$$= (a+3b)\begin{vmatrix} 1 & -b & -b \\ 1 & 4a & -2b \\ 1 & 3a & a \end{vmatrix} - 2b\begin{vmatrix} 1 & 4a-2b \\ 1 & 3a & a \\ 1 & 2a & a \end{vmatrix}$$

$$= (a+3b,\begin{vmatrix} 1 & -b & -b \\ 1 & 3a & -2b \\ 1 & 2a & a \end{vmatrix} + (a+3b)\begin{vmatrix} 1 & 0 & -b \\ 1 & a & -2b \\ 1 & a & a \end{vmatrix} - 2b\begin{vmatrix} 1 & 4a-2b \\ 1 & 3a & a \\ 1 & 2a & a \end{vmatrix}$$

$$= (a+3b,\begin{vmatrix} 1 & -b & -b \\ 1 & 3a & - \\ 1 & 2a & a \end{vmatrix} + (a+b)a(a+2b)\,,$$

for the last two determinants in the preceding line are each equal to $a(a+2b)$. Thus we have

$$S_4 = \frac{\begin{vmatrix} 1 & -b & -b \\ 1 & 3a & -2b \\ 1 & 2a & a \end{vmatrix}}{a(a+b)(a+2b)} + \frac{1}{a+3b}$$

$$= \quad S_3 \quad + \frac{1}{a+3b}$$

as it should be. And this method of showing that the validity of the fourth case is dependent on that of the third is applicable in other case.

8. Sevenfold Knottiness. By Prof. Tait.

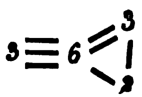
(Abstract.)

From the point of view of the Hypothesis of Vortex Atoms, it becomes a question of great importance to find how many distinct forms there are of knots with a given amount of knottiness. The enormous numbers of lines in the spectra of certain elementary substances show that the form of the corresponding Vortex Atoms cannot be regarded as very simple. But this is no objection against, it is rather an argument in favour of the truth of, the Hypothesis.

For not only are the great majority of possible knots not stable forms for vortices; but altogether independently of the question of kinetic stability, the number of distinct forms with each degree of knottiness is exceedingly small,—very much smaller than I was prepared to find it. I have already stated that for three, four, five, and sixfold knottiness, the numbers are only 1, 1, 2, 4. For a reason given in my first paper, knots whose number of crossings is a multiple of 6 form an exceptional class: so I thought it might be useful to discover and to figure all the distinct forms with seven-fold knottiness. Eight and higher numbers are not likely to be attacked by a rigorous process until the methods are immensely simplified. The method of partitions, supplemented by the graphic formulæ of my last paper, is to some extent tentative. I have verified the present results by means of it, and have extended it to 8-fold knottiness, but I am not certain that I have got *all* the possible forms of the latter.

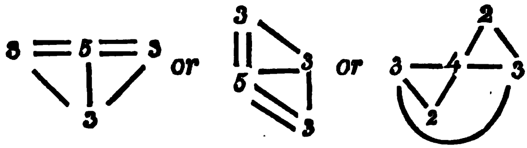
As I did not see how to abridge the process, I wrote out all the admissible permutations of the seven letters in the even places of the scheme. These I found to be 579, five of which were, of course, unique. The others (as 7 is a prime number) were divisible into 82 groups—those of each group being mutually equivalent. On examination, it was found that only 22 of the 87 selected arrangements satisfied the criterion for possible knots (see I § (*b*) of my paper, *ante* p. 238), and several even of these were repetitious. These repetitions were of two kinds—1st, the mere inversion of the order of the scheme; 2d, the relative positions of a 3-fold and a 4-fold knot which in certain cases were found combined as a 7-fold form. Clearing off these repetitions, and along with them a form really belonging to 6-fold knots (because consisting of two trefoil knots and one nugatory intersection), there remain only *eleven* distinct forms of the 7th order. These are as follows:—

1.



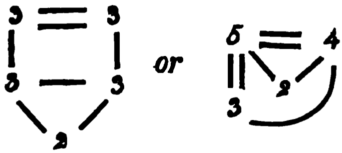
This has a great many forms, with correspondingly different symbols, being a mere compound of a 3-fold and a 4-fold knot, which may have *any* relative positions on the string.

2.

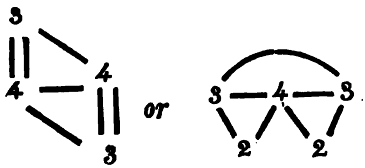


This is one of Listing's knots.

3.

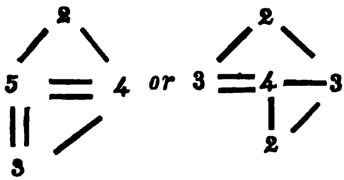


4.



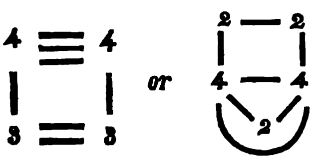
Listing has shown that this is deformable into 2 above.

5.

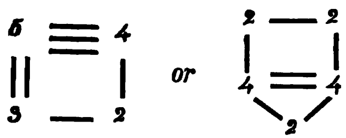


I find that this can be deformed into 3 above. It is figured in my paper on *Links*, *ante*, p. 325, first woodcut.

6.

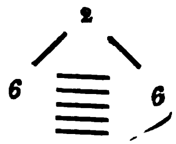


7.



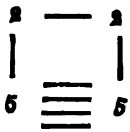
This can be deformed into 6 above

8.



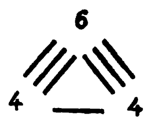
This species of knot occurs for *all* numbers of intersections greater than 2.

9.



This is the 7 knot which Listing does not sketch. *Ante*, p. 311.

10.



11.

This is the simple twist, which occurs for every *odd* number of intersections.

As 2 and 4, 3 and 5, 6 and 7 are capable of being deformed into one another, three of them are not independent forms, and thus the number of distinct forms of seven-fold knots is only *eight*.

Drawings of various forms of each of these knots were given, as well as indications of the modes in which they can be formed from knottinesses of lower orders.