Strings and the Geometry of Particle Physics

Cumrun Vafa

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A conference in honor of the 80th Birthday of Sir Michael Atiyah

This talk is based on the work of many physicists.

The more recent material I will present on F-theory is based on work I have done with my student Jonathan Heckman, and some of them include additional colleagues (Chris Beasley, Alireza Tavanfar, Vincent Bouchard, Jihye Seo, Miranda Cheng, Sergio Cecotti). Related work includes the work of Wijnholt and Donagi as well as Tatar etal.

Here I aim to draw a geometric picture of particle physics using modern ideas of theoretical physics as has been discovered in the context of string theory. I will start with a series of experimental facts and discuss how we can embed them in string theory and what this exercise teaches us.

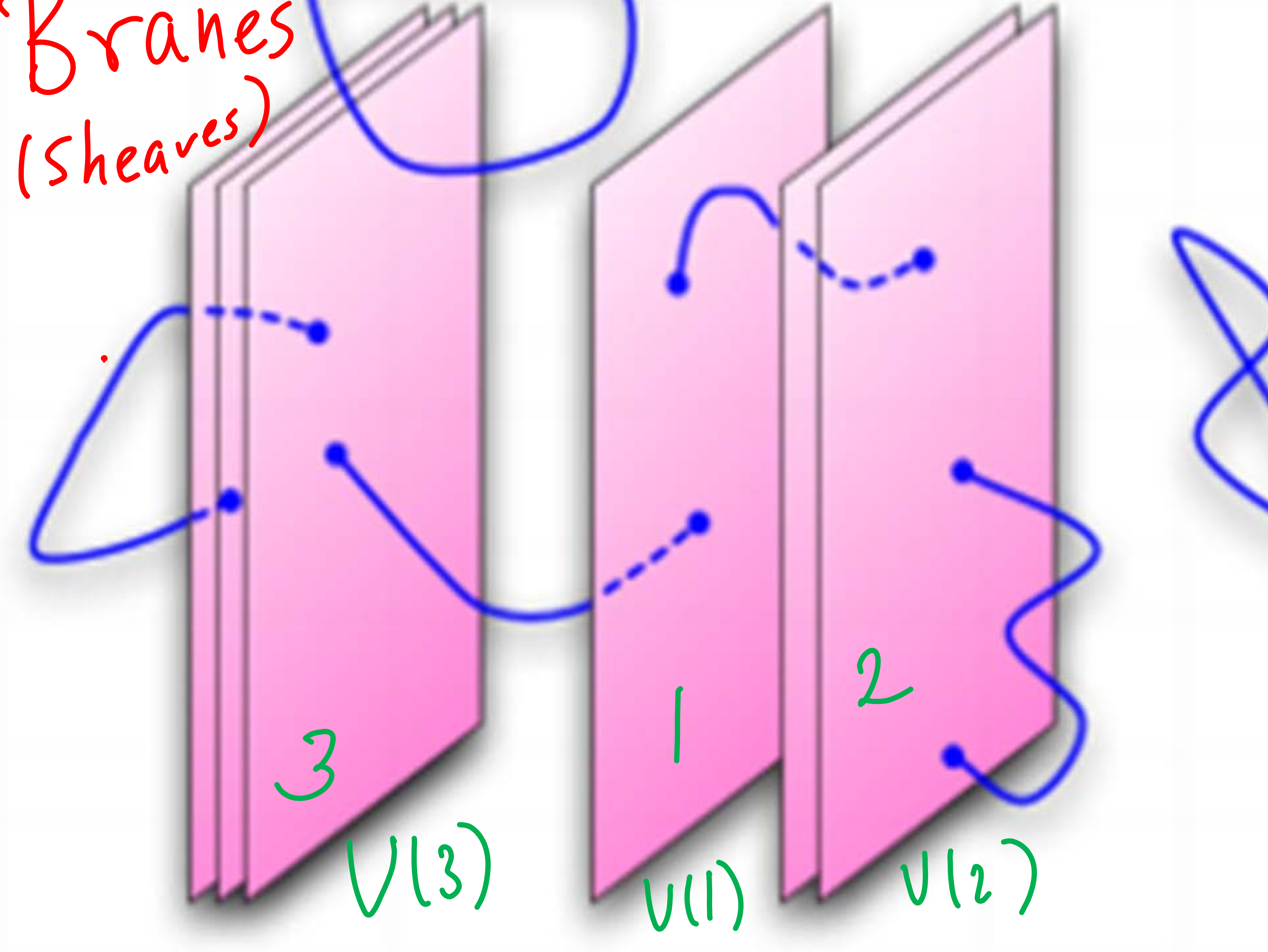
I will start with the main experimental fact, that has been known for a long time: The existence of gravitational force. Combining this fact with the modern age discovery of quantum theory leads to the natural question of how to understand quantum gravity.

This is precisely the unique defining property of string theory: It is currently our only consistent framework of a quantum theory of gravity.

Another important fact of nature is that gauge symmetry is an important principle of physics and is the underlying explanation of all forces in nature (with the exception of gravitational force). In particular we know that the gauge symmetry realized at energy scales which are presently probed in accelerators is:

$$SU(3)xSU(2)xU(1)$$

We ask how gauge symmetries are realized in string theory. It turns out we have a multitude of ways of doing this:

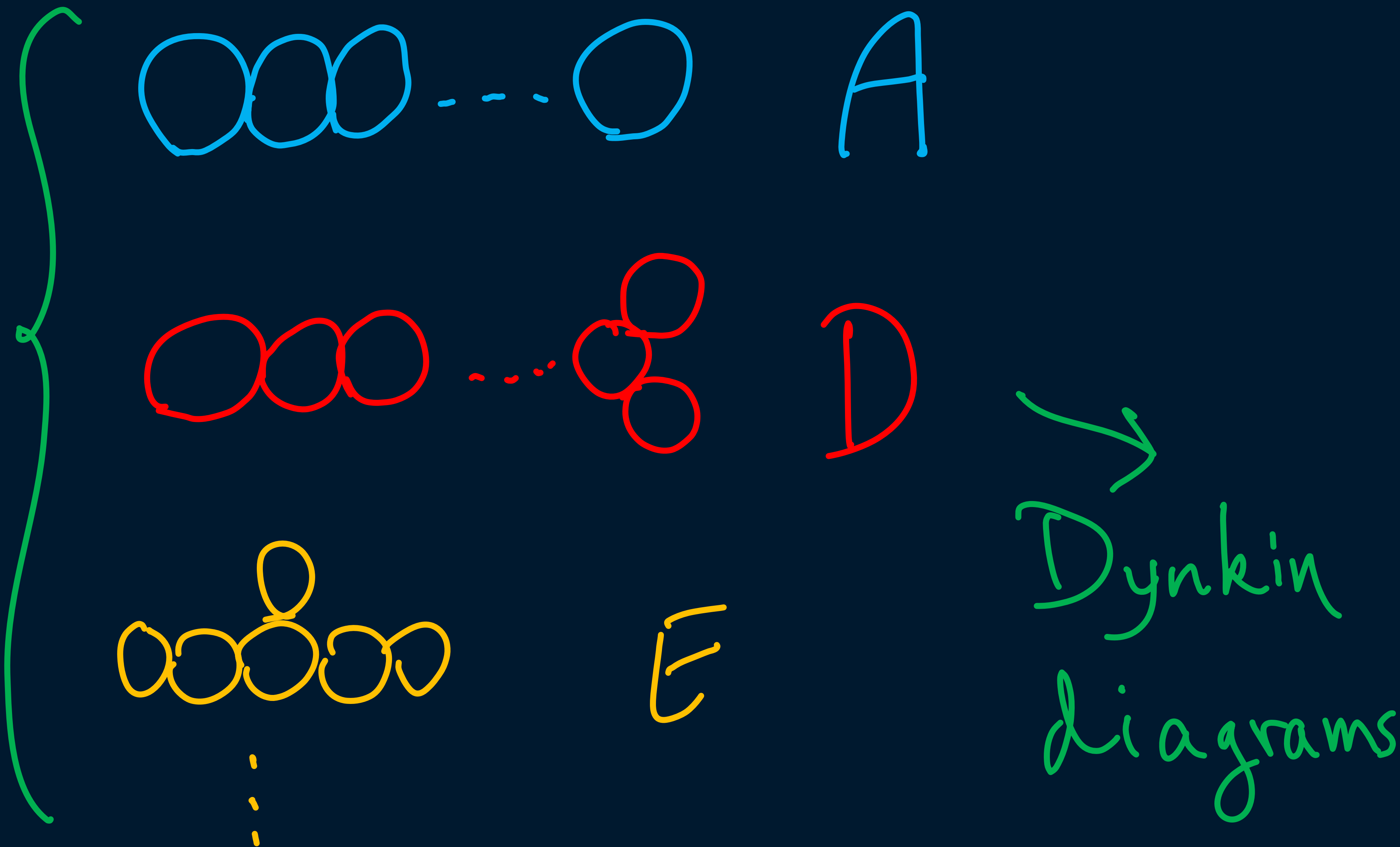

Branes
(Sheaves)
3
1
2
U(3)
U(1)
U(2)

Another way gauge theory arises in string theory is by having A-D-E singularities:

$$\mathbb{C}^2/\Gamma$$

$$\underbrace{\Gamma}_{\text{A-D-E}} \in SU(2)$$

blown up geometry

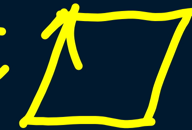


$$M^{10} = \underbrace{M^4}_{\text{Minkowski}} \times X^6$$

A-D-E sing. → X^6

codimension 4

F-theory

τ 

$$M^{10+2} = \underbrace{M^4}_{\text{Minkowski}} \times X^{6+2}$$

X^8: elliptic CY 4-fold with { A-D-E sing. codimension 4

Depending on what is the locus of the A-D-E singularity we obtain different theories in 4 dimensions. In the context of F-theory this locus is:

$$R^4 \times S^4$$

where $S^4 \subset X^{6+2}$

$$\cap \qquad \downarrow$$

$$B^6$$

d=4 SUSY

If

$S^4 = T^4 \Longrightarrow \mathcal{N}=4$

$S^4 = K3 \Longrightarrow \mathcal{N}=2$

$S^4 = \mathbb{P}^2 \Longrightarrow \mathcal{N}=1$

e.g.

physically most interesting

Grand Unification of Gauge Forces

The idea that we can potentially combine the three gauge groups into one, is an old idea, dating back to the work of Georgi and Glashow, and similar models by Pati and Salam:

SU(3) £ SU(2) £ U(1) ½ SU(5)

If the couplings of SU(3) and SU(2) and U(1) were equal, we could have imagined a simpler structure with a simple group being responsible for the gauge forces: $SU(3) \times SU(2) \times U(1) \subset SU(5)$

$$\sum_{i=1}^{3} \frac{1}{\alpha_i} \mathrm{tr}\, F_i \wedge * F_i$$

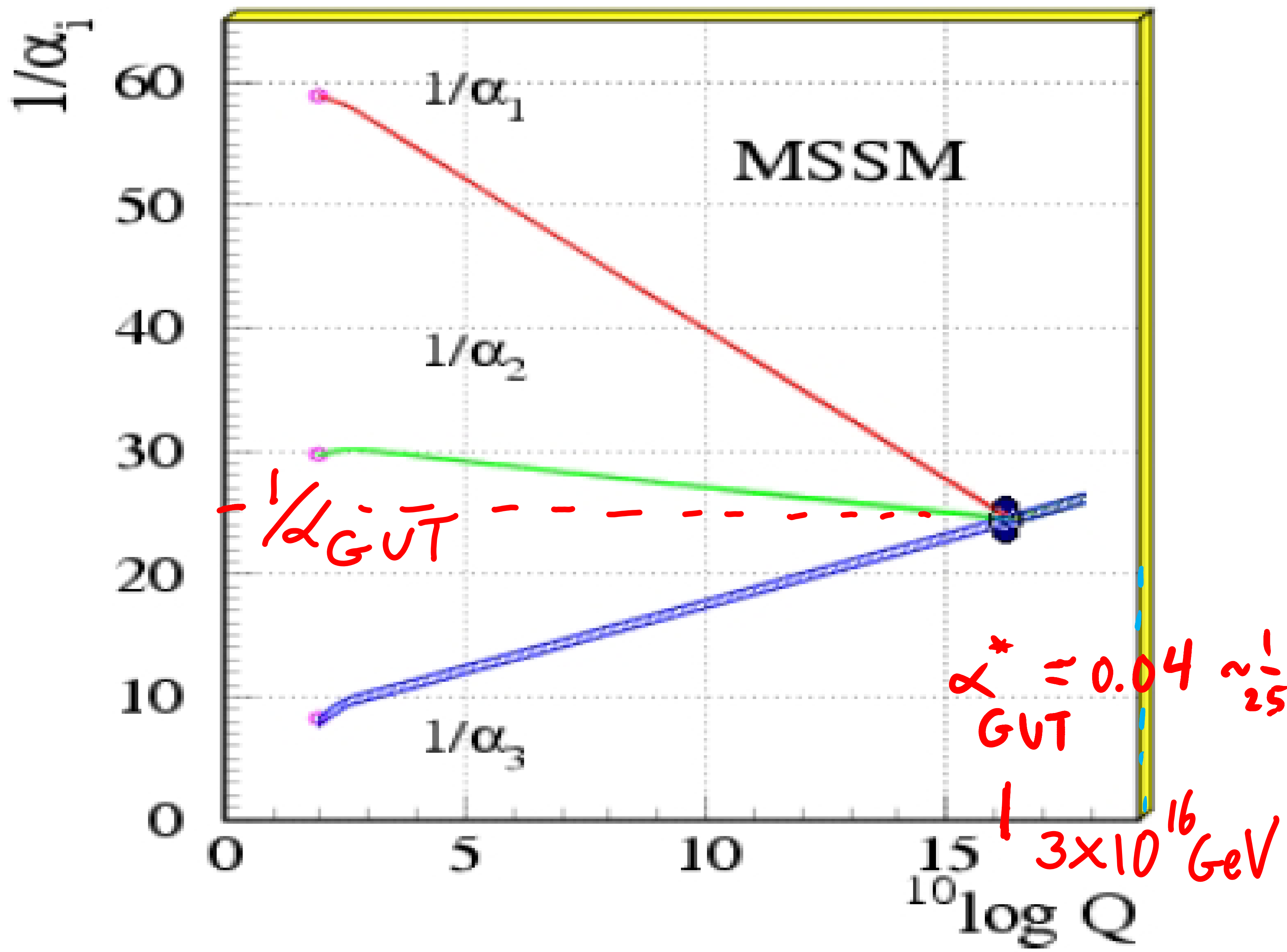
$$\downarrow \quad \alpha_i = \alpha$$

$$\frac{1}{\alpha} \mathrm{tr}\, F^{SU(5)} \wedge * F^{SU(5)}$$

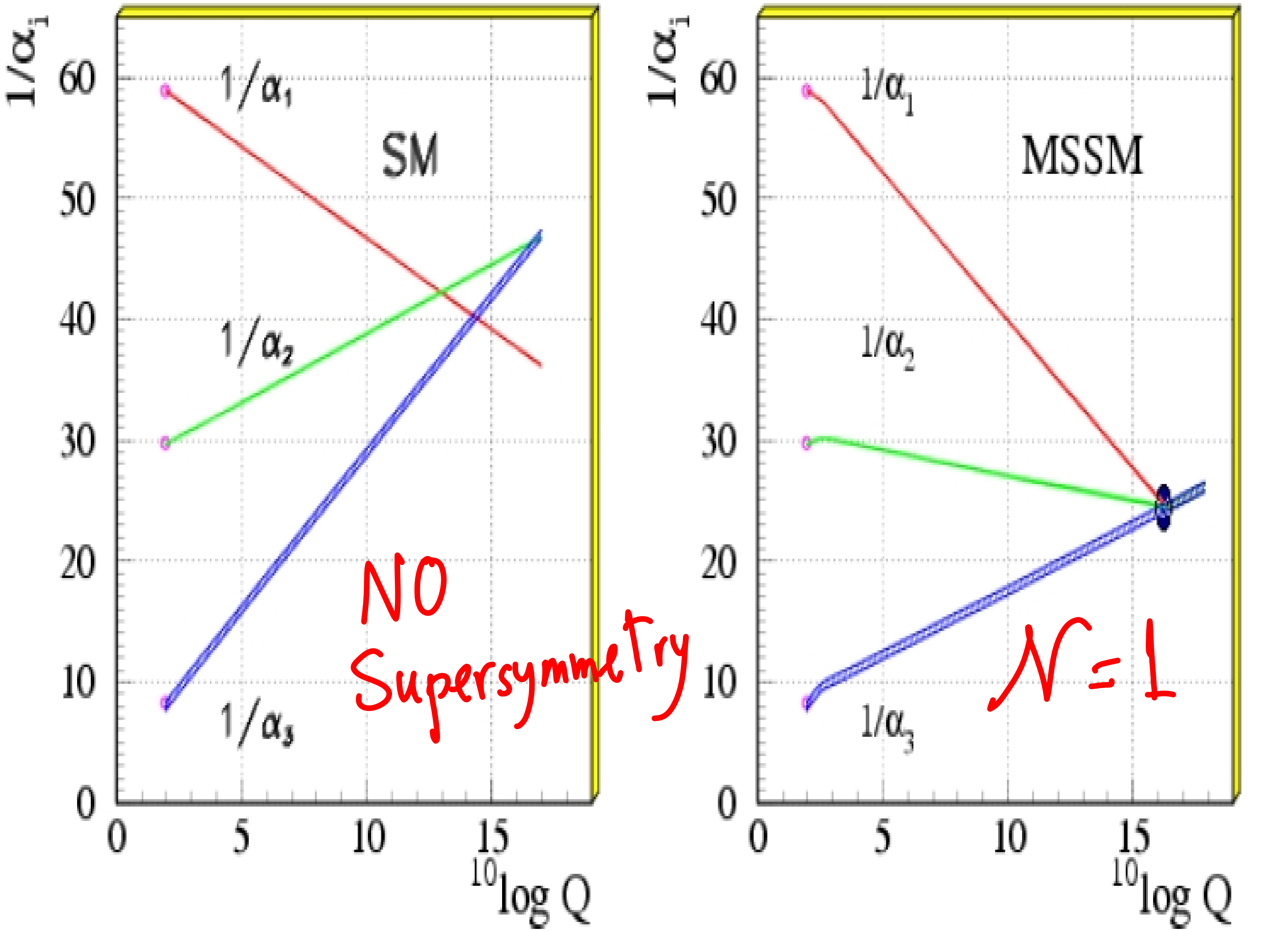
It is well known that the parameters that we measure in physics depends on scale. This is due to quantum corrections. This in particular applies to the coupling constants α_i.

$$\delta S \rightarrow \ln\left[\det\left(\not{D}_A^2 + m_k^2\right)\right] \rightarrow \text{Corrects } \alpha_i$$

Thus even though we start with fixed classical value for the couplings, in the quantum theory they vary. This is welcome as it is not true that at the energy scales available in labs coupling constants are equal.



$1/\alpha_i$
60
50
40
30
20
10
0
$1/\alpha_1$
MSSM
$1/\alpha_2$
$1/\alpha_3$
$1/\alpha_{GUT}$
$\alpha^*_{GUT} = 0.04 \sim \frac{1}{25}$
3×10^{16} GeV
0
5
10
15
$^{10}\log Q$



1/α_i
60
50
40
30
20
10
0
1/α_1
1/α_2
1/α_3
SM
NO
Supersymmetry
0
5
10
15
10log Q
1/α_i
1/α_1
1/α_2
1/α_3
MSSM
N = 1
10log Q

It is relatively simple to implement the idea of gauge symmetry breaking in string theory: We simply consider a configuration in the internal compact geometry of string theory where the gauge bundle is non-trivial (either by having non-trivial holonomies or field strengths), leaving a reduced symmetry group at lower scales.

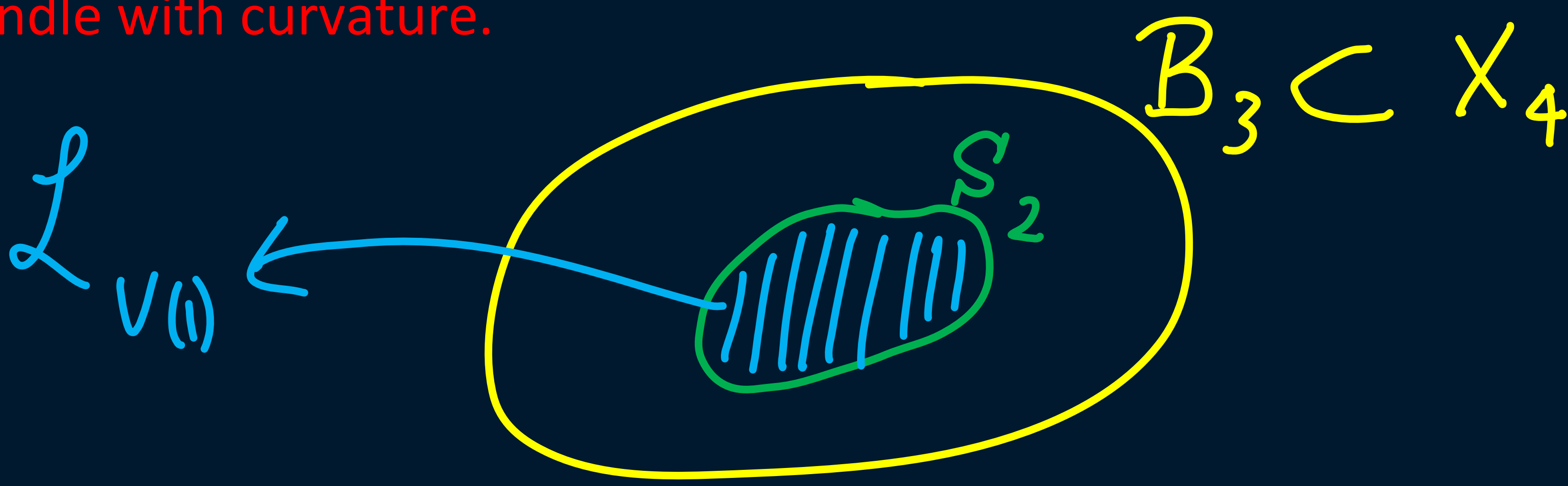
$$X \in i \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} \in U(1)$$

use $A_{U(1)} \neq 0 \Rightarrow SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$

There are two specific ways this idea has been implemented:

In the context of heterotic strings, it is natural to break the GUT group SU(5) by having a U(1) gauge bundle which is flat but with non-trivial holonomy. This requires the assumption that the compactification manifold has in particular a non-trivial fundamental group.

My main focus in this talk is on F-theory.
In the context of F-theory it is natural to consider non-trivial U(1) bundle with curvature.



In this context it is thus natural to identify the unification scale to be smaller than the scale at which the gauge bundle `breaks' the gauge symmetry to smaller group.
In other words the scale of unification of forces is a distance scale where we cannot distinguish the internal gauge bundle from that of a trivial SU(5) bundle.

$$R_S \sim \frac{1}{M_{GUT}}$$

$$F^{U(1)} \sim 1/R^2$$

The dictionary for F-theory thus far is the following:

The section of the elliptic fourfold = fano 3-fold

The locus where elliptic fiber degenerates = brane

The Kodaira-type of the singularity=gauge theory on the brane

This is very encouraging: We can `cook up' whatever gauge group we desire geometrically!

Matter Fields

In addition to gauge fields, there are also matter fields.
Quarks and Leptons transform as some representations of the gauge group SU(3) x SU(2) x U(1) and are sections of an associated vector bundle:

$$\text{Quarks}: \quad (u_L; d_L): (3; 2; \tfrac{1}{6}); \quad u_R: (3; 1; \tfrac{2}{3}); \quad d_R: (3; 1; -\tfrac{1}{3});$$

$$\text{Leptons}: \quad (e_L; \nu_L): (1; 2; -\tfrac{1}{2}); \quad e_R: (1; 1; -1); \quad \nu_R: (1; 1; 0);$$

Looks a little complicated!

Another evidence for unification of forces (and in my opinion a much stronger evidence) is that the matter representations dramatically simplify by going to a unifying gauge group:

$$10 + \bar{5} + 1$$

$$A_{ij} \,, \quad B_{\bar{i}} \,, \quad 1$$

$$\left(\begin{smallmatrix} \square \\ \square \end{smallmatrix} \,, \; \bar{\square} \,, \; \cdot \right)$$

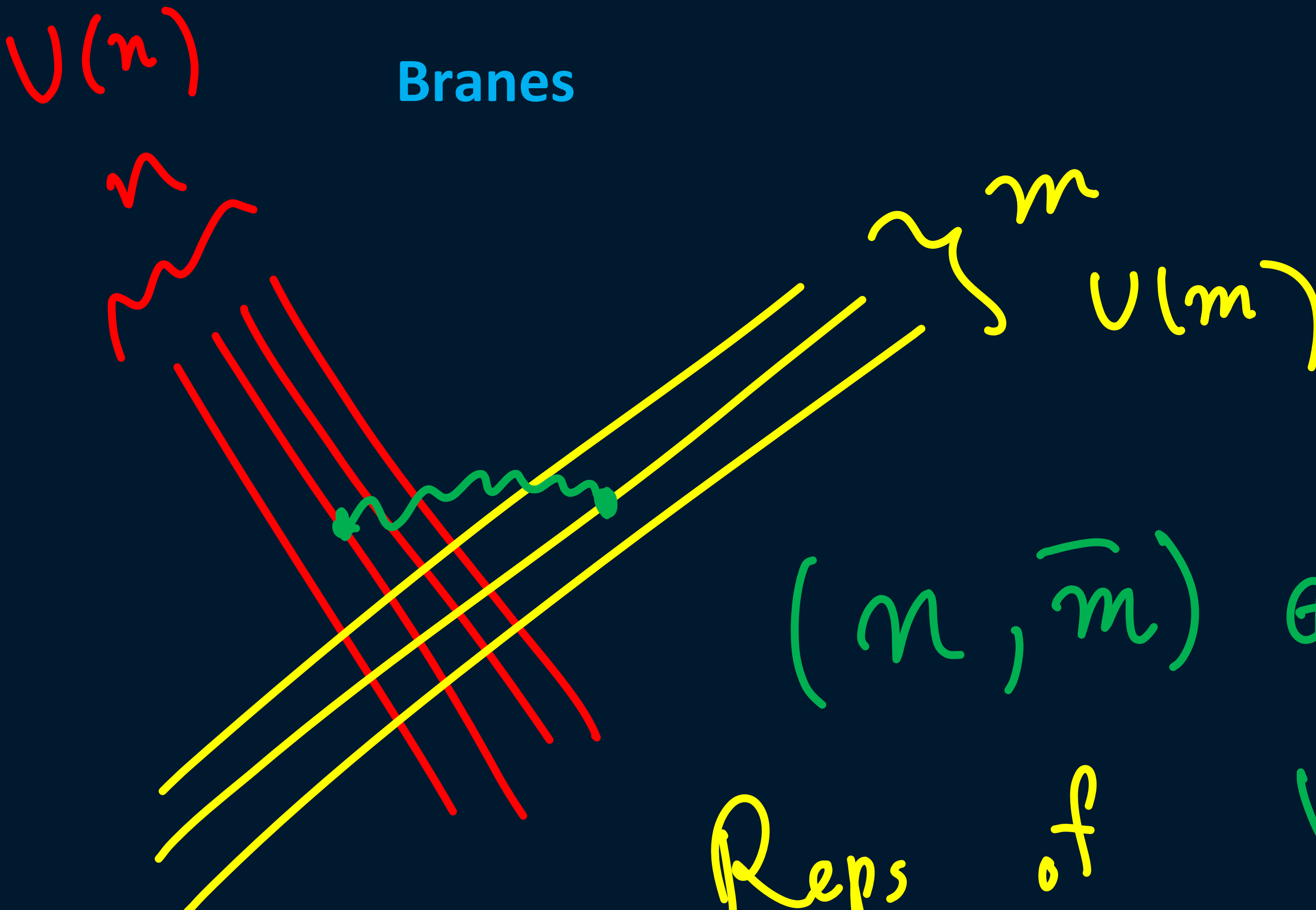
In fact by going to an even bigger unifying group the matter representations also unify:

$$SO(10) \supset SU(5)$$

$$16_{\text{spinor}} \rightarrow 10 + \bar{5} + 1$$

How do we get matter fields from string theory?

Branes



$$(n, \bar{m}) \oplus (\bar{n}, m)$$

Reps of $U(n) \times U(m)$

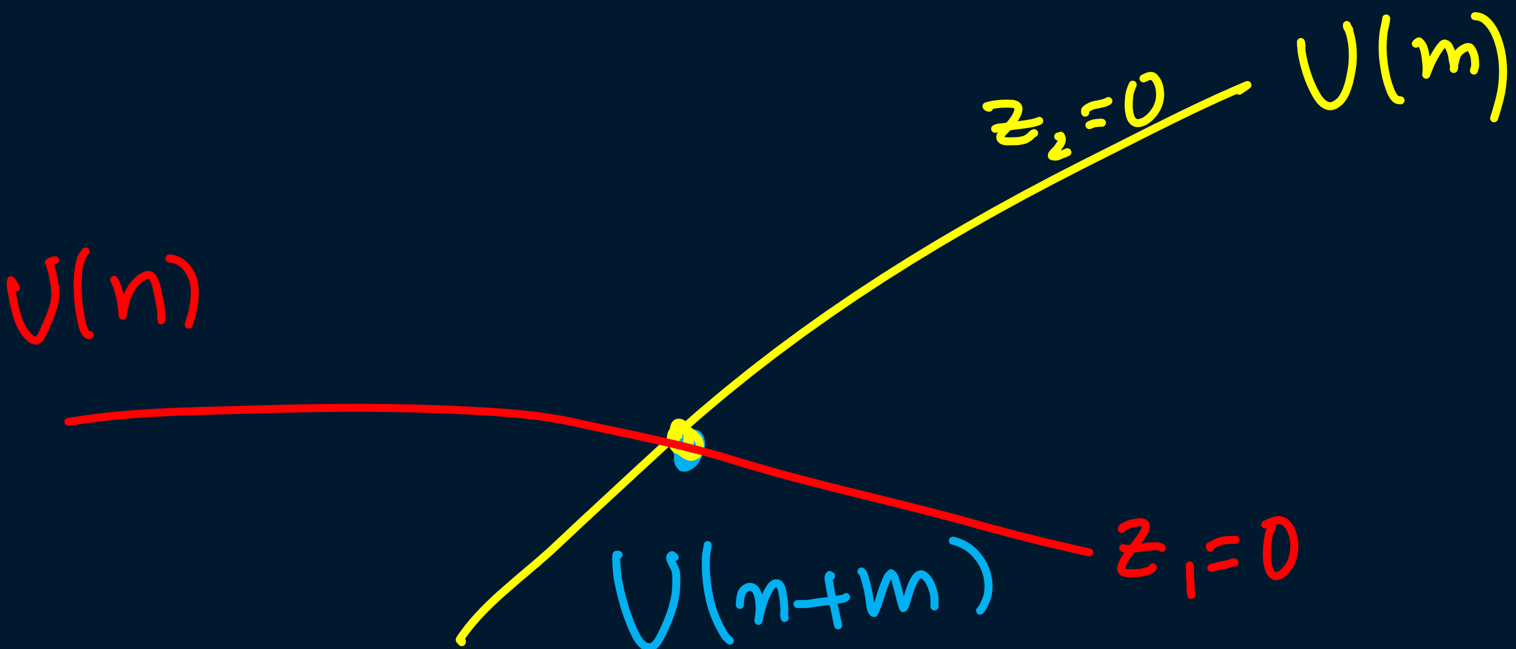
The same idea also works in the context of singularities: Intersecting singularities give rise to matter which lives on the intersection locus:

$$y^2 = x^2 + (z-a)^n (z-b)^m$$

$$x=y=0, \begin{cases} z=a & V(n) \\ z=b & V(m) \end{cases}$$

Intersection $\quad a=b \Rightarrow y^2 = x^2 + (z-a)^{n+m}$

$$\Rightarrow V(n+m)$$



Higgs

$$\phi_{n+m,n+m} = \begin{pmatrix} z_1 & 0 \\ 0 & z_2 \end{pmatrix}$$

$$U(n+m) \longrightarrow U(n) \times U(m)$$

Block diagonal elements of U(n+m) lead to connections of the U(n)xU(m) . The block off diagonal elements, become the matter field in

$$(n, \bar{m}) + (\bar{n}, m) .$$

The generalization of this story to other local Higgs bundles is simple: We have a codimension 2 locus where two singularities meet and give rise to a more singular locus, i.e., a bigger local gauge group, which is locally Higgsed.

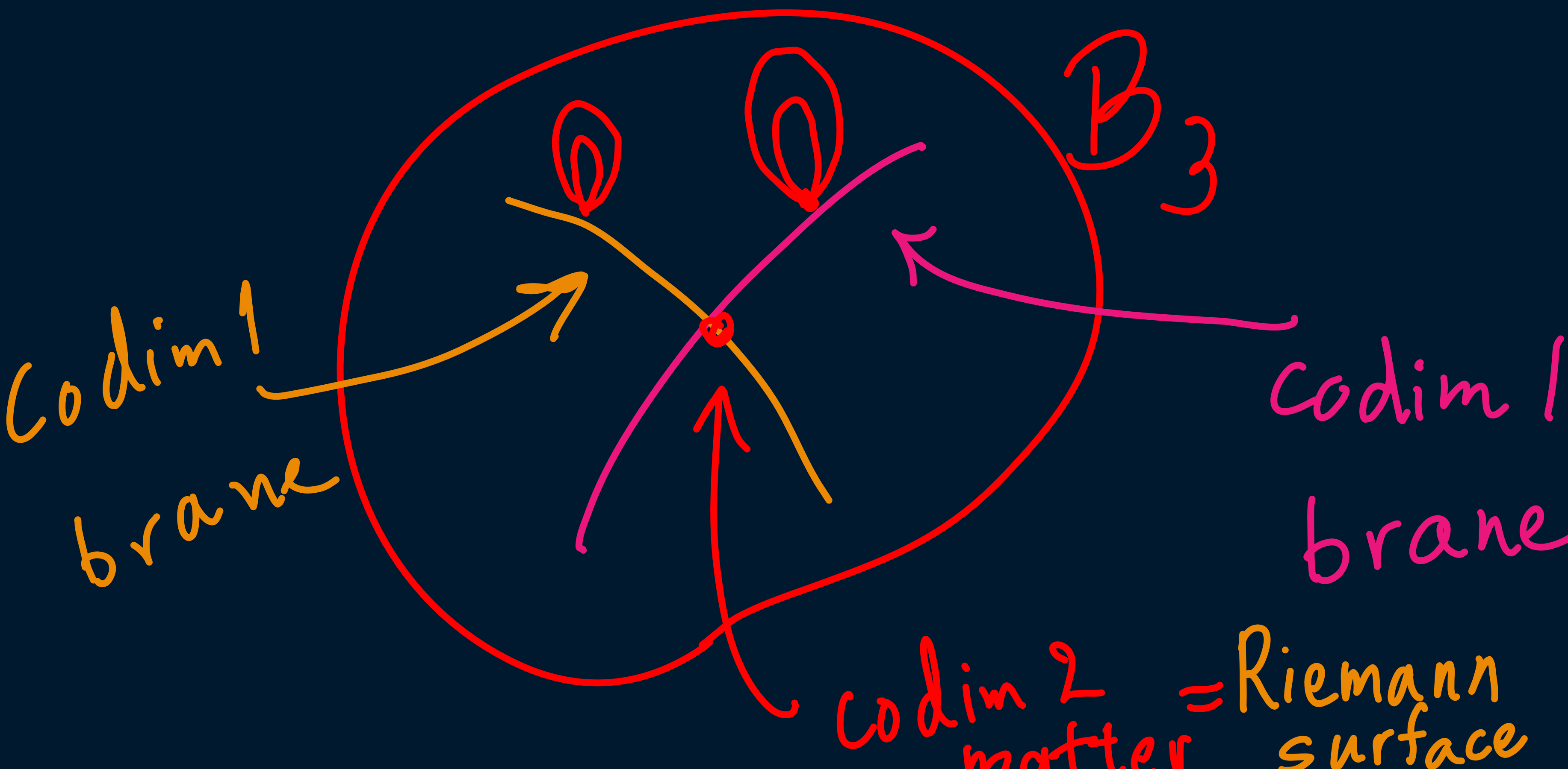
$$G \supset G_1 \times G_2$$

$$\mathrm{Adj}\, G \supset \mathrm{Adj}\, G_1 \oplus \mathrm{Adj}\, G_2 + \underbrace{R(G_1, G_2)}$$

matter rep.
living on the intersection

Matter

So for F-theory matter resides on the loci of colliding elliptic singularities.



It is relatively easy to get matter fields in the fundamental representations, or even the rank 2 representations of classical groups.

e.g. $$U(n) \subset SO(2n)$$

$$\Rightarrow A_{ij} : \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}$$

$$U(1) \times U(n) \subset U(n+1) \Rightarrow \underset{}{\overset{U(n)}{\begin{array}{|c|}\hline \\ \hline \end{array}}}$$

But how about the one of special interest for particle physics, namely the spinor of SO(10)?

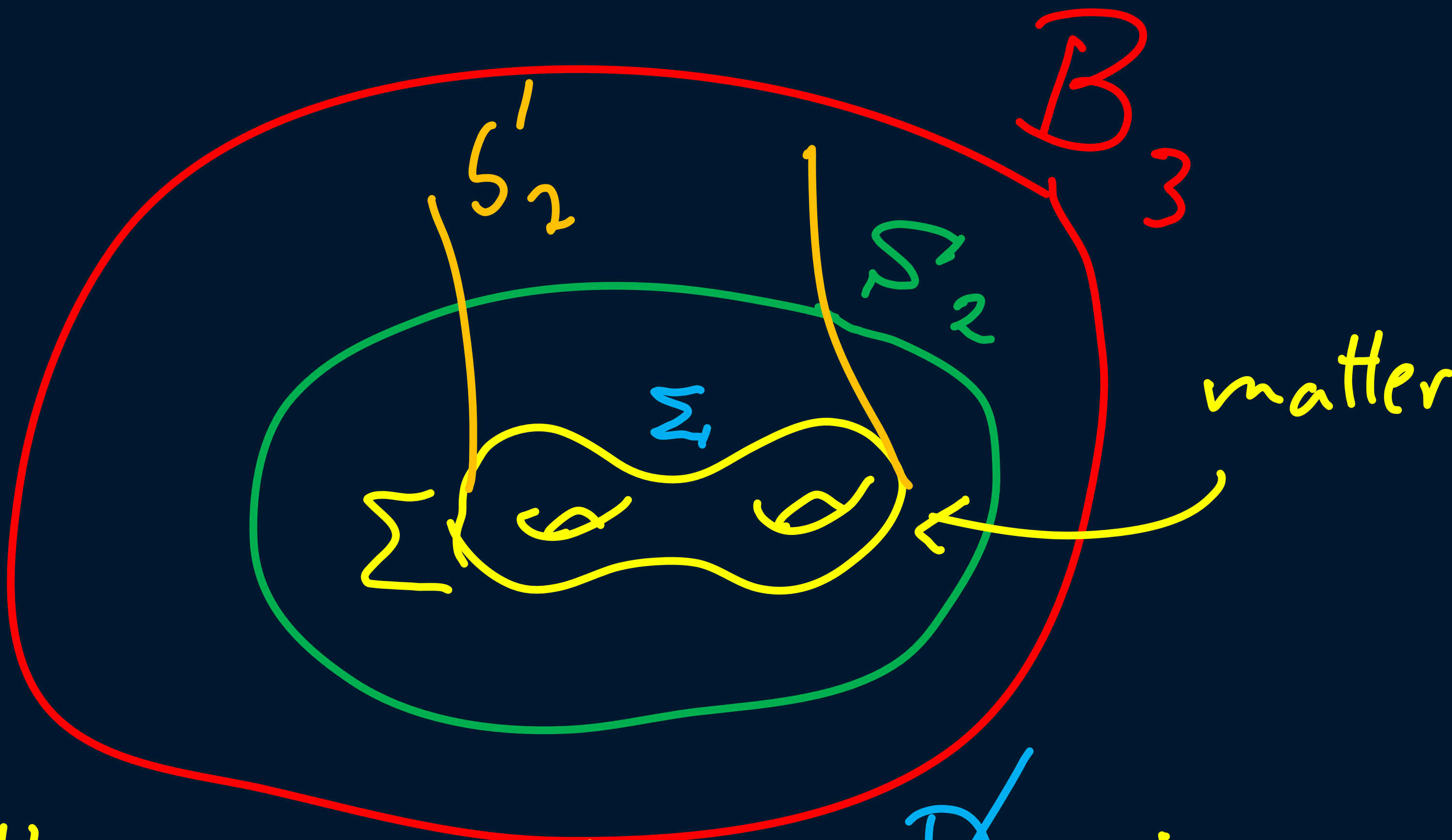
 $\subset$ 

$$U(1) \times SO(10) \subset E_6$$

$$\underbrace{45}_{Adj} + \underbrace{16 + \overline{16} + 1}_{\text{matter}} = \underbrace{78}_{Adj}$$

Exceptional singularities are needed for particle physics!

How many matter fields to we get?



$$\# \,\text{matter} = \text{index}(\not{D}_{L_\Sigma})$$

How many does the particle phenomenology suggest?

3 copies of the same representation, i.e. in the SO(10) context

$$3 \otimes \underline{16}_{\text{spinors}}$$

This repetitive structure of nature is very hard to explain from the viewpoint of particle theory in 4d. It is very satisfying that string theory offers an elegant explanation of this repetition.

Interactions

In addition to having a matter content we also need interactions. Of course there are interactions of matter fields with gauge fields, which simply follows from the fact that connections enter the covariant derivatives in the kinetic terms of the matter field Lagrangian. However, we need more: How does matter receive its mass?

For this to happen we need quadratic terms involving matter fields:

$$\mathcal{L} \supset m \Psi_{matter} \cdot \Psi_{matter}$$

$16 \otimes 16$ X NO

Instead we need to introduce an additional matter field (the Higgs field) and consider the cubic term:

$$H_\mu \bar{\Psi} \gamma^\mu \Psi$$

$$10 \otimes 16 \otimes 16$$

gauge invariant

$$\langle H_\mu \rangle : \bar{\Psi} \gamma_\mu \Psi$$

mass

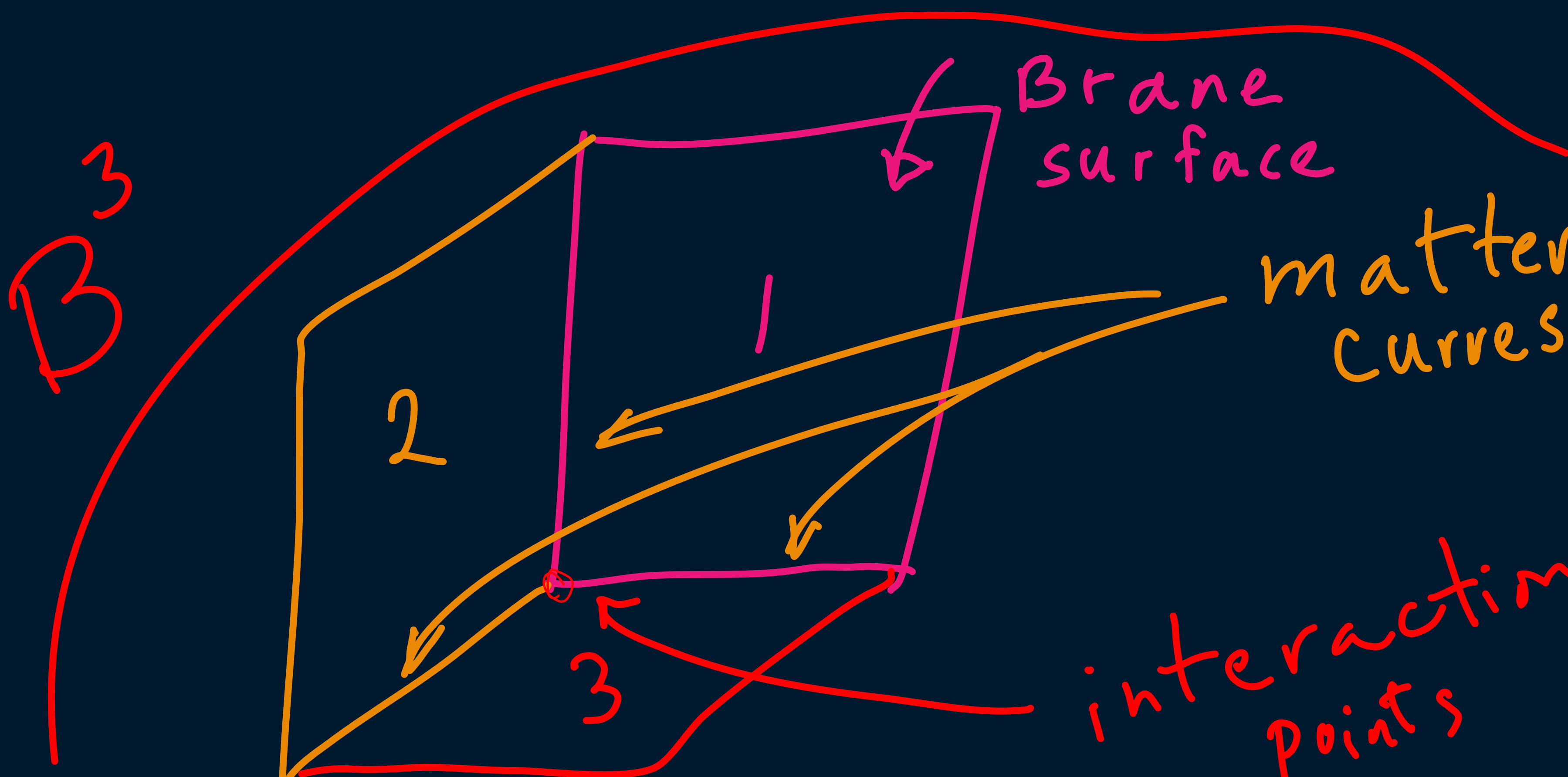
$$\text{iff } \langle H_\mu \rangle \neq 0$$

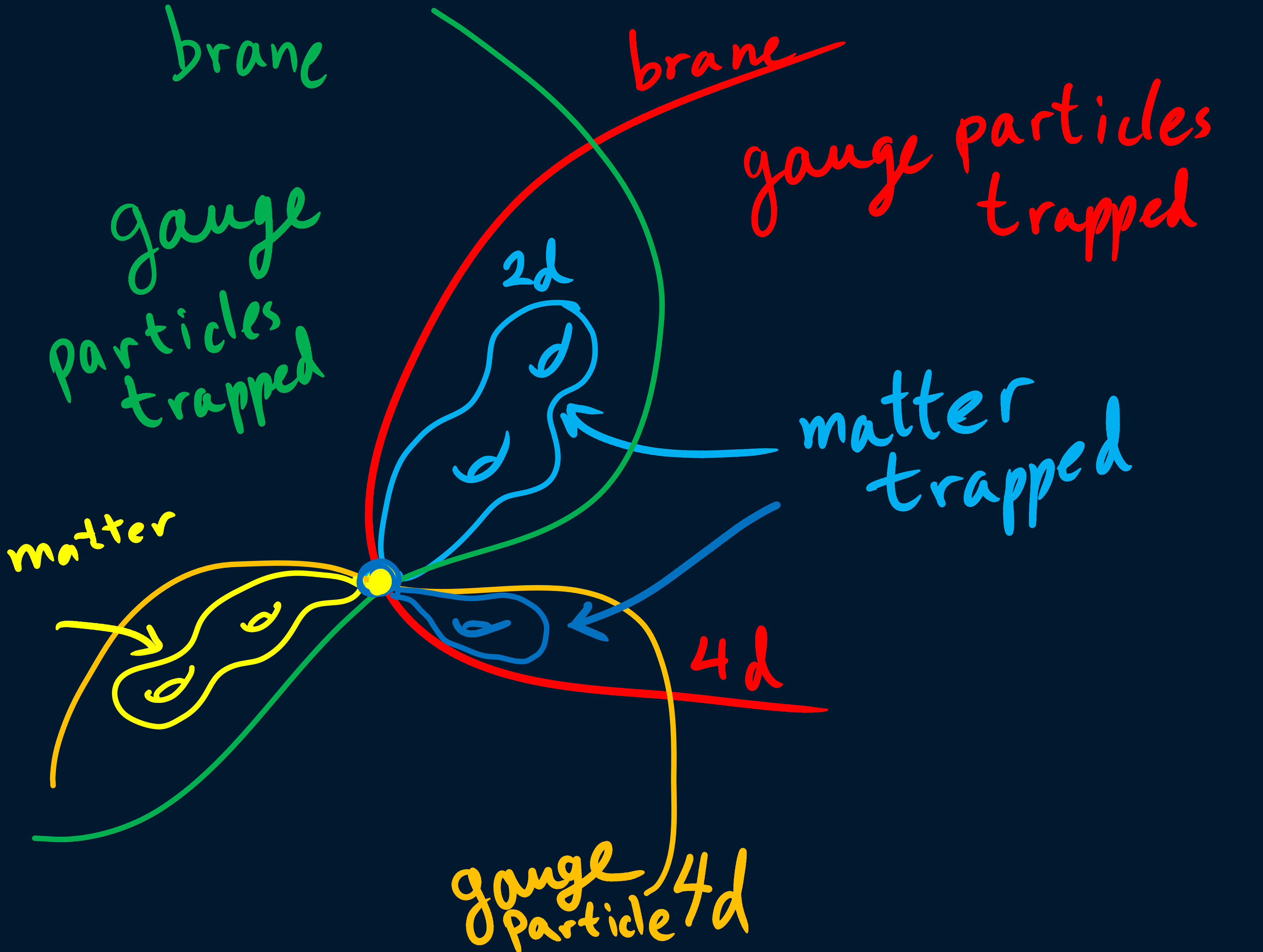
Yukawa Couplings in String Theory

For F-theory the interaction arises as a further enhancement of the singularity. Namely instead of just two singularities meeting on a curve to give matter fields, we have three singularities meeting pairwise on curves and all three meeting at a point. So we have an interesting hierarchy of structure:

Gravity in	10d = 4+6	
Gauge theory	8d = 4+4	surface
Matter	6d = 4+2	curve
Interaction	4d = 4+0	point

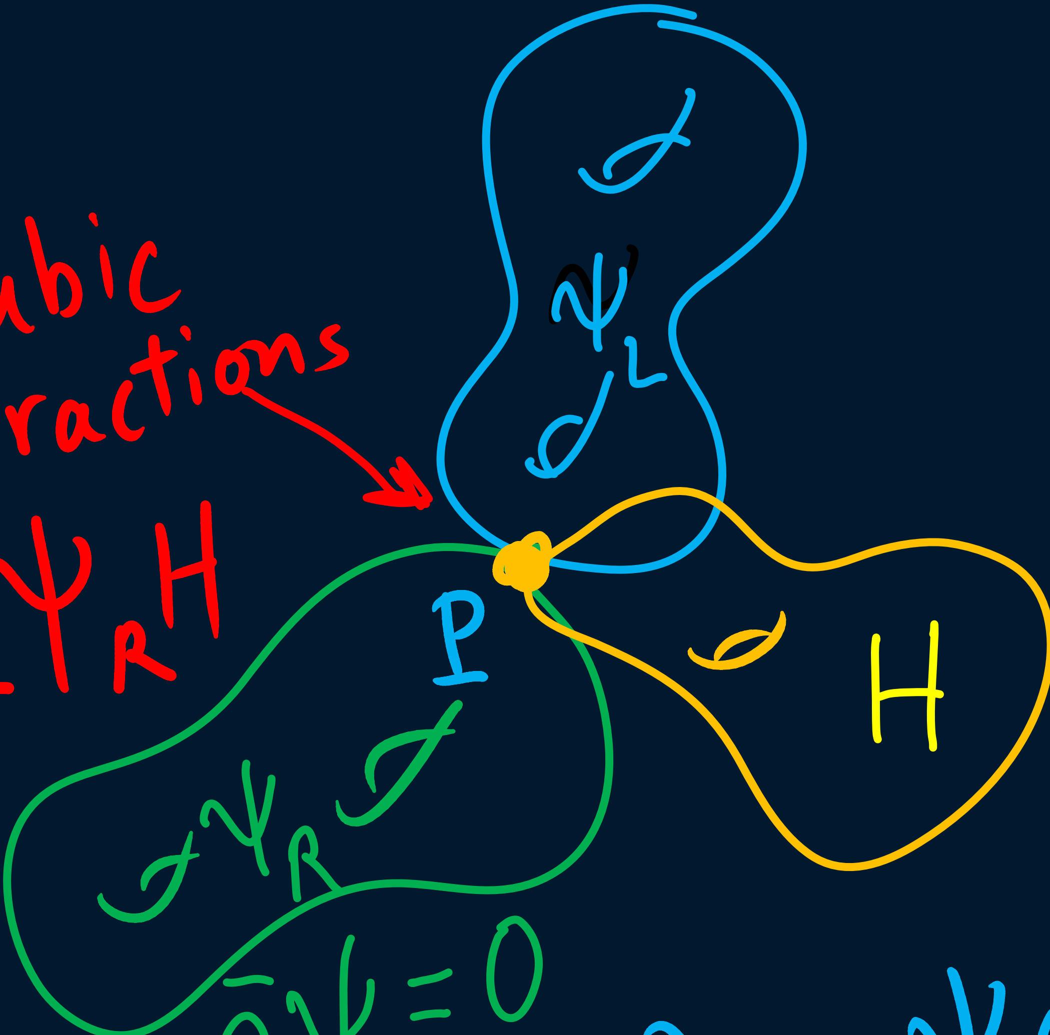
Cubic Interactions




brane
brane
gauge particles trapped
gauge particles trapped
2d
d
d
matter trapped
matter
d
d
d
4d
gauge particle
4d

Cubic interactions

$\psi_L \psi_R H$



$\bar{\partial} \psi_R = 0$

$\vdots$

$\lambda_{\text{Yuk.}} = \psi_L(\underline{p})\, \psi_R(p) H(p)$

gauge fields G_1, G_2, G_3

matter G_{12} G_{13} G_{23}

Yukawa G_{123}

Lie bracket on G_{123} induces Yukawa on G_{ij}

$$G_1 + G_2 + G_3 = E_7$$

$$\underbrace{10 \cdot 16 \cdot 16}$$

$$\cap$$

$$(\mathrm{Adj})^{\otimes 3} \text{ of } E_7 : (133) \cdot (133) \cdot (133)$$

Mathematics of Yukawa Coupling as a Local Obstruction Theory

The geometry of the interactions is captured by obstruction theory of a YM-Higgs bundle geometry characterized by the action:

$$L = \int F^{0,2} \wedge \varphi^{2,0}$$

$$\wr$$

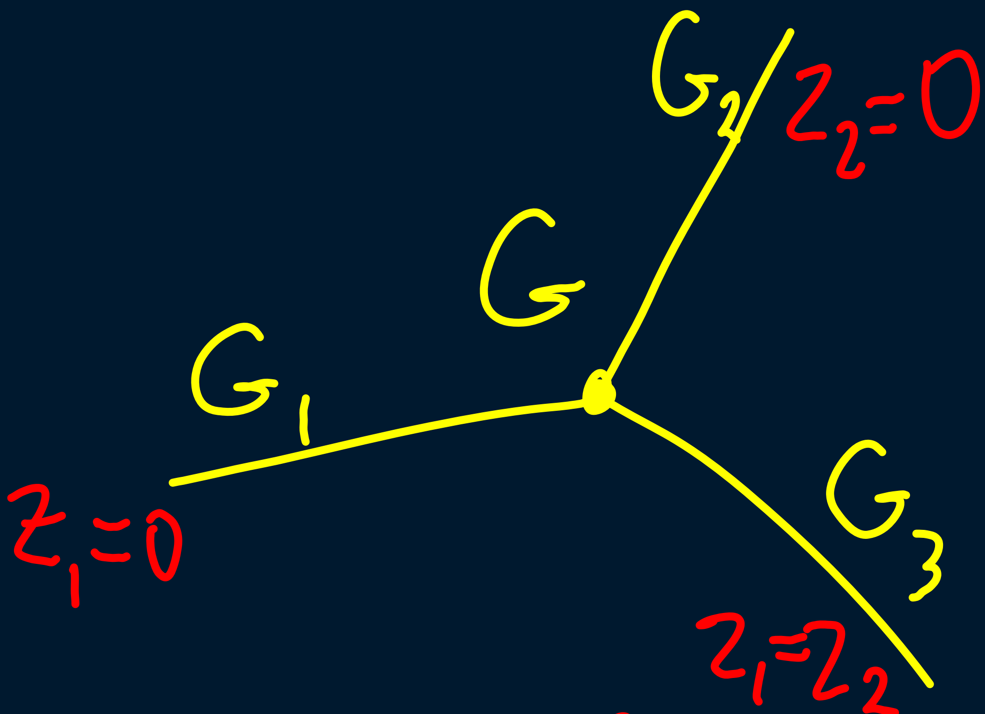
Holomorphic YM – Higgs bundle

equations $\rightarrow$

$$\delta_A \rightarrow \bar{\partial}_A \varphi^{2,0} = 0$$

$$\delta_\varphi \rightarrow F^{0,2} = 0$$

We start with a background A_0, φ_0



e.g. $A_0 = 0, \varphi_0 = \begin{pmatrix} -2z_1+z_2 & & \\ & z_1+z_2 & \\ & & z_1-2z_2 \end{pmatrix}$

$$A = A_0 + A_1 + A_2 + \cdots$$

$$\varphi = \varphi_0 + \varphi_1 + \cdots$$

$$\int F \wedge \varphi \Rightarrow \underbrace{\int A_1^a \wedge A_1^c \wedge \varphi_1^b}$$

$$\left(\begin{array}{c|c|c} * & a & b \\ \hline a & * & c \\ \hline b & c & * \end{array}\right)$$

↑ Yukawa Obstruction

+ higher order terms

An unexpected mass hierarchy:

1 2 3

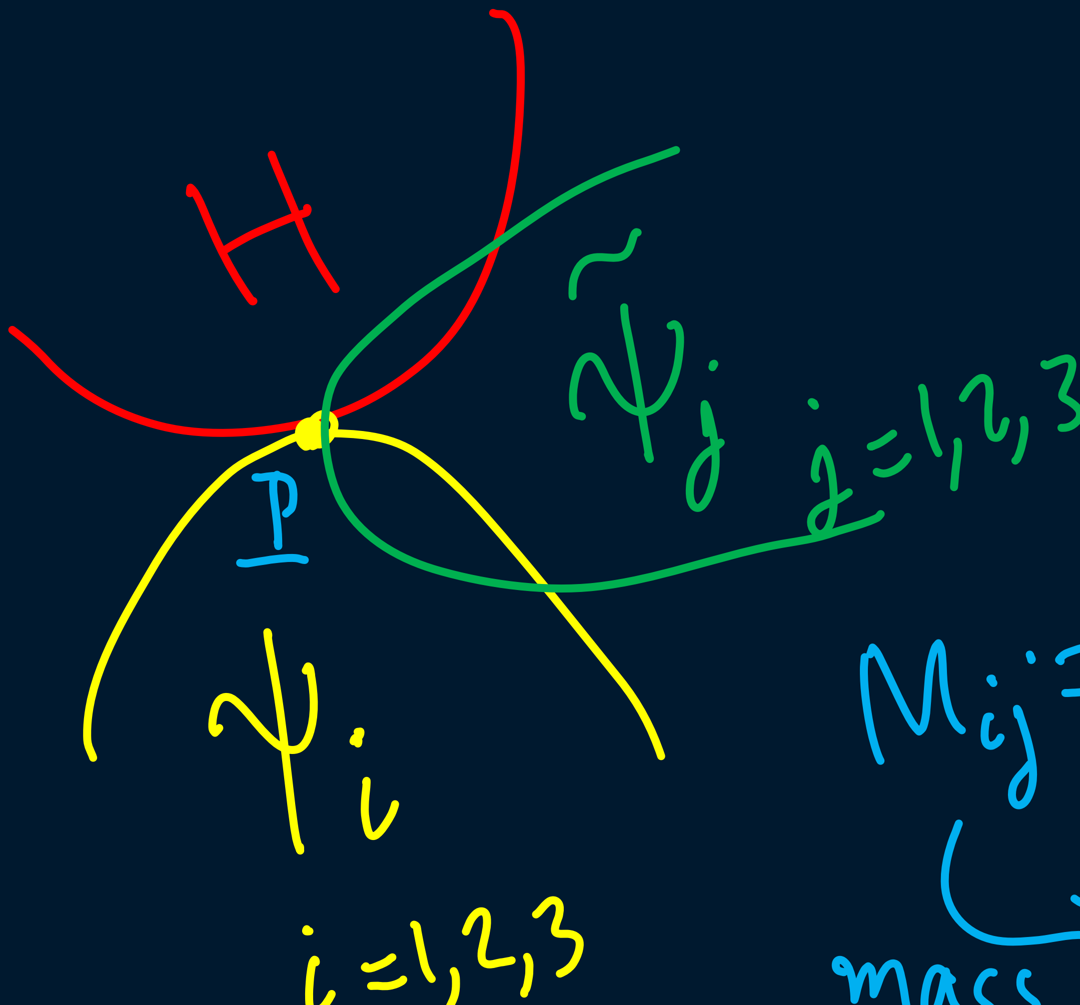
$$(m_u, m_c, m_t) \sim (0.003, 1.3, 170) \times \text{GeV}$$

$$(m_d, m_s, m_b) \sim (0.005, 0.1, 4) \times \text{GeV}$$

$$(m_e, m_\mu, m_\tau) \sim (0.0005, 0.1, 1.8) \times \text{GeV}$$

100 GeV : (.00003, .013, 1.7)

So to a good approximation we have one massive and two massless generations. Can we explain this bizarre fact?

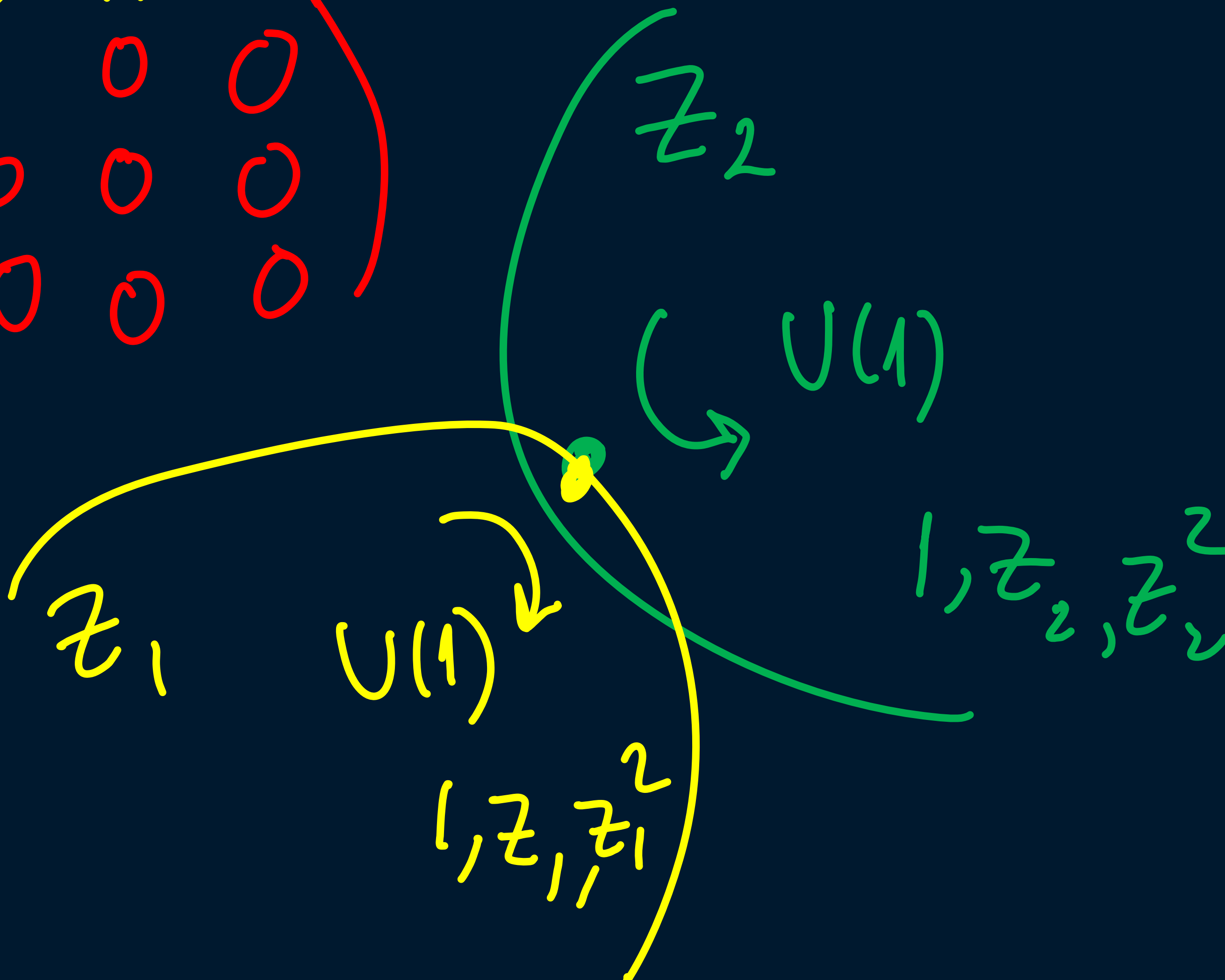


$$M_{ij} = H(p)\,\psi_i(p)\,\tilde{\psi}_j(p)$$

mass $\hookrightarrow$ rank 1 !

This rank one matrix can be organized as follows:

$$M = \begin{array}{c|ccc} & 0 & +1 & +2 \\ \hline 0 & 1 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ +2 & 0 & 0 & 0 \end{array}$$



$$M = \begin{pmatrix} 1 & \varepsilon & \varepsilon^2 \\ \varepsilon & \varepsilon^2 & \varepsilon^3 \\ \varepsilon^2 & \varepsilon^3 & \varepsilon^4 \end{pmatrix}$$

$$\varepsilon \approx \alpha_{GUT} = 0.04$$

$$\Rightarrow \quad m_1 : m_2 : m_3 \simeq \alpha_{GUT}^4 : \alpha_{GUT}^2 : 1$$


Happy
80th
Birthday


Happy
80th
Birthday
Sir Michael


Happy
80th
Birthday
مايكل عطيّة