DIFFERENTIABLE STRUCTURES ON SPHERES AND HOMOTOPY

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This talk was a report and brief survey of results obtained jointly by J. Milnor and the speaker on the groups of differentiable structures on spheres.

Let \( |\theta_n| \) denote the number of differentiable structures on \( S^n \) (up to diffeomorphism), then

\[
|\theta_{4k-1}| = |\pi_{4k-1}| \cdot 2^{2k-4}(2^{2k-1}-1) \cdot B_k a_k/k,
\]

and

\[
|\theta_{4k}| = |\pi_{4k}| \cdot a_k/2, \quad |\theta_{4}| = 1,
\]

for \( k > 1 \), where \( B_k \) is the \( k \)th Bernoulli number, and \( a_k \) equals 1 for \( k \) even and 2 for \( k \) odd. (\( B_1 = 1/6, B_2 = 1/30, B_3 = 1/42, B_4 = 1/30, B_5 = 5/66, B_6 = 691/2730 \), etc.)

For all non-negative integers \( m \), one has

\[
|\theta_{4m+1}| = |\pi_{4m+1}| \cdot a_m u_m/2
\]

and

\[
|\theta_{4m+2}| = |\pi_{4m+2}| \cdot u_m/2
\]

where \( a_m \) is as above, and \( u_m \) is either 1 or 2. The only known values of \( u_m \) are: \( u_0 = 1, u_1 = 1, u_2 = 2, u_3 = 1, u_4 = 2 \). If \( S^{2m+1} \) is parallelizable, then \( u_m = 1 \). It is conjectured that the converse also holds, i.e. that \( u_m = 2 \) except for \( m = 0, 1 \) and 3.

In the above formulae \( |\pi_n| \) denotes the order of the stable homotopy groups \( \pi_n = \pi_{n+N}(S^n), N > n+2 \). (\( |\pi_1| = |\pi_2| = 2, |\pi_3| = 24, |\pi_4| = |\pi_5| = 1, |\pi_6| = 2, |\pi_7| = 240, |\pi_8| = 4, |\pi_9| = 8, |\pi_{10}| = 6, |\pi_{11}| = 504, |\pi_{12}| = 1, |\pi_{13}| = 3, |\pi_{14}| = 4, |\pi_{15}| = 960, etc.)

The proofs will be included in two papers: Groups of Homotopy Spheres, I and II, to be published in the Annals of Mathematics.