DIFFERENTIABLE STRUCTURES ON SPHERES AND HOMOTOPY

By MICHEL A. KERVAIRE

This talk was a report and brief survey of results obtained jointly by J. Milnor and the speaker on the groups of differentiable structures on spheres.

Let $|\theta_n|$ denote the number of differentiable structures on S^n (up to diffeomorphism), then

$$|\theta_{4k-1}| = |\pi_{4k-1}| \cdot 2^{2k-4}(2^{2k-1}-1) \cdot B_k a_k/k,$$

and

$$|\theta_{4k}| = |\pi_{4k}| \cdot a_k/2, \quad |\theta_4| = 1,$$

for $k > 1$, where B_k is the kth Bernoulli number, and a_k equals 1 for k even and 2 for k odd. ($B_1 = 1/6$, $B_2 = 1/30$, $B_3 = 1/42$, $B_4 = 1/30$, $B_5 = 5/66$, $B_6 = 691/2730$, etc.)

For all non-negative integers m, one has

$$|\theta_{4m+1}| = |\pi_{4m+1}| \cdot a_m u_m/2$$

and

$$|\theta_{4m+2}| = |\pi_{4m+2}| \cdot u_m/2$$

where a_m is as above, and u_m is either 1 or 2. The only known values of u_m are: $u_0 = 1$, $u_1 = 1$, $u_2 = 2$, $u_3 = 1$, $u_4 = 2$. If S^{2m+1} is parallelizable, then $u_m = 1$. It is conjectured that the converse also holds, i.e. that $u_m = 2$ except for $m = 0$, 1 and 3.

In the above formulae $|\pi_n|$ denotes the order of the stable homotopy groups $\pi_n = \pi_{n+N}(S^N)$, $N \geqslant n+2$. ($|\pi_1| = |\pi_2| = 2$, $|\pi_3| = 24$, $|\pi_4| = |\pi_5| = 1$, $|\pi_6| = 2$, $|\pi_7| = 240$, $|\pi_8| = 4$, $|\pi_9| = 8$, $|\pi_{10}| = 6$, $|\pi_{11}| = 504$, $|\pi_{12}| = 1$, $|\pi_{13}| = 3$, $|\pi_{14}| = 4$, $|\pi_{15}| = 960$, etc.)

The proofs will be included in two papers: Groups of Homotopy Spheres, I and II, to be published in the *Annals of Mathematics*.