MILP Formulation for Islanding of Power Networks

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***Abstract*—In this paper, a mathematical formulation for the islanding of power networks is presented. Given an area of uncertainty in the network, the proposed approach uses mixed integer linear programming to isolate unhealthy components of the network and create islands, by (i) cutting lines, (ii) shedding loads and (iii) switching generators, while maximizing load supply. A key feature of the new method is that network constraints are explicitly included in the MILP problem, resulting in balanced, steady-state feasible DC solutions. A subsequent AC optimal load shedding optimization on the islanded network model provides a feasible AC solution. Numerical simulations on the 24-bus IEEE reliability test system and larger systems demonstrate the effectiveness of the method.**

***Index Terms*—Power system modeling, Power system security, Optimization, Integer programming, Blackouts, Islanding**

I. Introduction

In recent years, there has been an increase in the occurence of wide-area blackouts of power networks. In 2003, separate blackouts in Italy [1], Sweden/Denmark [2] and USA/Canada [3] affected millions of customers. The wide-area disturbance in 2006 to the UCTE system caused the system to split in an uncontrollable way [4], forming three islands. More recently, the UK network experienced a system-wide disturbance caused by an unexpected loss of generation; blackout was avoided by local load shedding [5].

While the exact causes of wide-area blackouts differ from case to case, some common driving factors emerge. Modern power systems are being operated closer to limits: liberalization of the markets, and the subsequent increased commercial pressures and change in expenditure priorities, has led to a reduction in security margins [6]–[8]. A more recently occurring factor is increased penetration of variable distributed generation, notably from wind power, which brings significant challenges to secure system operation [9].

For several large disturbance events, *e.g.*, [3], studies have shown that wide-area blackout could have been prevented by intentionally splitting the system into islands [10]. By isolating the faulty part of the network, the total load disconnected in the event of a cascading failure is reduced. *Controlled* islanding or system splitting is therefore attracting an increasing amount of attention. The problem is how to efficiently split the network into islands that are balanced in load and generation, and have stable steady-state operating points. This is a considerable challenge, since the search space of line cutsets grows combinatorially with network size, and is exacerbated by the requirement for strategies that obey non-linear power flow equations and satisfy operating constraints.

Approaches in the literature broadly differ according to the motive of islanding, and within that the search method employed to determine the splitting boundary. The simplest example of the former is forming balanced islands. In [11], a three-phase ordered binary decision diagram (OBDD) method is proposed that determines a set of islanding strategies. The approach uses a reduced graph-theoretical model of the network to minimize the search space for islanding; power flow analyses are subsequently executed on islands to exclude strategies that violate operating constraints, *e.g.*, line limits.

An alternative motive seeks to split the network into electromechanically stable islands, commonly by splitting so that generators with coherent oscillatory modes are grouped. Determing the optimal cutset of lines involves considerations of load-generation balance and other constraints; algorithms include exhaustive search [12], minimal-flow minimal-cutset determination using breadth-/depth-first search [13], and graph simplification and partioning [14]. The authors of [15] note that splitting based simply on slow coherency is not always effective under complex oscillatory conditions, and propose a framework that, iteratively, identifies the *controlling* group of machines and the contingencies that most severely impact system stability, and uses a heuristic method to search for a splitting strategy that maintains a desired margin. Wang et al. [16] employed a power flow tracing algorithm to first determine the domain of each generator, *i.e.* the set of load buses that 'belong' to each generator. Subsequently, the network is coarsely split along domain intersections before refinement of boundaries to minimize imbalances.

While several useful strategies exist for splitting a network into synchronous balanced islands, little attention has focused on islanding in response to particular contigencies. If, for example, a line failure occurs and subsequent cascading failures are likely, it may be desirable to isolate a small part of the network—the impacted area—from the rest. A method that does not take the impacted area into account when designing islands may leave this area within an arbitrary large section of the network, all of which may become insecure as a result.

In this paper, we propose an optimization-based approach to system islanding and load shedding. Given some uncertain set of buses and/or lines, solving an optimization determines (i) the optimal set of lines to cut, (ii) which generators to switch off, and (iii) which loads to shed. The solution isolates the suspected parts from the rest of the network while maximizing load supply. A key feature of the method is that any islands created are balanced and satisfy power flow equations, and also operating constraints are handled naturally by the constrained optimization framework. The approach uses two stages: solving a mixed-integer linear programming

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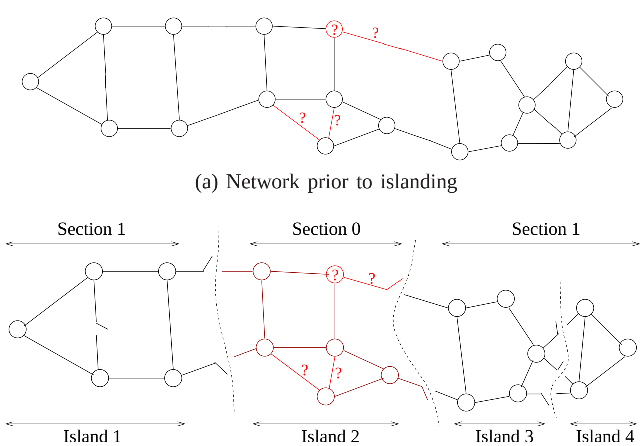
(MILP) islanding problem, which includes the linear DC flow equations, determines a DC-feasible solution, and an AC optimal load shedding optimization subsequently provides an AC-feasible operating point.

Integer programming has many applications in power systems, but its use in network splitting and blackout prevention is limited. Bienstock and Mattia [17] proposed an IP-based approach to the problem of designing networks that are robust to sets of cascading failures and thus avoid blackouts; whether to upgrade a line's capacity is a binary decision. Fisher et al. [18] propose a method for optimal transmission switching for the problem of minimizing the cost of generation despatch by selecting a network topology to suit a particular load. In common with the formulation presented here, binary variables represent switches that open or close each line and the DC power flow model is used, resulting in a MILP. However, in this paper sectioning constraints are present, and the problem is to create balanced islands while maximizing load supply.

The organization of the paper is as follows. The next section outlines the motivation and assumptions that underpin the approach. The islanding formulation is developed in Section III. The AC optimal load shedding problem is described in Section IV. In Section V, numerical simulations are presented. Finally, conclusions are drawn in Section VI

II. Motivation and Assumptions

The motivation for the formulation is stated as follows. Following some failure, we assume that limited information is available about the network and its exact state is uncertain; there are parts of the network that are suspected of having a fault and some where we are reasonably sure have no faults. We assume that in such a case, a robust solution to prevent cascading failures is to isolate the uncertain part of the network from the certain part, by forming one or more stable islands. Fig. 1(a) depicts such a situation for a fictional network; uncertain lines and buses are indicated.



(a) Network prior to islanding

(b) Network post islanding

Fig. 1. (a) Fictional network with uncertain buses and lines, and (b) the islanding of that network by disconnecting lines.

Our aim is to split the network into disconnected sections so that the possible faults are all in one section. It is desirable that this section be small, since it may be prone to failure, and that the other section is able to operate with little load shedding. We would also like the problem section to shed as little load as possible. Fig. 1(b) shows a possible islanding solution for this network, where all uncertain buses have been placed in a section 0 and all uncertain lines with at least one end in section 1 are disconnected. We make the following distinction between *sections* and *islands*.

- The optimized network consists of two sections, an "unhealthy" section 0 and a "healthy" section 1. No lines connect the two sections. On the other hand, neither section is required to be a single, connected component.
- An island is a connected component of the network.

Thus, either section may contain a number of islands, as in fig. 1(b), where section 1 comprises islands 1, 3 and 4, while section 0 is a single island. The boundaries of sections and the number of islands formed will depend on the optimization.

We will assume that generator outputs and load levels immediately after the initial fault are known. We have central control of generation, load shedding and line breakers; we may instantaneously reduce the demand and disconnect any lines. Furthermore, we assume that we have a certain degree of control over a generator's output. We require that after the adjustments the system is a feasible equilibrium.

III. MILP Islanding Formulation

In this section, we present a MILP formulation for islanding and minimizing the load shed in a network under stress.

Consider a network that comprises a set of buses $\mathcal{B} = \{1, 2, \ldots, n^{\mathrm{B}}\}$ and a set of lines $\mathcal{L} = \{1, 2, \ldots, n^{\mathrm{L}}\}$. The two vectors F and T describe the connection topology of the network: a line $l \in \mathcal{L}$ connects bus F_l to bus T_l. We assume there also exists a set of generators $\mathcal{G} = \{1, 2, \ldots, n^{\mathrm{G}}\}$ and a set of loads $\mathcal{D} = \{1, 2, \ldots, n^{\mathrm{D}}\}$. A subset $\mathcal{G}_b$ of generators is attached to bus $b \in \mathcal{B}$; similarly, $\mathcal{D}_b$ contains the subset of loads present at bus $b \in \mathcal{B}$.

A. Sectioning Constraints

We aim to allocate buses and lines into the two sections 0 and 1. We suspect that some subset $\mathcal{B}^0 \subseteq \mathcal{B}$ of buses and some subset $\mathcal{L}^0 \subseteq \mathcal{L}$ of lines have a possible fault. These subsets thus contain all "uncertain" buses and lines, while the remainder of buses/lines are defined as "certain". It is the uncertain components that we wish to confine to section 0.

We introduce a binary decision variable γ_b with each bus $b \in \mathcal{B}$; γ_b shall be set equal to 0 if b is placed in section 0 and $\gamma_b = 1$ otherwise. To partition the network in such a way, we need to disconnect lines. Accordingly, we define a binary decision variable ρ_l for each $l \in \mathcal{L}$; $\rho_l = 0$ if line l is disconnected and $\rho_l = 1$ otherwise.

Constraints (1a) and (1b) apply to each line l not assigned to $\mathcal{L}^0$. The line is cut if its two end buses are in different sections (*i.e.* $\gamma_{F_l} = 0$ and $\gamma_{T_l} = 1$, or $\gamma_{F_l} = 1$ and $\gamma_{T_l} = 0$). Otherwise, if the two end buses are in the same section then $\rho_l \leq 1$, and the line may or may not be disconnected. Thus, these constraints enforce the requirement that any certain line between sections 0 and 1 shall be disconnected.

$$\rho_l \leq 1 + \gamma_{F_l} - \gamma_{T_l}, \forall l \in \mathcal{L} \setminus \mathcal{L}^0, \quad (1a)$$
$$\rho_l \leq 1 - \gamma_{F_l} + \gamma_{T_l}, \forall l \in \mathcal{L} \setminus \mathcal{L}^0. \quad (1b)$$

Constraints (1c) and (1d) apply to lines assigned to $\mathcal{L}^0$. A line $l \in \mathcal{L}^0$ is disconnected if at least one of the ends is in section 1. Thus, an uncertain line either (i) shall be disconnected if entirely in section 1, (ii) shall be disconnected if between sections 0 and 1, or (iii) may remain connected if entirely in section 0.

$$\rho_l \le 1 - \gamma_{F_l}, \forall l \in \mathcal{L}^0, \tag{1c}$$
$$\rho_l \le 1 - \gamma_{T_l}, \forall l \in \mathcal{L}^0, \tag{1d}$$

Constraints (1e) and (1f) set the value of γ_b for a bus b depending on what section that bus was assigned to. We define $\mathcal{B}^1$ to be the set of buses that are desired to remain in section 1. It may be that we wish to exclude buses from the "unhealthy" section, and such an assignment will in general reduce computation time.

$$\gamma_b = 0, \forall b \in \mathcal{B}^0, \tag{1e}$$
$$\gamma_b = 1, \forall b \in \mathcal{B}^1. \tag{1f}$$

Given some assignments to $\mathcal{B}^0, \mathcal{B}^1$ and $\mathcal{L}^0$, the optimization will disconnect lines and place buses in sections 0 or 1, hence partitioning the network into sections 0 and 1. What else is placed in section 0, what other lines are cut, and which loads and generators are adjusted, are degrees of freedom for the optimization, and will depend on the objective function.

B. DC Power Flow Model

The power flow model we employ is a variant of the "DC" model, assuming unit voltage at each bus and small phase angle differences, but accouting for line losses. Kirchhoff's current law is applied at each bus $b \in \mathcal{B}$:

$$\sum_{g \in \mathcal{G}_b} p^{\mathrm{G}}_g = \sum_{d \in \mathcal{D}_b} p^{\mathrm{D}}_d + \sum_{l \in \mathcal{L}: F_l = b} p^{\mathrm{L}}_l - \sum_{l \in \mathcal{L}: T_l = b} (p^{\mathrm{L}}_l - \bar{h}^{\mathrm{L}}_l), \tag{2}$$

where p^{G}_g is the real power output of generator $g \in \mathcal{G}_b$ at bus b, p^{D}_d is the real power demand from load $d \in \mathcal{D}_b$. The variable p^{L}_l is the real power flow into the first end (bus F_l) of line l, and $p^{\mathrm{L}}_l - \bar{h}^{\mathrm{L}}_l$ is the flow into of the second end, reduced by the loss $\bar{h}^{\mathrm{L}}_l$. Loss modelling is described later in this section.

When a line l is connected, Kirchhoff's voltage law (KVL) demands that a flow of real power is established depending only on the difference in phase angle across the line. However, we may not equate p^{L}_l directly to this flow, since if a line is disconnected by the optimization, zero power will flow through that line. In this case, we must allow different phase angles at each end of the line. To achieve this, the KVL expression is equated to a variable $\hat{p}^{\mathrm{L}}_l$.

$$\hat{p}^{\mathrm{L}}_l = \frac{-B^{\mathrm{L}}_l}{\tau_l}\left(\delta_{F_l} - \delta_{T_l}\right), \tag{3}$$

where constants B^{L}_l, τ_l are, respectively, the susceptance and off-nominal turns ratio of line l. Then, when line l is connected we will set $p^{\mathrm{L}}_l = \hat{p}^{\mathrm{L}}_l$, and when l is disconnected $p^{\mathrm{L}}_l = 0$. We model this as follows.

Assume the maximum possible magnitude of real power flow through a line l is $P^{\mathrm{L\,max}}_l$. Then

$$-\rho_l P^{\mathrm{L\,max}}_l \le p^{\mathrm{L}}_l \le P^{\mathrm{L\,max}}_l \rho_l, \tag{4a}$$
$$-(1-\rho_l)\hat{P}^{\mathrm{L\,max}}_l \le \hat{p}^{\mathrm{L}}_l - p^{\mathrm{L}}_l \le \hat{P}^{\mathrm{L\,max}}_l (1-\rho_l). \tag{4b}$$

When the sectioning constraints set a particular $\rho_l = 0$, then $p^{\mathrm{L}}_l = 0$ but $\hat{p}^{\mathrm{L}}_l$ may take whatever value necessary to satisfy the KVL constraint (3). Conversely, if $\rho_l = 1$ then $p^{\mathrm{L}}_l = \hat{p}^{\mathrm{L}}_l$.

Line limits $P^{\mathrm{L\,max}}_l$ may be expressed either directly as MW ratings on real power for each line, or as a limit on the phase angle difference across a line. Since in the model the real power through a line is just a simple scaling of the phase difference across it, then any phase angle limit may be expressed as a corresponding MW limit. Note that at the very minimum $\hat{P}^{\mathrm{L\,max}}_l \ge P^{\mathrm{L\,max}}_l$, but these limits should be of large enough to allow two buses across a disconnected line to maintain sufficiently different phase angles.

C. Loss Modelling

While the DC power flow model allows the islanding problem to remain linear, one disadvantage is that real power losses in the network are assumed to be zero. The lossless DC model will under-estimate the amount of load that needs to be shed when forming islands, and thus could lead to poor islanding decisions. In this paper, three loss models are considered in addition to lossless DC. The actual loss function h^{L}_l is derived from the AC real power flows, and is then approximated by $\bar{h}^{\mathrm{L}}_l$.

1) In the standard lossless DC model, $\bar{h}^{\mathrm{L}}_l = 0$.
2) Constant loss. The loss for each line is determined from the current operating point of the network, in which line l has a flow $p^{\mathrm{L}*}_l$, voltages $v^*_{F_l}$ and $v^*_{T_l}$, and a corresponding loss $h^{\mathrm{L}*}_l = h^{\mathrm{L}}_l(p^{\mathrm{L}*}_l, v^*_{F_l}, v^*_{T_l})$.
$$\bar{h}^{\mathrm{L}}_l = \rho_l h^{\mathrm{L}*}_l,$$
The inclusion of ρ_l drives the loss to zero if the islanding optimization cuts the line.
3) Linear loss. The AC line loss function is linearized about the operating point, assuming constant voltages.
$$\bar{h}^{\mathrm{L}}_l = \rho_l h^{\mathrm{L}*}_l + \frac{\partial h^{\mathrm{L}*}_l}{\partial p^{\mathrm{L}}_l}\left(p^{\mathrm{L}}_l - \rho_l p^{\mathrm{L}*}_l\right).$$
Then if $\rho_l = 0$, $p^{\mathrm{L}}_l = 0$ and $\bar{h}^{\mathrm{L}}_l = 0$. The bound $\bar{h}^{\mathrm{L}}_l \ge 0$ is included to exclude the possibility of negative line losses in the solution. Consequently, the linear loss model restricts islanding solutions to a region around the pre-islanding operating point, and prohibits lines from generating real power.
4) Piecewise linear (PWL) loss. The AC loss function is first approximated by assuming $v_{F_l} = v_{T_l} = 1$ so that
$$h^{\mathrm{L}}_l \approx \frac{G^{\mathrm{L}}_l}{\tau_l}\left[\frac{1}{\tau_l} + \tau_l - 2\cos\left(\frac{\tau_l}{B^{\mathrm{L}}_l} p^{\mathrm{L}}_l\right)\right],$$
This function is then approximated over an interval by a number of line segments, to give $\bar{h}^{\mathrm{L}}_l$. The line binary variable ρ_l may be included in the PWL expression to set $\bar{h}^{\mathrm{L}}_l = 0$ when $\rho_l = 0$.

D. Generation constraints

In situations where there is a need to react quickly to an unplanned contingency, to prevent cascading failures the time available to island the network and adjust loads and generators will be short. Therefore, we must assume that full re-scheduling of generators and/or the addition of new units to the network will not be possible. On the other hand, a certain amount of spinning reserve will be available in the network for small-scale changes. For any unit, we will assume that a new setpoint, close to the current operating point, may be commanded. This setpoint should be reachable within a short time period, and also must not violate limits. In practice, fast governer action will quickly raise/lower real power output to the new setpoint, before the spinning reserve takes over.

A further assumption we make is that a generator obeys a binary regime: either it operates near its previous real power output, or it may have its output switched to zero. That is,

$$p_g^{\mathrm{G}} \in \left[P_g^{\mathrm{G}-}, P_g^{\mathrm{G}+}\right] \cup \{0\}.$$

This latter case models the removal of the source of mechanical input power; it is assumed that electrical power will fall to zero within the timeframe of islanding. Although the switched-off generating unit contributes no power in steady state to the network, it remains electrically connected to the network.

To model this disjoint set constraint, we introduce a binary variable $\zeta_d \in \{0, 1\}$ for each generator. 1

$$\zeta_g P_g^{\mathrm{G}-} \le p_g^{\mathrm{G}} \le \zeta_g P_g^{\mathrm{G}+}, \tag{5}$$

for all $g \in \mathcal{G}$. If $\zeta_g = 0$ then generator g is switched off; otherwise it outputs $p_g^{\mathrm{G}} \in \left[P_g^{\mathrm{G}-}, P_g^{\mathrm{G}+}\right]$. These limits depend on the ramp and output limits of the generator, and the amount of reserve available to the unit.

E. Load shedding

Following separation of the network into islands, and given the limits on generator power outputs, it follows that it may not be possible to fully supply all loads. However, the optimization is to determine a feasible steady-state for the islanded network, and thus it is necessary to permit some shedding of loads.

Suppose that a load $d \in \mathcal{D}$ has a constant real power demand P_d^{D}. We assume this load may be reduced by disconnecting a proportion $1 - \alpha_d$. For all $d \in \mathcal{D}$:

$$p_d^{\mathrm{D}} = \alpha_d P_d^{\mathrm{D}}, \tag{6}$$

where $0 \le \alpha_d \le 1$. In determining an feasible islanded network, it is in our interests to promote full load supply, and so load shedding is minimized in the objective function.

F. Objective function

The overall objective of islanding is to minimize the risk of system failure. In our motivation we assumed that there is some uncertainty associated with a particular subset of buses and/or lines; we suspect there may be a fault and so we wish to isolate these components from the rest of the network.

Suppose we associate a reward M_d per unit supply of load d. In islanding the uncertain components, we wish to maximize the total value of supplied load. However, in placing *any* load in section 0, we assume a risk of not being able to supply power to that load, since that section containts "unhealthy" components and may fail. Accordingly, we introduce a load loss penalty $0 \le \beta_d < 1$, which may be interpreted as the probability of being able to supply a load d if placed in section 0. If d is placed in section 1 we realize a reward M_d per unit supply, but if d is placed in section 0, with the uncertain components, we realize a reward of $\beta_d M_d < M_d$. The objective is to maximize the expected load supplied, J^*:

$$J^*_{\mathrm{DC}} = \max \sum_{d \in \mathcal{D}} M_d P_d \big(\beta_d \alpha_{0d} + \alpha_{1d}\big), \tag{7}$$

where,

$$\alpha_d = \alpha_{0d} + \alpha_{1d}, \forall d \in \mathcal{D}, \tag{8a}$$

$$0 \le \alpha_{0d} \le 1, \forall d \in \mathcal{D}, \tag{8b}$$

$$0 \le \alpha_{1d} \le \gamma_b, \forall b \in \mathcal{B}, d \in \mathcal{D}_b. \tag{8c}$$

Here we have introduced a new variable α_{sd} for the load d delivered in section $s \in \{0, 1\}$. If $\gamma_b = 0$, and the load at bus b is in section 0, then $\alpha_{1d} = 0, \alpha_{0d} = \alpha_d$ and the reward is $\beta_d M_d P_d \alpha_d$. On the other hand, if $\gamma_b = 1$ then $\alpha_{1d} = \alpha_d$ and $\alpha_{0d} = 0$, giving a higher reward $M_d P_d \alpha_d$. Thus the objective has a preference for $\gamma_b = 1$ and a smaller section 0.

G. Overall formulation

The overall formulation for islanding is to maximize (7) subject to (1)–(8). The resulting problem is an MILP.

Remark 1 (Penalizing line cuts and generator switching): While the sectioning constraints force certain lines to be cut, it may also be desirable to penalize the unnecessary disconnection of other, healthy lines in the network. To do so will also encourage binary variables ρ_l to take on integer values in the LP relaxations of the problem. This may be achieved by adding a small reward in the objective for non-zero values of ρ_l:

$$\epsilon_1 \sum_{l \in \mathcal{L} \setminus \mathcal{L}^0} \rho_l \tag{9}$$

For similar reasons, it may be desirable to penalize the switching-off of generators in the objective by rewarding non-zero values of ζ_g

$$\epsilon_2 \sum_{g \in \mathcal{G}} W_g \zeta_g, \tag{10}$$

where W_g is some weight. A uniform weight, *e.g.*, $W_g = 1, \forall g$, will encourage large generators to switch off, rather than several small units, for any given decrease in total generation. Generation disconnection can be more evenly penalized by instead setting W_g equal to the generator's capacity $P_g^{\mathrm{G}+}$.

IV. Post-Islanding AC Optimal Load Shedding

The solution of the DC islanding optimization includes a set of lines to disconnect, new generation levels, and the proportions of loads to be shed. In general, however, the predictions of the DC model will not match reality, and no consideration is given to reactive power and voltage. Therefore, to determine a feasible AC solution for the islanded network, we propose that an AC optimal load shedding (OLS) problem is solved immediately after the islanding optimization.

The AC-OLS optimization problem is a standard OPF problem albeit with load shedding. The AC-OLS is solved for the network in its islanded state. That is, the set $\mathcal{L}$ is modified by removing lines for which $\rho_l = 0$. Furthermore, any generator for which $\zeta_g = 0$ has its upper and lower bounds on real power set to zero; others are free to vary real power output within a restricted region, as described previously.

This problem also maximizes the value of total real power supplied to loads:

$$J^*_{\text{AC}} = \max \sum_{d \in \mathcal{D}} R_d \alpha_d P_d, \tag{11}$$

subject to,

$$f(x) = 0, \tag{12a}$$
$$g(x) \leq 0, \tag{12b}$$
$$(p^{\text{G}}_g, q^{\text{G}}_g) \in \mathcal{O}_g, \forall g \in \mathcal{G}, \tag{12c}$$
$$(p^{\text{D}}_d, q^{\text{D}}_d) = \alpha_d (P^{\text{D}}_d, Q^{\text{D}}_d), \forall d \in \mathcal{D}. \tag{12d}$$

Here, R_d is the reward for supplying load d, and is equal to M_d if the load has been placed in section 1 and $\beta_d M_d$ if placed in section 0. The equality constraint (12a) captures Kirchoff's current and voltage laws in a compact form; x denotes the collection of bus voltages, angles, and real/reactive power injections across the islanded network. The inequality constraint (12b) captures line limits and bus voltage limits.

$\mathcal{O}_g$ is the post-islanding region of operation for generator g, and depends on the solution of the islanding optimization and pre-islanded outputs of the generator. If $\zeta_d = 1$ the unit remains fully operational, and its output may vary within some region around the pre-islanded operating point; most generally $(p^{\text{G}}_g, q^{\text{G}}_g) \in \mathcal{O}_g\big(p^{\text{G}*}_g, q^{\text{G}*}_g\big)$, where $\big(p^{\text{G}*}_g, q^{\text{G}*}_g\big)$ is the pre-islanding operating point and $\mathcal{O}_g$ is defined by the output capabilities of the generating unit. If real and reactive power are independent, $p^{\text{G}}_g \in \big[P^{\text{G}-}_g, P^{\text{G}+}_g\big]$ and $q^{\text{G}}_g \in \big[Q^{\text{G}-}_g, Q^{\text{G}+}_g\big]$. If, conversely, the islanding optimization has set $\zeta_g = 0$, then real power output is set to zero: $p^{\text{G}}_g = 0$. In that case, the unit may remain electrically connected to the network, with reactive power output free vary within some specified interval $\big[Q^{\text{G}-}_g, Q^{\text{G}+}_g\big]$. Loads are assumed to be homogeneous; real and reactive components are shed in equal proportions.

The AC-OLS is a nonlinear programming (NLP) problem and may be solved efficiently by interior point methods.

V. Numerical Simulations

This section presents numerical simulation results using the above islanding formulation.

A. IEEE 24-bus Reliability Test System

The IEEE RTS [19] comprises 24 buses and 38 lines. Of the buses, 17 have loads attached. All loads are assumed to be constant, and total load demand is 2850 MW. Total generation capacity is 3405 MW from 32 synchronous generators.

The failure scenario we simulate is the consecutive tripping of line $(15, 24)$ followed by line $(3, 9)$. Hazra and Sinha [20] showed this to be the most probable collapse sequence for this network. We consider the network immediately after the first line trip, and our objective is to avoid total network failure by using controlled islanding.

TABLE I

Islanding solutions for different loss models, with $\beta_d = 0.75$.

Loss	Buses in section 0	Cut lines (F_l, T_l)	Disconnected generation
None	1, 3, 24	$(1,2)$, $(1,5)$, $(3,9)$	155 MW at bus 23
Constant	1, 3, 24	$(1,2)$, $(1,5)$, $(3,9)$, $(3,24)$	155 MW at bus 23
Linear	3, 24	$(1,2)$, $(1,3)$, $(3,9)$, $(3,24)$, $(9,12)$, $(15,24)$	None
PWL	1, 3, 24	$(1,2)$, $(1,5)$, $(3,9)$	155 MW at bus 23

Immediately following the failure of line $(15, 24)$ the flow through $(3, 9)$ rises quickly from 25 MVA to 123 MVA. Furthermore, the voltage at bus 3 falls from 1.014 p.u. to 0.883 p.u; similarly, bus 24 has falls from 1.006 p.u. to 0.857 p.u. We suspect that further failures may occur and are uncertain about the status of buses 3 and 24 and line $(3, 9)$.

In taking preventative action, the generator limits are set to allow a small movement from the pre-islanding operating point, $p^{\text{G}*}_g$. Ramp rates, R^{G}_g (MW/min), for the generators may be found in [21]. A time limit of two minutes is assumed for ramping to any new real power level. Thus, limits are set as

$$P^{\text{G}+}_g = \min\{p^{\text{G}*}_g + 2R^{\text{G}}_g, P^{\max}_g\}, \tag{13a}$$
$$P^{\text{G}-}_g = \max\{p^{\text{G}*}_g - 2R^{\text{G}}_g, P^{\min}_g\}. \tag{13b}$$

1) Load shedding without islanding: Solving an AC-OLS on the post-failure network sees 44.1 MW of the 180 MW load at bus 3 shed. The voltages at buses 3 and 24 rise to 0.979 p.u. and 0.950 p.u. respectively, and the power through line $(3, 9)$ falls to 93 MVA.

If, however, the uncertain line $(3, 9)$ subsequently trips, then more load must be shed. A second AC-OLS sheds a further 68.0 MW of the bus 3 load. However, line $(6, 10)$ is at capacity (175 MVA) and—moreover—the uncertain buses 3 and 24 have not been isolated, leaving the operation of the whole network prone to further failures.

2) Islanding: We assign buses 3 and 24 to $\mathcal{B}^0$ and line $(3, 9)$ to $\mathcal{L}^0$, and solve the islanding optimization. The solutions for the four different loss models are described in Table I for a load loss penalty $\beta_d = 0.75$. An 8-piece approximation was used for the PWL model. Common to all but one loss model is the islanding of buses 1, 3 and 24; the linear loss model opts to island only buses 3 and 24. The former choice retains bus 1's generation capability in section 0, while the latter does not. Nevertheless, in all cases the "unhealthy" section has been isolated; furthermore, line $(3, 9)$ has been cut, and so none of the solutions is sensitive to failure of this line.

Table II shows generation levels, load supplied, losses and objective values for the post-islanding DC and AC solutions, and for each of the loss models. Three of the four models elect to island bus 1 in addition to 3 and 24. The linear loss model islands only buses 3 and 24, with no generation capability in that island, and as a consequence the J^* values (both from the islanding optimization and the AC-OLS) are lower.

The PWL model best estimates the losses, with a small over-estimation, and shows the smallest mismatch between DC and AC objective values. The lossless DC and constant-loss models under-estimate losses. However, all three loss models deliver the same AC-OLS objective value.

TABLE II
DC AND AC SOLUTION DATA FOR THE ISLANDED NETWORK.

Loss model	Post-islanding DC $\sum_g p_g^{\mathrm{G}}$	$\sum_d p_d^{\mathrm{D}}$	$\sum_l h_l^{\mathrm{L}}$	J_{DC}^*	Post-islanding AC $\sum_g p_g^{\mathrm{G}}$	$\sum_d p_d^{\mathrm{D}}$	$\sum_l h_l^{\mathrm{L}}$	J_{AC}^*
None	2754.0	2754.0	0.0	2706.0	2800.6	2750.3	50.3	2703.3
Constant	2795.8	2753.9	41.9	2705.9	2800.6	2750.3	50.3	2703.3
Linear	2726.2	2670.0	56.2	2670.0	2723.4	2670.0	53.4	2670.0
PWL	2804.8	2749.9	55.0	2702.9	2800.6	2750.3	50.3	2703.3

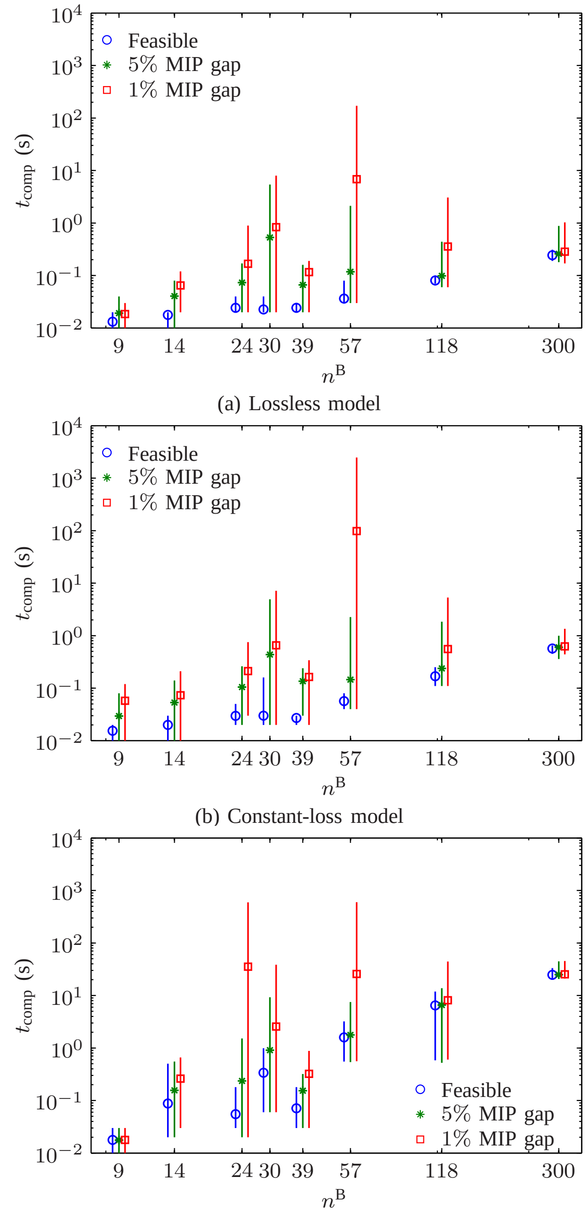
B. Larger networks

1) Computational results: The speed with which islanding decisions have to be made depends on whether the decision is being made before a fault has occurred as part of contingency planning within secure OPF, or after a problem has occurred, in which case the time scale depends on the cause of the contingency. Especially in the second case it is important to be able to produce feasible solutions within short time periods even if these are not necessarily optimal.

Fig. 2 shows the times required to find obtain feasible islanding solutions to varying proven levels of optimality. Times are recorded for different networks ranging from a 9-bus system to a 300-bus system. Three of the four loss models are compared; the linear model is omitted. For each network, 50 scenarios were generated by assigning a single randomly-chosen bus to $\mathcal{B}^0$. The same set of scenarios is simulated for each loss model. No pre-assignments were made to either $\mathcal{B}^1$ or $\mathcal{G}^1$. For the networks with no ramp rates or spinning reserve data available, it is assumed that each generator may vary its output by $\pm 5\%$ of the pre-islanding level. Where no line limits are present for a network, a maximum phase angle difference of 0.4 rad is imposed for each line. The PWL model assumes an 8-piece approximation to the line loss over the phase angle difference interval $[-0.4, 0.4]$ rad. In the objective function, the values of ϵ_1 and ϵ_2 in (9) and (10)—the penalties on line cuts and generator disconnection respectively—are 0.1 and 0.0001, with $W_g = P_g^{\mathrm{G}+}$ in the latter. This penalizes line disconnection more heavily.

Problems are solved on a dual quad-core 64-bit Linux machine with 8 GiB RAM, using AMPL 11.0 with parallel CPLEX 12.3 to formulate and solve MILP problems. Computation times quoted include only the time taken to solve the islanding optimization to the required level of optimality, and not the AC-OLS, and are obtained as total elapsed seconds used by CPLEX during the `solve` command. The required levels of optimality for each problem are 'feasible'—an integer feasible solution—and relative MIP gaps of 5% and 1%. The PWL loss model is implemented using AMPL's piecewise linear function builder notation and special ordered sets of type 2 (SOS-2) in CPLEX. An additional penalty of 10^{-4} times the total line loss (MW) is imposed in the PWL case, to encourage the SOS-2 conditions to be met in solutions of the LP relaxations at nodes in the branch and bound (B&B) tree. This was found to significantly aid computation.

Examining first the times required to find a feasible islanding solution, the results in fig. 2 show a rise in solve time as the network size increases. The lossless and constant-loss models perform well: all problems are solved to feasibility well within 1 s. In every case tested, a feasible solution is found at the root note, aided by CPLEX's cut generation, without requiring branching; thus, the rise in computation time is largely owing to the increasing size of the LP relaxation problem.



(a) Lossless model

(b) Constant-loss model

(c) PWL model

Fig. 2. Mean, max and min times for finding, to different levels of optimality, islanding solutions using each of the loss models.

For the PWL model, while times less than one second are recorded for networks up and including to the 39-bus system, solution times to feasibility rise thereafter. In particular, the mean and maximum times to feasibility for the 300-bus network are approaching 30 seconds. An immediate observation is that the LP relaxation problems are larger, since each line

TABLE III
RELATIVE ERRORS BETWEEN OPTIMAL AND RETURNED SOLUTIONS.

	Feasible	5% gap	1% gap
Lossless	10.79%	0.31%	0.03%
Constant loss	9.12%	0.37%	0.04%
PWL	2.31%	0.50%	0.05%

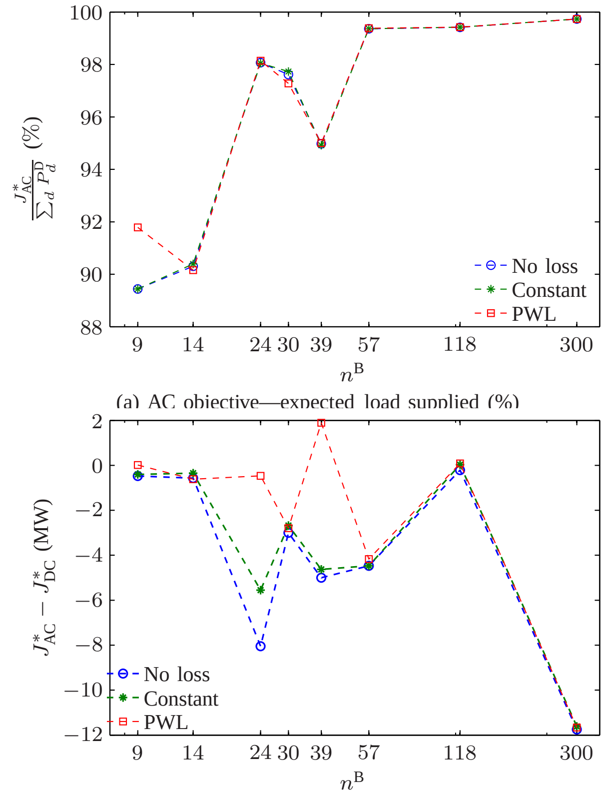
has an additional 8 SOS variables. Secondly, more branching is required, in order to satisfy the SOS-2 conditions on every line, as the network size increases. In the worst case, for example, $10,794$ B&B nodes were required to find a feasible solution for one 300-bus problem.

For a MILP problem solved by branching, the optimal integer solution is bounded from below (for maximization) by the highest integer objective value found so far during the solution process, and from above by an objective value deduced from all node subproblems solved so far. The relative MIP gap is the relative error between these two bounds. Fig. 2 indicates the progress made by the CPLEX solver, in terms of the times required to reach relative MIP gaps of 5% and 1% respectively. Performance of the lossless and constant-loss models is again good; the majority of problems are solved to 1% optimality within ten seconds. The exception is the 57-bus network. While all 57-bus lossless and constant-loss problems are solved to 5% optimality within two seconds, the times to 1% MIP gap can be significantly longer.

Future work will investigate heuristics and techniques for exploiting network topology and improving solution times. One practical consideration is that it may be desirable to make assignments to the sets $\mathcal{B}^1$ and $\mathcal{G}^1$, leaving fewer free variables in each optimization and reducing computation times further.

For practical application in real time—with the network in a stressed condition—a good, but possibly sub-optimal, integer solution may be acceptable, given that islanding is a last resort course of action and fast decision making is required. Moreover, because the DC model is an approximation of the AC model, it may make little sense to pursue proven optimal DC solutions. Table III shows the means of the relative errors between the solution value returned at termination of the solver and the actual optimum, where known. The 'real' gaps between early termination solutions and the true optima are nearer zero than 5% or 1%. Therefore, good islanding solutions—at least with respect to the DC model—can be provided even when the solver is terminated early. Moreover, these solutions can be found quickly with either the lossless or constant-loss models. The PWL model generally requires longer solve times; one question is whether the extra computation time, and more accurate loss modelling, provides better islanding solutions. In the following subsection, we investigate the quality of these solutions with respect to the AC model.

2) AC performance: Fig. 3 provides two comparisons; the mean values of the post-islanding AC objective and, secondly, the error between the objectives as predicted by the DC islanding optimization and the post-islanding AC-OLS. For the former, to enable easier comparison the AC objective is expressed relative to the total load; a value of 100% means that no load has been shed or assigned to section 0—the best possible outcome. The adopted islanding solution in each



(a) AC objective—expected load supplied (%)

(b) Difference between DC and AC objective values

Fig. 3. AC performance—absolute and relative objective values obtained from post-islanding DC and post-islanding AC optimizations.

TABLE IV
NUMBER OF UNIQUE PROBLEMS INCLUDED IN THE COMPARISONS.

n^{B}	9	14	24	30	39	57	118	300
Problems compared	8	11	10	16	15	20	30	23
MIP gap $> 0\%$	0	0	1	0	0	5	5	14
AC infeasible	0	2	10	6	8	4	5	8
Total	8	13	21	22	23	29	40	45

case is that from solving the problem to full optimality. Since some PWL problems require a long time to solve, only those problems solved to optimality within 10^4 seconds, for all models, are included. Furthermore, a number of islanding solutions were found to be AC infeasible, and so were removed from the comparison. (The AC infeasibility problem is discussed in the next section.) Tab. IV indicates the number of problems included in the comparison for each network.

Examining the relative performance of the loss models in fig. 3(a), with the exception of the 9-bus network, there is very little difference between the islands formed by the different models with respect to the AC objective.

The comparison of DC and AC objective values, shown in fig. 3(b), illustrates that the ability of the DC model to predict the AC objective depends on the loss model. The measure $J^*_{\mathrm{AC}} - J^*_{\mathrm{DC}}$ indicates an over-estimation of the load shed/lost if positive, and an under-estimation if negative. The PWL model is generally nearer zero and in one case is positive; on the other hand, the lossless and constant-loss models always under-estimate the load shed or lost.

In conclusion, despite the more accurate modelling of line losses, the PWL model rarely provides better islanding solutions with respect to AC power flow. In comparison, the simpler loss models allow solutions to be found quickly, without degradation in the quality of the resulting AC solution. Accurate modelling of line losses, then, should not be a primary consideration in designing network islands; a more pressing concern, as indicated by tab. IV, is the AC feasibility of the network after islanding.

C. Network voltage profile

A number of islanding solutions obtained by solving the MILP problem were subsequently found to be AC infeasible; that is, there was no solution to the AC-OLS lying within normal voltage bounds. In fact, by softening the normal voltage bounds a solution was found to all of the 'infeasible' instances in tab. IV. This subsection analyses a case study of such voltage-infeasible situation. The conclusion is that designing network partitions by consideration of real and reactive power balances in each island is not sufficient to produce solutions with a good voltage profile.

Consider the 24-bus network with bus 6 assigned to $\mathcal{B}^0$ and $\beta_d = 0.75$. The optimal islanding solution obtained isolates buses 1, 2 and 6 by disconnecting lines $(1,3)$, $(1,5)$, $(2,4)$ and $(6,10)$. Though two 19-MW units are switched off at buses 1 and 2 respectively, there remains sufficient real power capacity in both islands to meet demand, and no load is shed. Moreover, there is sufficient reactive power capacity in each island to meet the total reactive power demand. Even though the islands are balanced, the AC-OLS fails to find a feasible solution. Softening the voltage constraints allows a solution to be recovered, but with out-of-bound voltages at buses 2 and 6 ($v_2 = 1.1461$ and $v_6 = 0.8452$). This results in 95.5 MVAr being extracted from the line $(2,6)$ at bus 6, yet the power demand there is only 24.1 MVAr. However, a shunt reactor at bus 6 consumes $100v_6^2$ MVAr. To meet this demand, an abnormally large voltage drop is required across the line $(2,6)$. If the shunt reactor is removed, or if a synchronous condenser is placed at the bus, a feasible AC-OLS solution with voltages within limits can be found.

Further inspection of the network reveals that this situation has arisen because of the disconnection of line $(6,10)$, an underground cable with high shunt capacitance. In normal operation, the passive reactor at bus 6 would locally balance the reactive power and maintain a satisfactory voltage profile.

This is just one example of where an islanding solution formed by considering only real power—even if network constraints are included—is unsatisfactory. However, it also shows that even if a global reactive power balance is achieved, local shortages or surpluses of reactive power can lead to an abnormal voltage profile. Many of the IEEE test networks are prone to the same problem, as observed from our results.

VI. Conclusion

In this paper, an optimization-based approach to controlled islanding and load shedding has been presented. The proposed method uses MILP to determine which lines to cut, loads to shed, and generators to switch in order to isolate an uncertain or failure-prone region of the network. The optimization framework allows linear network constraints—a loss-modified DC power flow model, line limits, generator outputs—to be explicitly included in decision making, and produces balanced, steady-state feasible DC islands. AC islanding solutions are found via the subsequent solving of an AC optimal load shedding problem. The approach has been demonstrated through simulations on the 24-bus IEEE system. Simulations on larger networks have indicated the practicality of the method, in terms of computational time, and have shown that the quality of islanding solutions does not benefit from the accurate modelling of real power losses. Thus, line loss modelling has been found to be less important than the modelling of reactive power to ensure a healthy voltage profile in all parts of the network after islanding. Future research will investigate methods for improving computation times for islanding, and techniques for finding feasible and optimal AC solutions.

References

[1] "Final Report of the Investigation Committee on the 28 September 2003 Blackout in Italy," UCTE, Final Report, April 2004.

[2] S. Larsson and A. Danell, "The black-out in southern Sweden and eastern Denmark, September 23, 2003," in *IEEE PSCE*, 2006.

[3] U.S.-Canada Power System Outage Task Force, "Final Report on the August 14, 2003 Blackout in the United States and Canada: Causes and Recommendations," Final Report, Apr 2004.

[4] "Final Report System Disturbance on 4 November 2006," UCTE, Final Report, 2007.

[5] "Report of the National Grid Investigation into the Frequency Deviation and Automatic Demand Disconnection that occurred on the 27th May 2008," National Grid, Final Report, February 2009.

[6] J. W. Bialek, "Are blackouts contagious?" *IEE Power Engineer*, vol. 17, no. 6, p. 10, Dec/Jan 2003.

[7] ——, "Blackouts in the US/Canada and continental Europe in 2003: is liberalisation to blame?" in *IEEE PowerTech Conf.*, 2005.

[8] G. Andersson et al., "Causes of the 2003 major grid blackouts in North America and Europe, and recommended means to improve system dynamic performance," *IEEE T. Power Syst.*, pp. 1922–1928, 2005.

[9] D. E. Newman, B. A. Carreras, M. Kirchner, and I. Dobson, "The impact of distributed generation on power transmission grid dynamics," in *Int. Conf. Syst. Sci.*, 2011.

[10] B. Yang, V. Vittal, and G. T. Heydt, "Slow-coherency-based controlled islanding—a demonstration of the approach on the August 14, 2003 blackout scenario," *IEEE T. Power Syst.*, vol. 21, pp. 1840–1847, 2006.

[11] K. Sun, D.-Z. Zheng, and Q. Lu, "Splitting strategies for islanding operation of large-scale power systems using obdd-based methods," *IEEE T. Power Syst.*, vol. 18, pp. 912–923, 2003.

[12] H. You, V. Vittal, and X. Wang, "Slow coherency-based islanding," *IEEE T. Power Syst.*, vol. 19, no. 1, pp. 483–491, Feb 2004.

[13] X. Wang and V. Vittal, "System islanding using minimal cutsets with minimum net flow," in *IEEE PSCE*, 2004.

[14] G. Xu and V. Vittal, "Slow coherency based cutset determination algorithm for large power systems," *IEEE T. Power Syst.*, vol. 25, no. 2, pp. 877–884, May 2010.

[15] M. Jin, T. S. Sidhu, and K. Sun, "A new system splitting scheme based on the unified stability control framework," *IEEE T. Power Syst.*, vol. 22, pp. 433–441, 2007.

[16] C. G. Wang, B. H. Zhang, Z. G. Hao, J. Shu, P. Li, and Z. Q. Bo, "A novel real-time searching method for power system splitting boundary," *IEEE T. Power Syst.*, vol. 25, pp. 1902–1909, 2010.

[17] D. Bienstock and S. Mattia, "Using mixed-integer programming to solve power grid blackout problems," *Discrete Optim.*, pp. 115–141, 2007.

[18] E. B. Fisher, R. P. O'Neill, and M. C. Ferris, "Optimal transmission switching," *IEEE T. Power Syst.*, vol. 23, no. 3, pp. 1346–1355, 2008.

[19] Reliability Test System Task Force of the Application of Probability Methods Subcommittee, "IEEE reliability test system," *IEEE T. Power Ap. Syst.*, vol. PAS-98, no. 6, pp. 2047–2054, 1979.

[20] J. Hazra and A. K. Sinha, "Prognosis of catastrophic failures in electric power systems," in *IEEE Int. Conf. Ind. Tech.*, 2006, pp. 1349–1354.

[21] Reliability Test System Task Force of the Application of Probability Methods Subcommittee, "IEEE reliability test system – 1996," *IEEE T. Power Syst.*, vol. 14, no. 3, pp. 1010–1020, 1999.