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**Ranicki, Andrew****Finite domination and Novikov rings.** (English)

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57Q12 Wall finiteness obstruction for CW-complexes

55U15 Chain complexes

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This paper is a pleasant mixture of a survey, an announcement and a standard research paper, all of it centered around the notions finitely dominated and homotopy finite. Both of these notions exist in a geometric context, involving CW complexes and manifolds, and an algebraic one, involving chain complexes over a ring  $R$ , and there is a rich classical interplay between the geometry and the algebra. Among the early results surveyed here are Wall's finiteness obstruction [*C. T. C. Wall*, Ann. Math., II. Ser. 81, 56-69 (1965; Zbl 152.21902)]; Siebenmann's end obstruction [*L. C. Siebenmann*, The obstruction to finding a boundary of an open manifold, PhD thesis (1965)]; and Mather's mapping torus trick [*M. Mather*, Topology 4, 93-94 (1965; Zbl 134.42702)]. The main new result applies to the following situation. One has two covering maps  $\tilde{X} \rightarrow \bar{X} \rightarrow X$  where  $X$  is a finite, connected CW complex,  $\bar{X}$  is a connected, infinite cyclic covering of  $X$ , and  $\tilde{X}$  is the universal covering of  $X$ . In that situation, it is shown that the vanishing of all homology groups  $H_*(X; [\pi]((z)))$  and  $H_*(X; [\pi]((z^{-1})))$  is necessary and sufficient for  $\bar{X}$  to be finitely dominated. Here,  $\pi = \pi_1(\bar{X})$ , and for any ring,  $A$ , the Novikov rings  $A((z^{\pm 1}))$  are the power series rings

$$A((z)) = \left\{ \sum_{j=-\infty}^{\infty} a_j z^j \mid \text{each } a_j \in A \text{ and } \{j \leq 0 \mid a_j \neq 0\} \text{ is finite} \right\},$$

$$A((z^{-1})) = \left\{ \sum_{j=-\infty}^{\infty} a_j z^j \mid \text{each } a_j \in A \text{ and } \{j \geq 0 \mid a_j \neq 0\} \text{ is finite} \right\}.$$

Finally, on the announcement side, we list the verification of a conjecture posed in [*A. V. Pazhitnov*, Surgery on the Novikov complex, *K*-theory 10, 323-412 (1996)]. A proof is briefly outlined. A full proof appears in the author's preprint [The algebraic theory of bands]. This paper has been available on the internet (<http://math.uiuc.edu/K-theory/>). The manager of these *K*-theory Archives (Dan Grayson) reports that the paper has been "often looked at by people who are searching for web pages with the words domination or dominatrix on them." This reviewer cannot help wondering whether such traffic would have been even more intense if the title of the paper had also revealed the author's long standing and well known preoccupation with chains.

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