d^{\mathrm{D}}\eta_d^{D}, \tag{4a}$$
$$0 \le p_{d,2}^{\mathrm{D}} \le P_d^{\mathrm{D}}(1-\eta_d^{\mathrm{D}}), \tag{4b}$$
$$p_d^{\mathrm{D}} = p_{d,1}^{\mathrm{D}} + p_{d,2}^{\mathrm{D}}. \tag{4c}$$

3) Line connections to busbars: At each end of a line, a switch exists at the bus to connect the line to one of the two busbars. This requires two binary variables for each line, one for each end. A third binary variable, ρ_l, is used to break the line completely, if desired. The arrangement is shown in fig. 3.

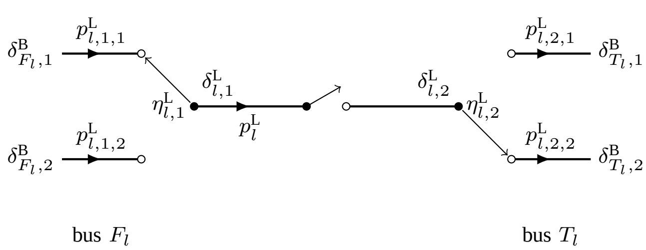


Fig. 3. Line between buses F_l and T_l. The switches at each end (controlled by $\eta_{l,1}^{\mathrm{L}}$ and $\eta_{l,2}^{\mathrm{L}}$) connect the line to one of two busbars. The line disconnection switch, controlled by ρ_l, allows the line to be broken.

Supposing we denote the 'from' end (bus F_l) as 1 and the 'to' end (bus T_l) as 2, a line l has two binary variables, $\eta_{l,1}^{\mathrm{L}}$ and $\eta_{l,2}^{\mathrm{L}}$. A positive power flow through the line corresponds to a flow from bus F_l to bus T_l. At each end, the real power flow is the sum of the real power flows from/to each busbar, as was done for the generators, and is limited by $P_l^{\mathrm{L}+}$, the maximum possible real power flow through the line. Power flow switching constraints at each end $e \in \{1,2\}$ of the line $l \in \mathcal{L}$ are then

$$-P_l^{\mathrm{L}+}\eta_{l,e}^{\mathrm{L}} \le p_{l,e,1}^{\mathrm{L}} \le P_l^{\mathrm{L}+}\eta_{l,e}^{\mathrm{L}}, \tag{5a}$$
$$-P_l^{\mathrm{L}+}(1-\eta_{l,e}^{\mathrm{L}}) \le p_{l,e,2}^{\mathrm{L}} \le P_l^{\mathrm{L}+}(1-\eta_{l,e}^{\mathrm{L}}), \tag{5b}$$
$$p_l^{\mathrm{L}} = p_{l,e,1}^{\mathrm{L}} + p_{l,e,2}^{\mathrm{L}}. \tag{5c}$$

The real power flow through the line depends on the phase angle difference across it. We have the phase angle at each of the busbars—$\delta_{b,1}^{\mathrm{B}}$ and $\delta_{b,2}^{\mathrm{B}}$ for a bus b—but the phase angle that the line will adopt depends on the switch $\eta_{l,e}^{\mathrm{L}}$. Thus, define line-end phase angles $\delta_{l,e}^{\mathrm{L}}$, with $e \in \{1,2\}$, for a line l. Then these line-end phase angles are connected to bus phase angles by the constraints.

$$-\Delta^+(1-\eta_{l,1}^{\mathrm{L}}) \le \delta_{l,1}^{\mathrm{L}} - \delta_{F_l,1}^{\mathrm{B}} \le \Delta^+(1-\eta_{l,1}^{\mathrm{L}}), \tag{6a}$$
$$-\Delta^+\eta_{l,1}^{\mathrm{L}} \le \delta_{l,1}^{\mathrm{L}} - \delta_{F_l,2}^{\mathrm{B}} \le \Delta^+\eta_{l,1}^{\mathrm{L}}, \tag{6b}$$
$$-\Delta^+(1-\eta_{l,2}^{\mathrm{L}}) \le \delta_{l,2}^{\mathrm{L}} - \delta_{T_l,1}^{\mathrm{B}} \le \Delta^+(1-\eta_{l,2}^{\mathrm{L}}), \tag{6c}$$
$$-\Delta^+\eta_{l,2}^{\mathrm{L}} \le \delta_{l,2}^{\mathrm{L}} - \delta_{T_l,2}^{\mathrm{B}} \le \Delta^+\eta_{l,2}^{\mathrm{L}}. \tag{6d}$$

That is, if at the 'from' end ($e = 1$) the switch $\eta_{l,e}^{\mathrm{L}} = 1$ then $\delta_{l,e}^{\mathrm{L}}$ will be equal to the $\delta_{F_l,1}^{\mathrm{B}}$ of busbar 1; otherwise, $\delta_{l,e}^{\mathrm{L}}$ is equal to the $\delta_{F_l,2}^{\mathrm{B}}$ of busbar 2. The same constraints are applied at the 'to' end ($e = 2$), which is connected to bus T_l.

4) DC power flow—Kirchhoff's voltage law: When a line l is connected, Kirchhoff's voltage law (KVL) demands that a flow of real power is established depending only on the difference in phase angle across the line. However, we may not equate p_l^{L} directly to this flow, since if a line is disconnected by the optimization, zero power will flow through that line. In this case, we must allow different phase angles at each end of the line. To achieve this, the KVL expression is equated to a variable $\hat{p}_l^{\mathrm{L}}$.

$$\hat{p}_l^{\mathrm{L}} = \frac{-B_l^{\mathrm{L}}}{\tau_l}\left(\delta_{l,1}^{\mathrm{L}} - \delta_{l,2}^{\mathrm{L}}\right). \tag{7}$$

where constants B_l^{L}, τ_l are, respectively, the susceptance and off-nominal turns ratio of line l. Then, when line l is connected we will set $p_l^{\mathrm{L}} = \hat{p}_l^{\mathrm{L}}$, and when l is disconnected $p_l^{\mathrm{L}} = 0$. We model this as follows.

Assume the maximum possible magnitude of real power flow through a line l is $P_l^{\mathrm{L}+}$. Then

$$-\rho_l P_l^{\mathrm{L}+} \le p_l^{\mathrm{L}} \le P_l^{\mathrm{L}+}\rho_l, \tag{8a}$$
$$-(1-\rho_l)\hat{P}_l^{\mathrm{L}+} \le \hat{p}_l^{\mathrm{L}} - p_l^{\mathrm{L}} \le \hat{P}_l^{\mathrm{L}+}(1-\rho_l). \tag{8b}$$

When the sectioning constraints set a particular $\rho_l = 0$, then $p_l^{\mathrm{L}} = 0$ but $\hat{p}_l^{\mathrm{L}}$ may take whatever value is necessary to satisfy the KVL constraint (7). Conversely, if $\rho_l = 1$ then $p_l^{\mathrm{L}} = \hat{p}_l^{\mathrm{L}}$. Note that at the very minimum $\hat{P}_l^{\mathrm{L}+} \ge P_l^{\mathrm{L}+}$, but these limits should be of large enough to allow two buses across a disconnected line to maintain sufficiently different phase angles.

5) Kirchhoff's current law: All flows must balance at each busbar. For all $b \in \mathcal{B}, i \in \{1,2\}$:

$$\sum_{(b,g)\in\mathcal{BG}} p^{\mathrm{G}}_{g,i} = \sum_{(b,d)\in\mathcal{BD}} p^{\mathrm{D}}_{d,i} + \sum_{l\in\mathcal{L}:F_l=b} p^{\mathrm{L}}_{l,1,i} - \sum_{l\in\mathcal{L}:T_l=b} p^{\mathrm{L}}_{l,2,i} - (-1)^i p^{\mathrm{B}}_b. \quad (9)$$

The final term is the busbar-to-busbar flow p^{B}_b, a positive value of which flows from busbar $i = 1$ to busbar $i = 2$ of bus b.

B. Operating constraints

1) Generator outputs: In situations where there is a need to react quickly to an unplanned contingency, to prevent cascading failures the time available to island the network and adjust loads and generators will be short. Therefore, we must assume that full re-scheduling of generators and/or the addition of new units to the network will not be possible. On the other hand, a certain amount of spinning reserve will be available in the network for small-scale changes. For any unit, we will assume that a new set-point, close to the current operating point, may be commanded. This set-point should be reachable within a short time period, and also must not violate limits. In practice, fast governor action will quickly raise/lower real power output to the new set-point, before the spinning reserve takes over.

A further assumption we make is that a generator obeys a binary regime: either it operates near its previous real power output, or it may have its output switched to zero. That is,

$$p^{\mathrm{G}}_g \in \left[P^{\mathrm{G}-}_g, P^{\mathrm{G}+}_g\right] \cup \{0\}.$$

This latter case models the removal of the source of mechanical input power; it is assumed that electrical power will fall to zero within the time-frame of islanding. Although the switched-off generating unit contributes no power in steady state to the network, it remains electrically connected to the network.

To model this disjoint set constraint, we introduce a binary variable $\zeta_d \in \{0,1\}$ for each generator.

$$\zeta_g P^{\mathrm{G}-}_g \le p^{\mathrm{G}}_g \le \zeta_g P^{\mathrm{G}+}_g, \quad (10)$$

for all $g \in \mathcal{G}$. If $\zeta_g = 0$ then generator g is switched off; otherwise it outputs $p^{\mathrm{G}}_g \in \left[P^{\mathrm{G}-}_g, P^{\mathrm{G}+}_g\right]$. We may protect any generator g from switch-off by assigning it to a set $\mathcal{G}^1 \subseteq \mathcal{G}$ and including the constraint

$$\zeta_g = 1, \forall g \in \mathcal{G}^1. \quad (11)$$

2) Load shedding: Following separation of the network into islands, and given the limits on generator power outputs, it follows that it may not be possible to fully supply all loads. However, the optimization is to determine a feasible steady-state for the islanded network, and thus it is necessary to permit some shedding of loads.

Suppose that a load $d \in \mathcal{D}$ has a constant real power demand P^{D}_d. We assume this load may be reduced by disconnecting a proportion $1 - \alpha_d$. For all $d \in \mathcal{D}$:

$$p^{\mathrm{D}}_d = \alpha_d P^{\mathrm{D}}_d, \quad (12)$$

where $0 \le \alpha_d \le 1$. In determining a feasible islanded network, it is in our interests to promote full load supply, and so load shedding is minimized in the objective function.

3) Line limits: Line limits $P^{\mathrm{L}+}_l$ may be expressed either directly as MW ratings on real power for each line, using (8), or as a limit on the phase angle difference across a line. Since in the model the real power through a line is just a simple scaling of the phase difference across it, then any phase angle limit may be expressed as a corresponding MW limit.

C. Sectioning constraints

We aim to allocate buses and lines into the two sections 0 and 1. We suspect that some subset $\mathcal{B}^0 \subseteq \mathcal{B}$ of buses and some subset $\mathcal{L}^0 \subseteq \mathcal{L}$ of lines have a possible fault. These subsets thus contain all "uncertain" buses and lines, while the remainder of buses/lines are defined as "certain". It is the uncertain components that we wish to confine to section 0. The constraints developed in the sequel achieve this by forcing busbar splits and line disconnections.

1) Bus assignment: In [16], we introduced a binary decision variable γ_b for each bus $b \in \mathcal{B}$; γ_b is set equal to 0 if b is placed in section 0 and $\gamma_b = 1$ otherwise. With the bus-splitting formulation, we may now place the two busbars at a bus in different sections, thus we define two binary variables for each bus, $\gamma^{\mathrm{B}}_{b,1}$ and $\gamma^{\mathrm{B}}_{b,2}$.

Constraints (13) set the values of $\gamma^{\mathrm{B}}_{b,i}$ for a bus b depending on what section that bus was assigned to. We define $\mathcal{B}^1$ to be the set of buses that are desired to remain in section 1. It may be that we wish to exclude buses from the "unhealthy" section, and such an assignment will in general reduce computation time. If any bus is assigned to the sets $\mathcal{B}^0$ or $\mathcal{B}^1$ then both busbars at that bus will lie in the same section.

$$\gamma^{\mathrm{B}}_{b,i} = 0, \forall i \in \{1,2\}, b \in \mathcal{B}^0, \quad (13a)$$

$$\gamma^{\mathrm{B}}_{b,i} = 1, \forall i \in \{1,2\}, b \in \mathcal{B}^1. \quad (13b)$$

Constraints (14) apply to all buses not assigned to $\mathcal{B}^0$ or $\mathcal{B}^1$, and state that if the two busbars at a bus b are placed in different sections then the interconnection between them must be opened. For all $b \in \mathcal{B} \setminus (\mathcal{B}^0 \cup \mathcal{B}^1)$,

$$\eta^{\mathrm{B}}_b \le 1 + \gamma^{\mathrm{B}}_{b,1} - \gamma^{\mathrm{B}}_{b,2}, \quad (14a)$$

$$\eta^{\mathrm{B}}_b \le 1 - \gamma^{\mathrm{B}}_{b,1} + \gamma^{\mathrm{B}}_{b,2}. \quad (14b)$$

2) Line disconnection: We must disconnect a line l (by setting $\rho_l = 0$) if its two ends lie in different sections. However, an end of a line l may be switched between the two busbars, as we saw in the previous section. Thus, define variables $\gamma^{\mathrm{L}}_{l,e} \in \{0,1\}$, for the 'from' and 'to' ends, $e \in \{1,2\}$, of each line $l \in \mathcal{L}$, such that

$$\gamma^{\mathrm{L}}_{l,1} \Leftrightarrow \eta^{\mathrm{L}}_{l,1}\gamma^{\mathrm{B}}_{F_l,1} + \overline{\eta^{\mathrm{L}}_{l,1}}\gamma^{\mathrm{B}}_{F_l,2},$$

$$\gamma^{\mathrm{L}}_{l,2} \Leftrightarrow \eta^{\mathrm{L}}_{l,2}\gamma^{\mathrm{B}}_{T_l,1} + \overline{\eta^{\mathrm{L}}_{l,2}}\gamma^{\mathrm{B}}_{T_l,2}.$$

where the over-bar denotes logical 'not'. These may be reformulated as the following linear constraints

$$\gamma^{\mathrm{B}}_{b,1} + \eta^{\mathrm{L}}_{l,e} - 1 \le \gamma^{\mathrm{L}}_{l,e} \le 1 + \gamma^{\mathrm{B}}_{b,1} - \eta^{\mathrm{L}}_{l,e}, \tag{15a}$$

$$\gamma^{\mathrm{B}}_{b,2} - \eta^{\mathrm{L}}_{l,e} \le \gamma^{\mathrm{L}}_{l,e} \le \gamma^{\mathrm{B}}_{b,2} + \eta^{\mathrm{L}}_{l,e}, \tag{15b}$$

for all $e \in \{1,2\}, l \in \mathcal{L}$, and where $b = F_l$ if $e = 1$ and $b = T_l$ if $e = 2$. These constraints force $\gamma^{\mathrm{L}}_{l,e}$ at end e of line l to take on the value of either $\gamma^{\mathrm{B}}_{b,1}$ or $\gamma^{\mathrm{B}}_{b,2}$ depending on whether $\eta^{\mathrm{L}}_{l,e}$ is 1 or 0.

Subsequently, lines are disconnected in the following way. Any line l not assigned to $\mathcal{L}^0$ is disconnected if its two ends lie in different sections, as indicated by non-equal values of $\gamma^{\mathrm{L}}_{l,1}$ and $\gamma^{\mathrm{L}}_{l,2}$. For all $l \in \mathcal{L}\backslash\mathcal{L}^0$,

$$\rho_l \le 1 + \gamma^{\mathrm{L}}_{l,1} - \gamma^{\mathrm{L}}_{l,2}, \tag{16a}$$

$$\rho_l \le 1 - \gamma^{\mathrm{L}}_{l,1} + \gamma^{\mathrm{L}}_{l,2}. \tag{16b}$$

Secondly, any line assigned to $\mathcal{L}^0$ is disconnected if at least one of its ends is in section 1. For all $l \in \mathcal{L}^0$,

$$\rho_l \le 1 - \gamma^{\mathrm{L}}_{l,1}, \tag{17a}$$

$$\rho_l \le 1 - \gamma^{\mathrm{L}}_{l,2}. \tag{17b}$$

Aside from constraints (16) and (17), the decision of whether to cut a line that lies wholly within a section is free. Although research has shown that disconnecting lines in an intact network can lower generation cost or increase load supply [17], line disconnections in addition to those necessary to create islands may be undesirable in terms of security. However, we may not simply limit the total number of disconnections, since we do not know, *a priori*, how many line cuts are required to create islands. Instead, the following constraints, when included, prohibit the disconnection of any line not assigned to $\mathcal{L}^0$, and whose both ends lie within the same section.

$$\rho_l \ge 1 - \gamma^{\mathrm{L}}_{l,1} - \gamma^{\mathrm{L}}_{l,2}, \tag{18a}$$

$$\rho_l \ge -1 + \gamma^{\mathrm{L}}_{l,1} + \gamma^{\mathrm{L}}_{l,2}, \tag{18b}$$

for all $l \in \mathcal{L}\backslash\mathcal{L}^0$. Alternatively, the number of such disconnections may be limited to within some number n^{cuts}. Introduce a binary variable ρ^{X}_l for each line l. Then (18) is modified to

$$\rho_l + \rho^{\mathrm{X}}_l \ge 1 - \gamma^{\mathrm{L}}_{l,1} - \gamma^{\mathrm{L}}_{l,2}, \tag{19a}$$

$$\rho_l + \rho^{\mathrm{X}}_l \ge -1 + \gamma^{\mathrm{L}}_{l,1} + \gamma^{\mathrm{L}}_{l,2}, \tag{19b}$$

for all $l \in \mathcal{L}\backslash\mathcal{L}^0$, and with the additional constraint

$$\sum_{l \in \mathcal{L}\backslash\mathcal{L}^0} \rho^{\mathrm{X}}_l \le n^{\mathrm{cuts}}. \tag{19c}$$

3) Load placement: A load will be placed in either section 0 or section 1 depending on the placement of the busbar to which it is connected. In a way similar to the line sectioning approach, we define variables $\gamma^{\mathrm{D}}_d \in \{0,1\}$ for each $d \in \mathcal{D}$, whose value will be equal to the value of $\gamma^{\mathrm{B}}_{b,i}$ if the load is connected to busbar i of bus b. For all $(b,d) \in \mathcal{BD}$,

$$\gamma^{\mathrm{B}}_{b,1} + \eta^{\mathrm{D}}_d - 1 \le \gamma^{\mathrm{D}}_d \le 1 + \gamma^{\mathrm{B}}_{b,1} - \eta^{\mathrm{D}}_d, \tag{20a}$$

$$\gamma^{\mathrm{B}}_{b,2} - \eta^{\mathrm{D}}_d \le \gamma^{\mathrm{D}}_d \le \gamma^{\mathrm{B}}_{b,2} + \eta^{\mathrm{D}}_d. \tag{20b}$$

The value of γ^{D}_d will be used in the definition of the objective.

D. Objective function

The overall objective of islanding is to minimize the risk of system failure. In our motivation we assumed that there is some uncertainty associated with a particular subset of buses and/or lines; we suspect there may be a fault and so we wish to isolate these components from the rest of the network.

Suppose we associate a reward M_d per unit supply of load d. In islanding the uncertain components, we wish to maximize the total value of supplied load. However, in placing *any* load in section 0, we assume a risk of not being able to supply power to that load, since that section contains "unhealthy" components and may fail. Accordingly, we introduce a load loss penalty $0 \le \beta_d < 1$, which may be interpreted as the probability of being able to supply a load d if placed in section 0. If d is placed in section 1 we realize a reward M_d per unit supply, but if d is placed in section 0, with the uncertain components, we realize a reward of $\beta_d M_d < M_d$. The objective is to maximize the expected load supplied, J^*:

$$J^* = \max \sum_{d \in \mathcal{D}} M_d P_d \big(\beta_d \alpha_{0d} + \alpha_{1d}\big). \tag{21}$$

where,

$$\alpha_d = \alpha_{0d} + \alpha_{1d}, \tag{22a}$$

$$0 \le \alpha_{1d} \le \gamma^{\mathrm{D}}_d, \tag{22b}$$

Here we have introduced a new variable $\alpha_{sd} \ge 0$ for the load d delivered in section $s \in \{0,1\}$. If $\gamma^{\mathrm{D}}_d = 0$, and the load d is in section 0, then $\alpha_{1d} = 0, \alpha_{0d} = \alpha_d$; otherwise, because $\beta_d < 1$, $\alpha_{0d} = 0$ and $\alpha_{1d} = \alpha_d$. Thus α_{0d} and α_{1d} may not be simultaneously non-zero.

Remark 1: While the sectioning constraints force the values of certain binary variables, it may be desirable encourage other binary variables to take on integer values in the LP relaxations of the problem. To do so will also discourage the unnecessary disconnection of switches and breakers. For example, we may wish to discourage the cutting of lines in the healthy part of the network, which we may do so by subtracting a small penalty from the objective for zero values of ρ_l:

$$\epsilon_1 \sum_{l \in \mathcal{L}\backslash\mathcal{L}^0} 1 - \rho_l \tag{23}$$

As another example, it may be desirable to penalize the switching-off of generators in the objective by penalizing zero values of ζ_g

$$\epsilon_2 \sum_{g \in \mathcal{G}} W_g (1 - \zeta_g), \tag{24}$$

where W_g is some weight. A uniform weight, *e.g.*, $W_g = 1, \forall g$, will encourage large generators to switch off, rather than several small units, for any given decrease in total generation. Generation disconnection can be more evenly penalized by instead setting W_g equal to the generator's capacity $P_g^{\mathrm{G+}}$.

E. Overall formulation

The overall formulation for islanding by bus splitting is to maximize (21) subject to (1)–(17), and (18) or (19).

IV. Cuts and symmetry-breaking constraints

Redundancy is inherent in the network as modelled, since similar bus configurations can be represented by different binary variable settings. For example, a bus with all binary switches for connected components set to 1 is equivalent to one with all set to 0. Such redundancy is likely to add to computation time, and therefore it is desirable, where possible, to include additional constraints that break the symmetry of problems.

The next constraint eliminates the example case just described. We hard-set one of the component switches at every bus, so that all other switches at the bus are set relative to this. The only component sure to be present at each bus is a line. Define $\mathcal{BLE} \subset \mathcal{B} \times \mathcal{L} \times \{1, 2\}$ as the set that lists, for each bus, a single line connected to that bus and which end (1 or 2) is incident. Then, without loss of generality, we can connect that line end to busbar 1 of bus b.

$$\eta^{\mathrm{L}}_{l,e} = 1, \forall (b, l, e) \in \mathcal{BLE}. \tag{25}$$

Simulations show that this constraint can significantly reduce computation time.

Next, manipulation of the logical relations that gave rise to constraints (15) yields the constraints

$$0 \le \gamma^{\mathrm{B}}_{b,1} + \gamma^{\mathrm{B}}_{b,2} - \gamma^{\mathrm{L}}_{l,e} \le 1, \tag{26}$$

for all $l \in \mathcal{L}$, at each end $e \in \{1, 2\}$, and where $b = F_l$ if $e = 1$ and $b = T_l$ if $e = 2$. Similarly, the constraint (20) is complemented by

$$0 \le \gamma^{\mathrm{B}}_{b,1} + \gamma^{\mathrm{B}}_{b,2} - \gamma^{\mathrm{D}}_{d} \le 1, \tag{27}$$

for all $(b, d) \in \mathcal{BD}$. It is simple to show these constraints are facets of the convex relaxation of the set of feasible $(\gamma^{\mathrm{B}}_{b,1}, \gamma^{\mathrm{B}}_{b,2}, \eta^{\mathrm{L}}_{l,e}, \gamma^{\mathrm{L}}_{l,e})$ and $(\gamma^{\mathrm{B}}_{b,1}, \gamma^{\mathrm{B}}_{b,2}, \eta^{\mathrm{D}}_{d}, \gamma^{\mathrm{D}}_{d})$ respectively. Investigation of all facets for these constraints found no further facets other than the trivial (*e.g.* $0 \le \gamma^{\mathrm{B}}_{b,1} \le 1$).

Finally, consideration of the generation capability in the network allows an upper bound on the objective, the expected load supplied, to be derived. The best possible solution for any network will have each generator operating at its maximum output. Any unit attached to a bus $b \in \mathcal{B}^0$ will be confined to supplying loads in section 0, while all others could supply loads in section 1. This implies the constraint

$$\begin{aligned}\sum_{d \in \mathcal{D}} P_d\big(\beta_d \alpha_{0d} + \alpha_{1d}\big) \le \sum_{(b,g) \in \mathcal{BG}: b \notin \mathcal{B}^0} & P_g^{\mathrm{G+}} \\ + \max_{d \in \mathcal{D}}\{\beta_d\} \sum_{(b,g) \in \mathcal{BG}: b \in \mathcal{B}^0} & P_g^{\mathrm{G+}}\end{aligned} \tag{28}$$

This constraint cuts off no feasible integer solutions. While apparently trivial, tests show that its inclusion can for some problems offer a significantly lower best upper bound than that deduced by the solver during the MILP solution process.

Further constraints that attempt to eliminate redundancy are possible, but may just add to the size of the MILP problem. Simulations have shown that the constraints presented here have the most profound effect on reducing computation time. It may be helpful to include additional small terms in the objective—in the way outlined in Remark 1—to encourage binary variables to take binary variables in the solution of the LP relaxation.

V. Numerical simulations

This section presents preliminary simulation results using the new formulation. Comparisons with the line-cutting approach of [16] show that the bus-splitting approach has the potential to significantly lower the amount of load that need be shed when islanding.

A. 14-bus network case study

The test network is the IEEE 14-bus system, shown in fig. 4, which comprises two synchronous generators (indicated by single circles), three synchronous condensers (double circles), and ten loads. The total generation capacity is 400 MW against a total demand of 259 MW.

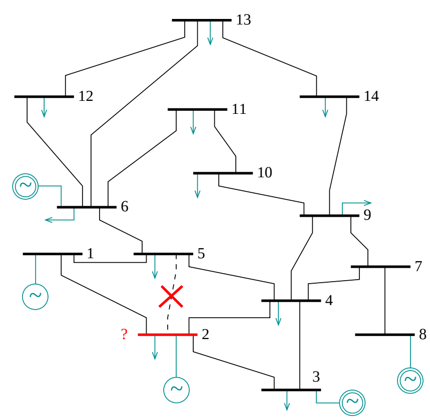


Fig. 4. 14-bus test network with a failure line $(2, 5)$ and bus 2 uncertain.

Firstly, a steady-state operating point for the network is established by solving an AC OPF. Generator real and reactive power output limits as set to the values reported in [22]. The resulting OPF solution sets the generator outputs as in tab. I. The largest phase angle difference across any line in the solution is 8.1 degrees, for line $(1, 5)$.

The scenario to be simulated is described as follows. While operating at this point, line $(2, 5)$ breaks and bus 2 is assigned to the uncertain set $\mathcal{B}^0$ for islanding. We then seek islanding solutions by (i) cutting lines only, (ii) splitting buses only, and (iii) a combination of both. For the islanding optimizations, the two generators are permitted to vary outputs by up to 5% of their pre-islanding levels or switch off, as per (10). The value

TABLE I
REAL AND REACTIVE POWER OUTPUTS OF GENERATORS AND CONDENSERS IN THE AC-OPF SOLUTION.

Gen, g	Bus, b	P_g^{G} (MW)	Q_g^{G} (MVAr)
1	1	200.00	−12.62
2	2	70.92	40.94
3	3	0.00	30.23
4	6	0.00	10.83
5	8	0.00	8.42

of β_d, for placing loads in section 0, is 0.5. Since line limits are not present in the network data, a phase angle limit of $\pi/7$ radians (25.71 degrees)—far in excess of the AC-OPF flows—is applied to each line, giving a corresponding maximum MW limit, for (8), of

$$P_l^{\mathrm{L}+} = \frac{\pi B_l^{\mathrm{L}}}{7\tau_l}.$$

1) Line cutting only: An optimal islanding solution by using only line cuts is obtained using the method of [16]. In the problem, the objective assumes a reward of $M_d = 1$ per unit supply of load. Line disconnections are unlimited in number but penalized, using (23), with a weight $\epsilon_1 = 0.1$, while generator switch-offs are discouraged by imposing the penalty (24) with $\epsilon_2 = 10^{-3}$ and $W_g = P_g^{\mathrm{G}+}$. These penalties make up less than 1% of the overall objective value.

The islanded network is shown in fig. 5. Bus 2 has been isolated by disconnecting lines $(1,2)$, $(2,3)$, $(2,4)$ in addition to the failed line $(2,5)$. No lines have been cut extra to those required to island bus 2. As the demand at bus 2 is only 21.7 MW but the pre-islanding output of generator 2 was 70.92 MW, the generator has been switched off and the load shed. In section 1, 50.1% of the 94.2 MW load at bus 3 has been shed, but all other loads are fully served. In total, 68.93 MW of the 259 MW load has been shed, and all of the load remaining is in the healthy section 1. The objective value—the expected load supplied—is 190.07 MW.

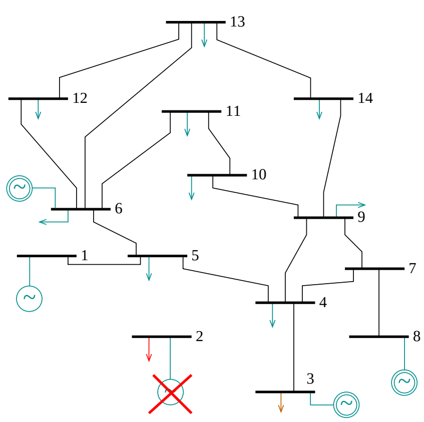


Fig. 5. The 14-bus network as islanded by line cuts.

An interesting feature of the solution is that the remaining generator, at bus 1, is not operating at its maximum output. Inspection of the line flows shows that line $(1,5)$—the sole remaining path from the generator to the rest of the network—is at its phase angle limit, transferring 190.07 MW of real power. Thus, this line is acting as a 'bottleneck' in the islanded network.

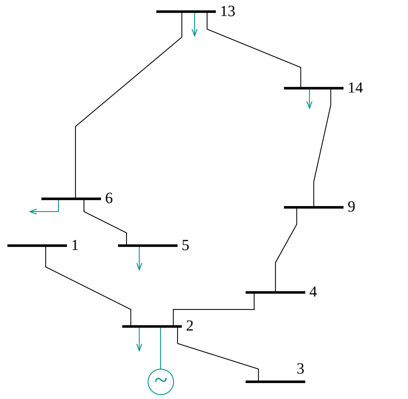
2) Bus-splitting only: To obtain an islanding solution using only bus splitting, we permit no line disconnections by imposing the constraint $\rho_l = 1, \forall l \in \mathcal{L}$. Other parameters in the problem are set as follows. The maximum phase angle difference and real power flow between two busbars at a bus are π radians and 250 MW respectively. In the objective function, as well as the generator switch-off penalty already mentioned, we penalize zero values of all binary variables bar $\gamma_{b,i}^{\mathrm{B}}$ with a weighting of 0.1. This will change the optimal solution only negligibly, if at all, but will encourage binary variables to take integer values in the LP relaxations during the solution process, aiding computation time. The line disconnection penalty subtracts nothing from the objective, since line cuts are not permitted.

Using AMPL 11.0 with Parallel CPLEX 12.2 to model and solve the islanding MILP problem, on a 2.66 GHz quad-core Linux machine with 4 GiB RAM, the solver finds an optimal solution in around 4 seconds.

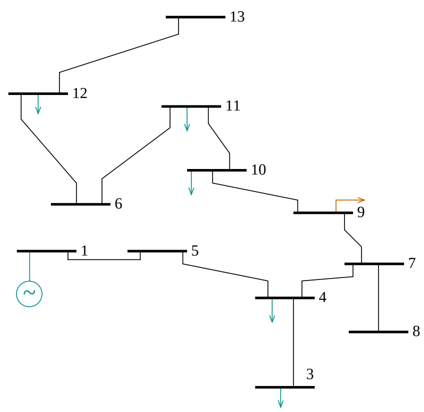
The islanded network is shown in fig. 6. The sections 0 and 1 are overlapping in this solution; since no line cuts were permitted, some buses have been split and have one busbar in section 0 and the other in section 1. Section 0 contains the peripheral buses of the network, with loads served by the generator at bus 2. Section 1 contains more of the central buses, served by the generator at bus 1. Though the sections appear to be of equal size, section 0 contains 68.9 MW of demand compared with 190.1 MW in section 1; thus, the healthy section contains the largest loads. The solution sheds 0.03 MW at bus 3 in section 1 and nothing in section 0. The objective value—the expected load supply taking into account the 'probability' β_d—is 224.52 MW, which is 34 MW higher than that obtained with line cuts only. Therefore, the expectation is that less load is lost by islanding in this way.

Inspection of the power flows in the solution shows that the generator outputs have changed little from their pre-fault AC values, with $p_1^{\mathrm{G}} = 190.07$ MW and $p_2^{\mathrm{G}} = 68.90$ MW. The output of generator 1 is again limited by the maximum power that can be transferred along line $(1,5)$. However, the generator at 2 has not been switched off in this solution, which enables more load to be supplied.

3) Bus splitting and line cutting: The islanding optimization was re-solved, now permitting any number of line disconnections as well as bus splitting. To penalize line cutting more heavily than bus splitting, the line disconnection penalty ϵ_1 was increased to 0.5 while the bus splitting penalty was held at 0.1. The optimal islanding solution is identical to that obtained using bus splitting only—no lines are cut, and the objective value is the same. Restoring the line cut penalty to 0.1 finds an optimal solution that splits fewer buses; buses 1 and 3 are instead isolated from bus 2 by cutting lines $(1,2)$ and $(2,3)$. The optimal objective value is identical, confirming that the maximum expected supply can be obtained by a number of



(a) Section 0



(b) Section 1

Fig. 6. Sections of the islanded 14-bus network as islanded by bus splitting.

different islanding solutions, and suggesting that line cutting and bus splitting should be given different priorities.

Allowing line cuts raised the solve time from 4 to 40 seconds. However, firstly, note that the optimal integer solution was obtained near the start of the solution process, with the majority of time spent proving optimality. For practical application, with the network in an emergency state, sub-optimal feasible islands are likely to be satisfactory. Secondly, it may not be necessary to model the entire network in full detail; for example, buses far away from the disturbance need not be modelled as double busbars with a full complement of switches. Thus, complexity of the problem for larger networks may be minimized.

VI. Conclusions

In this paper, a new MILP-based approach to islanding of power networks has been presented. The formulation models each bus in the network as a double busbar arrangement, with interconnecting switches and switches to lines, loads and generators. Islanding is then by a combination of bus splitting and line disconnections. Preliminary simulations on the 14-bus test network show that partitioning the network by splitting buses, rather than cutting lines, can lead to islands with significantly smaller amounts of load shedding. Future research will investigate the application of the method to larger networks and techniques and heuristics for shortening computation time.

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