IX.—*Alternate* ± *Knots of Order Eleven.* By Professor C. N. LITTLE. (With Two Plates.)

(Read 21st July; Revised December 1890.)

1. A year ago last April, Prof. TAIT proposed that I should undertake to derive from Mr KIRKMAN'S polyhedral drawings the alternate ± knots of eleven crossings, thus doing for order 11 what had been done so admirably by himself in orders 8, 9, and 10.

2. The work has been a very protracted one, because of the great number of forms involved—more than three times as many as in all preceding orders combined. Mr KIRKMAN'S manuscript contains 1581 forms, of which 22 are bifilar and 16 duplicates. I find from the remainder 357 knots with 1595 forms as shown in the following table:—

CLASS.	Un.	Two.	Three.	Four.	Five.	Six.	Seven.	Eight.	Nine.	Ten.	Eleven.	Twelve.	Fourteen.	Fifteen.	Sixteen.	Eighteen.	Twenty-four.	
II.	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
III.	4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
IV.	8	14	3	6	...	2	...	...	...	...	...	...	...	...	...	...	...	...
V.	26	25	6	14	1	19	1	8	...	2	...	3	...	...	1	...	...	...
VI.	44	48	18	26	4	17	1	15	5	3	1	16	1	2	3	6	3	...
Total Knots	83	87	27	46	5	38	2	23	5	5	1	19	1	2	4	6	3	357

As this is an odd order, perversion doubles these numbers, making 714 elevenfold knots, with crossings alternately over and under.

3. It has been thought unnecessary to show upon the Plates more than one form of each knot; all, however, have been drawn. Knots of each class having the same number of forms are grouped together to make more simple the identification of a particular elevenfold. A small figure following the series number upon the plates indicates how many distinct forms each knot can assume. Knots 84, 357, and 238_6 are misplaced.

4. Below each knot-form figured will be found the number of the corresponding form in Mr KIRKMAN'S manuscript, and partition symbols to which the following table gives the key:—

Class VI.

A_2	$7^2 2^4$	o	$6^2 2^5$
B_2	7632^3	π	6532^4
C_2	7542^3	ρ	$64^2 3^4$
D_2	$753^2 2^2$	σ	$643^2 2^3$
E_2	$743^3 2$	τ	$63^4 2^2$
F_2	73^5	ν	$5^2 42^4$
G_2	$74^2 32^2$	ϕ	$5^2 3^2 2^3$
H_2	$6^2 42^3$	χ	$54^2 32^3$
I_2	$6^2 3^2 2^2$	ψ	$543^3 2^2$
K_2	$65^2 2^3$	ω	$53^5 2$
L_2	65432^2	Γ	$4^4 2^3$
M_2	$653^3 2$	Δ	$4^3 3^2 2^2$
N_2	$64^2 3^2 2$	Λ	$4^2 3^4 2$
S_2	$6^4 3^4$	Ξ	43^6
T_2	$5^3 32^2$		
U_2	$5^2 4^2 2^2$		
V_2	$5^2 43^2 2$		
W_2	$5^2 3^4$		
X_2	$54^3 32$		
Y_2	$54^2 3^3$		
Z_2	$4^4 3^2$		

Class V.

A	$8^2 2^3$	υ	$5^2 2^6$
B	8732^2	ζ	5432^5
C	8642^2	η	$53^3 2^4$
D	$863^2 2$	θ	$4^3 2^5$
F	$85^2 2^2$	κ	$4^2 3^2 2^4$
G	85432	λ	$43^4 2^3$
H	853^3	μ	$3^6 2^2$
I	$84^3 2$		
K	$84^2 3^2$		
L	$7^2 42^2$		
M	$7^2 3^2 2$		
N	7652^2		
O	76432		
P	763^3		
Q	$75^2 32$		
R	$754^2 2$		
S	7543^2		
T	$74^3 3$		
U	$6^3 2^2$		
V	$6^2 532$		
W	$6^2 4^2 2$		
X	$6^2 43^2$		

Class V.—*continued.*

Y	$65^2 42$
Z	$65^2 3^2$
E_1	$654^2 3$
A_1	64^4
B_1	$5^4 2$
C_1	$5^3 43$
D_1	$5^2 4^3$

Class IV.

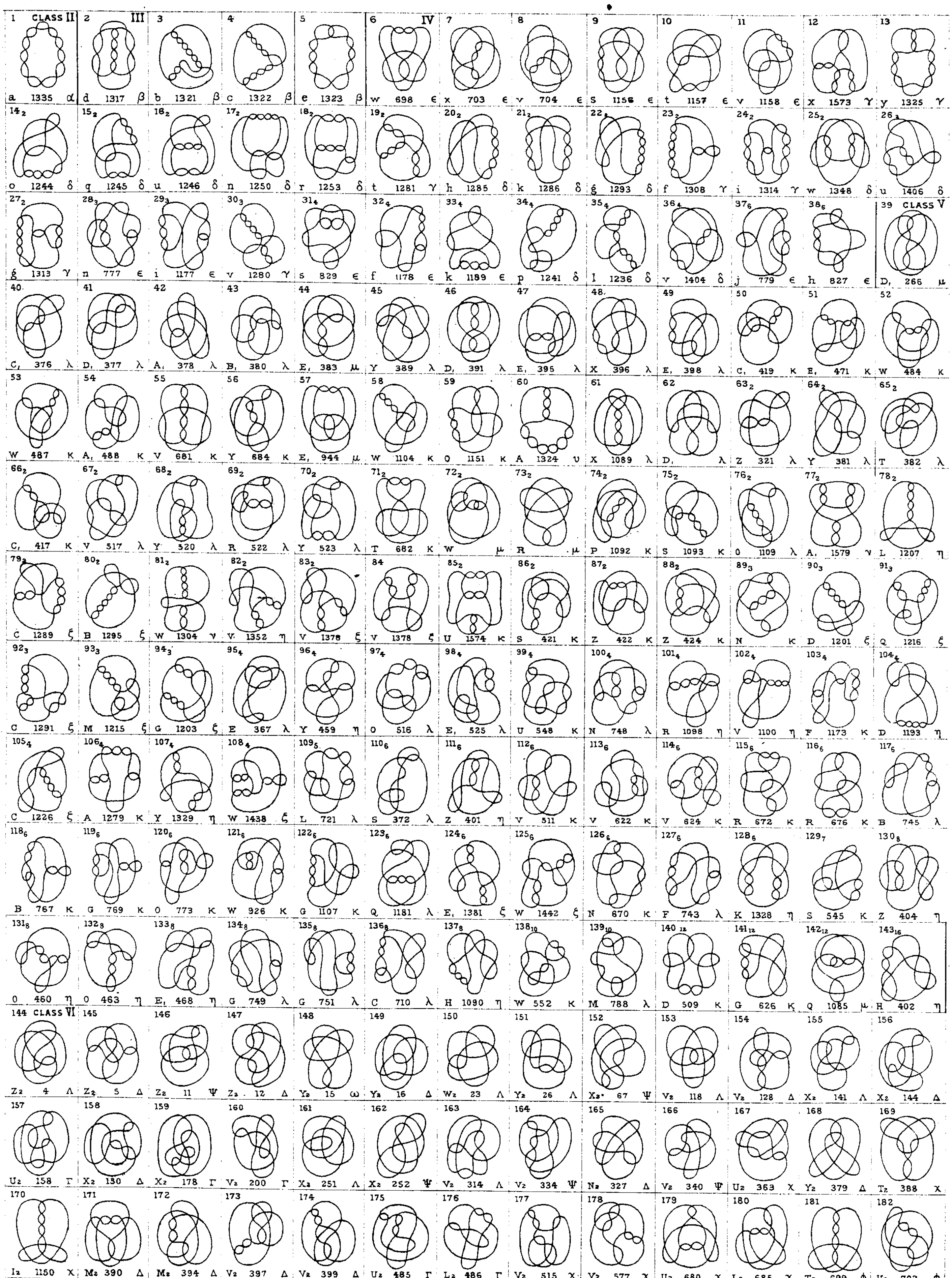
y	$9^2 2^2$	γ	$4^2 2^7$
f	9832	δ	$43^2 2^6$
g	9742	ϵ	$3^4 2^5$
h	973^2		
i	9652		
j	9643		
k	$95^2 3$		
l	954^2		
m	$8^2 3^2$		
n	8752		
o	8743		
p	8653		
q	$85^2 4$		
r	$7^2 62$		
s	$7^2 53$		
t	$7^2 4^2$		
u	$76^2 3$		
v	7654		
w	75^3		

Class III.

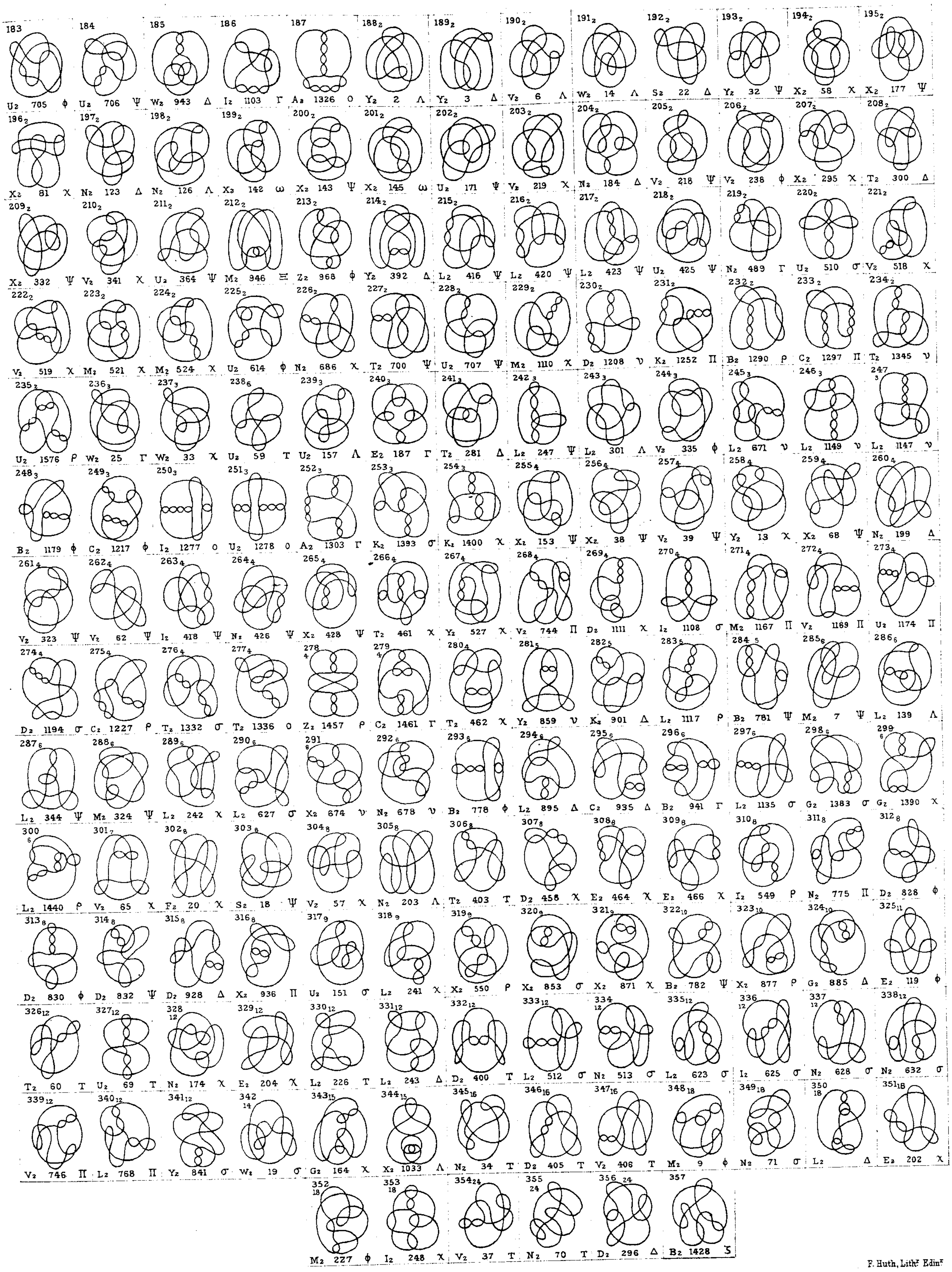
e	$10^2 2$	β	$3^2 2^8$
b	1084		
c	106^2		
d	$8^2 6$		

Class II.

a	11^2	α	2^{11}



C. N. Little del. F. Huth, Lith^r Edin^r



C.N. Little del.

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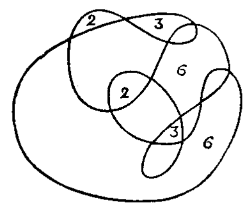
5. The manner of using these plates to identify a given elevenfold knot can be seen from the following example. Having drawn at random the figure in the margin, it is to be noticed that it is a *reduced, non-composite* form of eleven crossings. Mark the parts of the leading partition, and write down LISTING'S type symbol—



$$\left\{\begin{array}{l}6^2 3^2 2^2\\ 5 4^2 3 2^3\end{array}\right.$$

As the leading partition has six parts, the knot belongs to Class VI. Write now a graph of the leading partition showing how the parts are arranged :—

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  ⁄ 2 − 3 ⟍
6  − 2 −  6
  ⟍  3  ⁄
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The two 6-gons have six connections, a 3-gon and 2-gon, a 2-gon, a 3-gon, and two single crossings, which may be represented, in order, by a, b, c, d, d, and these letters have six circular arrangements as follows :—

abcdd	*bcdad*
acbdd	*acdbd*
cabdd	*abdcd*

But for each of these arrangements a may have three forms since c is asymmetrical, as follows :—

−2−3=
−3=3−
=3−2−

There are, therefore, eighteen distinct forms of this knot. A glance at Plate II. shows that it is the last of the six eighteen-form knots there given—No. 353.