The Intertwining of Two Lifelines

in memoriam of V. I. Arnold

V. P. Maslov

Received June 28, 2010

Abstract. In memoriam of V. I. Arnold.

DOI 10.1134/S1061920810040011

I simply cannot understand that Arnold is no longer. For me, he is “more alive than all the living.” And death did not creep up to him from the side across which his wife Elya (a medical doctor) had put up a secure barrier.

My life as a mathematician and my personal life was intertwined in a strange way with Arnold’s life. In our careers first one, then the other would take the lead, and we would always help each other. At one moment our lifelines met at one point, when both of us (a rare occasion, namely, two Russian mathematicians) were invited to give plenary talks at the same International Congress of Mathematicians (in Warsaw).

At the most tragic moment of my life, when my wife was dying, I sped to see Arnold, and Elya told me what the doctors should do to save her.

At Arnold’s funeral, Ludvig Faddeev spoke of our group of friends and called us the “Warsaw fraternity.”

When we were young, Arnold, Yuri Manin, and me, we felt it necessary to carry out an “overthrow of the government” in mathematics similar to the one that had occurred in physics when quantum mechanics, which many venerable professors of the Physics Department of Moscow State University did not understand at the time, became an obligatory part of the syllabus for all fourth year physics students. At the beginning of the 20th century, V. A. Pyast published an article entitled “Overthrow of the Government” in which he pointed out that the representatives of the modernistic trends in literature had shifted from the position of “banned” to that of “established.”

At the time I had observed that a number of students had been awarded A’s at the entrance examination in mathematics and, nevertheless, were not accepted to the Physics Department, and I felt that these “banned” students should be collected in a study group in order to carry out an “overthrow of the government” of that type with them. I reached an agreement with the President of MIEM (The Moscow Institute of Electronics and Mathematics), E. V. Armenskii, namely, I would head the Chair of Applied Mathematics at that institute, provided I would be allowed to accept the “banned group” as first-year students. Arnold, Manin, Kostyuchenko, and Fedoryuk in math and Bogolyubov’s pupil Medvedev in physics agreed to give courses in contemporary mathematics to the “banned group”, without any fee, following a completely new syllabus. We were all experiencing the inspiration of futurists in mathematics.

Each of Arnold’s or Manin’s lectures was an inimitable masterpiece. There were only 15 listeners who understood perfectly how lucky they were. Manin would spend some 10 hours preparing for each lecture, and this was his way of competing with the best lecturer in the world, Arnold. On the basis of these courses, Manin, Arnold, Medvedev, and me published textbooks of a new type.

Unfortunately, organizing a second group of “banned students” turned out to be impossible: the President of MIEM Armenskii was summoned to the Central Committee, where he was severely reprimanded, so the experiment was limited to that group only. It was also unfortunate that, no matter how hard I tried, and despite the help of several research institutes, the rules of the so-called *raspredelenie* (allocation of students to various institutions at graduation by the communist party administration) dispersed the group in different directions. There was no Landau minimum which would gather the most talented young people around one leader. At Moscow University this was easier, and Arnold created a whole school there; in MIEM, after that reprimand, it became more difficult.

Arnold obtained his Doctor of State degree (the next degree after the PhD) earlier than I did. Just before the defense of my State Doctorate, one of the reviewers fell ill, and I asked Arnold to replace him, although little time until the defense remained. In five days, I explained the geometric

part to him, and on the basis of these talks, he wrote a brilliant text for the review, crossing all the t's and dotting all the i's in the geometry of the intersection index with the singularities of Lagrangian manifolds. I had a more algebraic approach, and its geometric interpretation was presented only for specific examples. The algebraic proof took up a whole chapter of the book [1].

For quite a while I tried to convince Arnold to publish the text of his review, and finally it appeared under the title "On the characteristic class appearing in the quantization condition." The number of citations of that paper was enormous. As far as I remember, in the article he changed very little as compared to the review. Historians of mathematics, who will undoubtedly study the scientific heritage of such a great scientist, can compare the texts by consulting the dissertation archives.

The abstract algebraic approach was not lost. S. P. Novikov applied it a series of brilliant papers on K -theory, which were then developed by his outstanding school. The approach is also used by physicists in order to compute the so-called Maslov index.

Arnold was not even a corresponding member of the Academy when I was elected to full membership directly, despite the opposition from the Central Committee of the Communist Party. Not having yet been officially given the title of academician by the General Meeting of the Academy, I came to the meeting nevertheless (not very legally) for the election of the corresponding members.

And I did not come in vain. The influence on the corresponding members, who don't yet know how the newly elected academician will vote, cannot be disregarded, but in fact the main role in these elections was played by Ludvig Faddeev.

Now the election to full membership is a different affair. Here painstaking work was needed, one needed analyze who will vote how, what scientists eventually likely to vote for Arnold have a good chance to be elected. This principle (will he vote for Arnold) was my main guideline in further academic elections.

When Arnold finally became a serious candidate, he was doubtful about how one of the greatest academicians (we both considered him a genius), with respect to whom Arnold had a kind of inferiority complex, would vote. In secret, he told me about his doubts, and I promised to take control of this issue.

Finally, Arnold was elected to full membership in the Academy. One would think that for such an important scientist, this would be of no importance; indeed, after all, Mendeleev was only a corresponding member. However, a common friend of ours told me that for Dima (as we all called Arnold) with his complexes his election played a decisive role. From my point of view, however, this was more of an indication of the level of the Academy, than that of Arnold.

Arnold was never much of a diplomat, he was always sincere and even naive. For example, when he spoke in favor of one of his brilliant pupils, a candidate for a corresponding membership in the Academy, he explained that this person was a much stronger mathematician than all of the corresponding members running for full membership. Later I explained him how this had displeased those who voted, and even counted the number of votes that he had lost. How he repented and was angry at himself for that!

Arnold was helpless, when he was attacked by a deputy dean of the math department or by the chairperson of the differential equations chair (O. A. Oleinik) where he worked. Returning to the Warsaw Congress (I have written about our adventures there in [2]), let me only mention that Oleinik had asked (if not ordered) him to mention her name in the talk. I laughed and told him "Please don't mention me, mind you." He didn't, but began his talk with the words: "all these problems come from MIEM." During our presentations, we helped each other in changing the transparencies, and when it was my turn to help Arnold, I would encircle or underline the name Oleinik whenever it appeared in the slides.

At the Congress, Arnold was describing his famous catastrophe theory. Actually, to some extent, I was the one who initiated this work. I have a strong disgust of all panegyrics about me (especially told to my face and especially if they include unequivocal hints about my age, rather advanced now). For example, to the unexpected laudatory remarks about my achievements in *Novaya Gazeta* (# 36, May 22, 2008), I reacted immediately in my article "On Risks and Acquisitions" (*Novaya Gazeta*, Addendum *Centaur*, # 11, May 29, 2008); in it, I added (and had to throw out an important part of the original text to do this) the following: "I categorically disagree with the exaggerated assessment of my contribution to science given by the Editors of *Novaya Gazeta* in a previous issue of NG (# 36)."

As to the problem solved in Arnold's famous catastrophe theory, it tormented me until 1965, when I gave a detailed description of caustics and focal points by means of the canonical operator (now often called "Fourier integral operator"). In the paper [1], I had been able to reduce the problem to a minimal number of integrals, but it was desirable to express these integrals in a simpler form, to specify them.

Asymptotic formulas are asymptotic precisely because they can be expressed in different ways. Thus, the expression for the asymptotic distribution $p_k(n)$ of the prime numbers, obtained by Temperley and studied in detail in a remarkable paper by A. M. Vershik, differs from the unified global form that was proposed in my paper [3]. The latter form is more convenient for computations and can be generalized to smaller dimensions.

Lars Hörmander, having learned about my theory from Yu. Egorov, gave another representation of the canonical operator, which was worse than my original version in the sense explained above. At the Congress of Mathematicians in Nice, I personally explained to Hörmander that his representation coincides with my notion of canonical operator. After that he sent a correction to the text of his talk to many mathematicians (the correction was added on p. 4a of the Congress proceedings; it was incorporated in the main text in the Russian version). Here is this correction.

"The work of Egorov is actually an application of ideas from Maslov [20]. The author regrets his lack of direct familiarity with this book, which according to a lecture by Dr. Maslov at the International Congress in Nice, actually contains the ideas attributed here to Egorov [7] and Arnold [1] and even a more general and precise operator calculus than the one that we shall describe. However, since the book is highly inaccessible and perhaps not quite rigorous we hope this paper will still prove to be useful."

I was asked (in particular by O. A. Oleinik) if I agree with this correction. I answered that I was not. First of all, it is not correct to say "not quite rigorous." The book was checked by such first class mathematicians as G. Eskin and O. A. Ladyzhenskaya, and I gave exhaustive answers to all their questions in the presence of such experts as V. P. Palomodov and S. P. Novikov. Their questions helped me in my lectures, and I understood which parts should be explained in more detail.

Second, the work of Yu. Egorov deals with a transformation discovered long ago by V. A. Fock. Whereas Arnold's contribution and his brilliant interpretation, as I mentioned above, cannot be overestimated¹.

At the same time, during the Nice congress, I asked whether the canonical operator could be presented by means of simpler formulas to the great mathematician Sir Michael Atiyah². I demonstrated the answer to the question in the trivial case of the sum of powers of the exponent (action) and showed that the degree of the singularity in the parameter h (the Planck constant) does not change under changes of coordinates, and is therefore invariant of the singularity of the projection of the Lagrangian manifold on the plane. But he was not interested. I also brought this fact to the attention of other topologists, as well as A. S. Mishchenko and Arnold, but no one reacted.

Finally I was able to generate the interest of engineers from the Ministry of Radio-industry in this question. I proposed finding the solution to the problem of the reflection of radio location waves from the Heaviside stratum: a problem in which the appearance of complicated focal points is very rare and one can consider focal points and caustics in general position.

I signed a contract and hired Arnold's team to solve the very cumbersome problem of classifying caustics in general position. This problem was brilliantly solved by Arnold and his team (my role was only that of general supervision and encouragement). Arnold, however, thanked me with such enthusiasm in his articles, and the book "Catastrophe Theory" [4] ascribes to me something like 50% of the contribution to that question, while in fact I did not do much more than the general who put his signature on the articles dealing with the reflection of radio waves published in classified journals.

Finally Pervukhin, the minister himself, wrote a presentation for me to membership in the Academy, which in no way compensated the negative opinion of the Central Committee of

¹As I was told, Hörmander learned about the disagreement of Russian mathematicians with him on this issue and adequately reacted to it by writing a phrase of the type "sort this out by yourselves."

²His title was awarded to him by the Queen of the United Kingdom of Great Britain and Ireland for his scientific achievements.

the Communist Party. Let me stress that all things considered, it was Arnold who deserved this reaction from the higher levels of the administration of the Radio Industry Ministry, and not me, and not even the above mentioned general.

When I had calculated the inevitability of the default in Russia in 1998, I decided to sell my country house, located not far from Arnold's, and visited him to explain my understanding of chaos. He read the nuances of my talk very well, and told me: "You are telling me this as if you were asking me write an official review of your doctorate thesis."

He had guessed correctly. I explained that I want to leave for England, at least for some time, until the consequences of the default will be overcome provided they do not lead to the disintegration of the Russian Federation. My second daughter would continue her high school education (13 years in the UK); she had graduated from high school in Moscow, but had not yet chosen how she would continue her studies. Arnold told me that he had just received a request from the University of Hong Kong to convince me to accept a position there, for at least a year, under very advantageous conditions. Hong Kong has the same educational system as the UK.

Besides this, Arnold wrote to our common friend Sir Michael Berry who had then sent me an invitation to Bristol.

The invitation to Hong Kong had many positive aspects, which Arnold and other scientists described to me. At the last moment, I chose England, London is merely 3 hours away from Moscow by plane. Fortunately, the default was rapidly overcome, and I returned to Moscow with my family, leaving my older daughter in England to continue her university education.

The last mystic coincidence of our lifelines was that the day of Arnold's funeral coincided with my birthday, and a very round date at that. There is a Russian expression *on prikazal dolgo zhit'* (literally: "he ordered to live for long"), which is a euphemism for "he died." A common friend of ours sadly said: "He ordered to live for long; he ordered you." For Arnold and myself, to live meant to create. When he was taken away to the hospital, sheets of paper with partially completed formulas remained on his pillow.

REFERENCES

1. V. P. Maslov, *Perturbation Theory and Asymptotic Methods* (Izd-vo MGU, Moscow, 1965) [in Russian].
2. V. Maslov. *Daring to Touch Radha* (Academic Express, Lviv, 1991), also at <http://www.viktor-maslov.narod.ru>.
3. V. P. Maslov, *Threshold Levels in Economics*, arXiv:0903.4783v2 [q-fin. ST] 3 Apr 2009.
4. T. Poston and I. Stewart, *Catastrophe Theory and Its Applications* (Pitman, London–San Francisco–Melbourne, 1978).