

IX.—*Alternate ± Knots of Order Eleven.* By Professor C. N. LITTLE.
(With Two Plates.)

(Read 21st July ; Revised December 1890.)

1. A year ago last April, Prof. TAIT proposed that I should undertake to derive from Mr KIRKMAN'S polyhedral drawings the alternate \pm knots of eleven crossings, thus doing for order 11 what had been done so admirably by himself in orders 8, 9, and 10.

2. The work has been a very protracted one, because of the great number of forms involved—more than three times as many as in all preceding orders combined. Mr KIRKMAN'S manuscript contains 1581 forms, of which 22 are bifilar and 16 duplicates. I find from the remainder 357 knots with 1595 forms as shown in the following table :—

CLASS.	Un.	Two.	Three.	Four.	Five.	Six.	Seven.	Eight.	Nine.	Ten.	Eleven.	Twelve.	Fourteen.	Fifteen.	Sixteen.	Eighteen.	Twenty-four.	
II.	1
III.	4
IV.	8	14	3	6	...	2
V.	26	25	6	14	1	19	1	8	...	2	...	3	1
VI.	44	48	18	26	4	17	1	15	5	3	1	16	1	2	3	6	3	...
Total Knots }	83	87	27	46	5	38	2	23	5	5	1	19	1	2	4	6	3	357

As this is an odd order, perversion doubles these numbers, making 714 elevenfold knots, with crossings alternately over and under.

3. It has been thought unnecessary to show upon the Plates more than one form of each knot ; all, however, have been drawn. Knots of each class having the same number of forms are grouped together to make more simple the identification of a particular elevenfold. A small figure following the series number upon the plates indicates how many distinct forms each knot can assume. Knots 84, 357, and 238, are misplaced.

4. Below each knot-form figured will be found the number of the corresponding form in Mr KIRKMAN'S manuscript, and partition symbols to which the following table gives the key :—

CLASS VI.

A ₂	7 ² 2 ⁴	o	6 ² 2 ⁵
B ₂	7632 ³	π	6532 ⁴
C ₂	7542 ³	ρ	64 ² 3 ⁴
D ₂	753 ² 2 ²	σ	643 ² 2 ³
E ₂	743 ³ 2	τ	63 ⁴ 2 ²
F ₂	73 ⁵	ν	5 ² 42 ⁴
G ₂	74 ² 32 ²	φ	5 ² 3 ² 2 ³
H ₂	6 ² 42 ³	χ	54 ² 32 ³
I ₂	6 ² 3 ² 2 ²	ψ	543 ³ 2 ²
K ₂	65 ² 2 ³	ω	53 ⁵ 2
L ₂	65432 ²	Γ	4 ⁴ 2 ³
M ₂	653 ³ 2	Δ	4 ³ 3 ² 2 ²
N ₂	64 ² 3 ² 2	Λ	4 ² 3 ⁴ 2
S ₂	643 ⁴	Ξ	43 ⁶
T ₂	5 ³ 3 ²		
U ₂	5 ² 4 ² 2 ²		
V ₂	5 ² 43 ² 2		
W ₂	5 ² 3 ⁴		
X ₂	54 ³ 3 ²		
Y ₂	54 ² 3 ³		
Z ₂	4 ⁴ 3 ²		

CLASS V.

A	8 ² 2 ³	ν	5 ² 2 ⁶
B	8732 ²	ξ	5432 ⁵
C	8642 ²	η	53 ³ 2 ⁴
D	863 ³ 2	θ	4 ³ 2 ⁵
F	85 ² 2 ²	κ	4 ² 3 ² 2 ⁴
G	85432	λ	43 ⁴ 2 ³
H	853 ³	μ	3 ⁶ 2 ²
I	84 ³ 2		
K	84 ² 3 ²		
L	7 ² 42 ²		
M	7 ² 3 ² 2		
N	7652 ²		
O	76432		
P	763 ³		
Q	75 ² 3 ²		
R	754 ² 2		
S	7543 ²		
T	74 ³ 3		
U	6 ³ 2 ²		
V	6 ² 532		
W	6 ² 4 ² 2		
X	6 ² 43 ²		

CLASS V.—continued.

Y	65 ² 42
Z	65 ² 3 ²
E ₁	654 ² 3
A ₁	64 ⁴
B ₁	5 ⁴ 2
C ₁	5 ³ 43
D ₁	5 ² 4 ³

CLASS IV.

y	9 ² 2 ²	γ	4 ² 2 ⁷
f	9832	δ	43 ² 2 ⁶
g	9742	ε	3 ⁴ 2 ⁵
h	973 ²		
i	9652		
j	9643		
k	95 ² 3		
l	954 ²		
m	8 ² 3 ²		
n	8752		
o	8743		
p	8653		
q	85 ² 4		
r	7 ² 62		
s	7 ² 53		
t	7 ² 4 ²		
u	76 ² 3		
v	7654		
w	75 ³		

CLASS III.

e	10 ² 2	β	3 ² 2 ³
b	1084		
c	106 ²		
d	8 ² 6		

CLASS II.

a	11 ²	a	2 ¹¹
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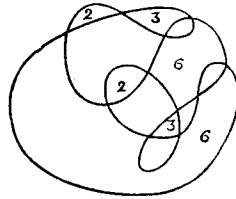
1 CLASS II	2 III	3	4	5	6 IV	7	8	9	10	11	12	13
a 1335 α	d 1317 β	b 1321 β	c 1322 β	e 1323 β	w 698 ε	x 703 ε	v 704 ε	s 1156 ε	t 1157 ε	v 1158 ε	x 1573 γ	y 1325 γ
14 ₂	15 ₂	16 ₂	17 ₂	18 ₂	19 ₂	20 ₂	21 ₂	22 ₂	23 ₂	24 ₂	25 ₂	26 ₂
o 1244 δ	q 1245 δ	u 1246 δ	n 1250 δ	r 1253 δ	t 1281 γ	h 1285 δ	k 1286 δ	g 1293 δ	f 1308 γ	i 1314 γ	w 1348 δ	u 1406 δ
27 ₂	28 ₂	29 ₂	30 ₂	31 ₂	32 ₂	33 ₂	34 ₂	35 ₂	36 ₂	37 ₂	38 ₂	39 CLASS V
g 1313 γ	n 777 ε	i 1177 ε	v 1280 γ	s 829 ε	f 1178 ε	k 1189 ε	p 1241 δ	l 1236 δ	v 1404 δ	j 779 ε	h 827 ε	D, 266 μ
40	41	42	43	44	45	46	47	48	49	50	51	52
C, 376 λ	D, 377 λ	A, 378 λ	B, 380 λ	E, 383 μ	Y, 389 λ	D, 391 λ	E, 395 λ	X, 396 λ	E, 398 λ	C, 419 κ	E, 471 κ	W, 484 κ
53	54	55	56	57	58	59	60	61	62	63 ₂	64 ₂	65 ₂
W, 487 κ	A, 488 κ	V, 681 κ	Y, 684 κ	E, 944 μ	W, 1104 κ	O, 1151 κ	A, 1324 υ	X, 1089 λ	D, 1321 λ	Z, 321 λ	Y, 381 λ	T, 382 λ
66 ₂	67 ₂	68 ₂	69 ₂	70 ₂	71 ₂	72 ₂	73 ₂	74 ₂	75 ₂	76 ₂	77 ₂	78 ₂
C, 417 κ	V, 517 λ	Y, 520 λ	R, 522 λ	Y, 523 λ	T, 682 κ	W, 682 μ	R, 682 μ	P, 1092 κ	S, 1093 κ	O, 1109 λ	A, 1579 γ	L, 1207 η
79 ₂	80 ₂	81 ₂	82 ₂	83 ₂	84 ₂	85 ₂	86 ₂	87 ₂	88 ₂	89 ₂	90 ₂	91 ₂
C, 1289 ζ	B, 1295 ζ	W, 1304 υ	V, 1352 η	V, 1378 ζ	V, 1378 ζ	U, 1574 κ	S, 421 κ	Z, 422 κ	Z, 424 κ	N, 424 κ	D, 1201 ε	Q, 1216 ζ
92 ₂	93 ₂	94 ₂	95 ₂	96 ₂	97 ₂	98 ₂	99 ₂	100 ₂	101 ₂	102 ₂	103 ₂	104 ₂
C, 1291 ζ	M, 1215 ζ	G, 1203 ζ	E, 387 λ	Y, 459 η	O, 516 λ	E, 525 λ	U, 548 κ	N, 748 λ	R, 1088 η	V, 1100 η	F, 1173 κ	D, 1193 η
105 ₂	106 ₂	107 ₂	108 ₂	109 ₂	110 ₂	111 ₂	112 ₂	113 ₂	114 ₂	115 ₂	116 ₂	117 ₂
C, 1226 ε	A, 1279 κ	Y, 1329 η	W, 1438 ζ	L, 721 λ	S, 372 λ	Z, 401 η	V, 511 κ	V, 622 κ	V, 624 κ	R, 672 κ	R, 676 κ	B, 745 λ
118 ₂	119 ₂	120 ₂	121 ₂	122 ₂	123 ₂	124 ₂	125 ₂	126 ₂	127 ₂	128 ₂	129 ₂	130 ₂
B, 767 κ	G, 769 κ	O, 773 κ	W, 926 κ	G, 1107 κ	Q, 1181 λ	E, 1381 ε	W, 1442 ζ	N, 670 κ	F, 743 λ	K, 1328 η	S, 545 κ	Z, 404 η
131 ₂	132 ₂	133 ₂	134 ₂	135 ₂	136 ₂	137 ₂	138 ₂	139 ₂	140 ₂	141 ₂	142 ₂	143 ₂
O, 460 η	O, 463 η	E, 468 η	G, 749 λ	G, 751 λ	C, 710 λ	H, 1090 η	W, 552 κ	M, 788 λ	D, 509 κ	G, 626 κ	Q, 1085 μ	H, 402 η
144 CLASS VI	145	146	147	148	149	150	151	152	153	154	155	156
Z ₂ 4 Δ	Z ₂ 5 Δ	Z ₂ 11 Ψ	Z ₂ 12 Δ	Y ₂ 15 ω	Y ₂ 16 Δ	W ₂ 23 Δ	Y ₂ 26 Δ	X ₂ 67 Ψ	V ₂ 118 Δ	V ₂ 128 Δ	X ₂ 141 Δ	X ₂ 144 Δ
157	158	159	160	161	162	163	164	165	166	167	168	169
U ₂ 158 Γ	X ₂ 130 Δ	X ₂ 178 Γ	V ₂ 200 Γ	X ₂ 251 Δ	X ₂ 252 Ψ	V ₂ 314 Δ	V ₂ 334 Ψ	N ₂ 327 Δ	V ₂ 340 Ψ	U ₂ 363 X	Y ₂ 379 Δ	T ₂ 388 X
170	171	172	173	174	175	176	177	178	179	180	181	182
I ₂ 1150 X	M ₂ 390 Δ	M ₂ 394 Δ	V ₂ 397 Δ	V ₂ 399 Δ	U ₂ 485 Γ	L ₂ 486 Γ	V ₂ 515 X	V ₂ 577 X	U ₂ 680 X	L ₂ 685 X	T ₂ 699 φ	U ₂ 702 φ

C. N. Little del.

F. Hutch. Lith^r Edin^r

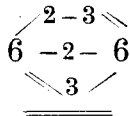
U_2 705 ϕ	U_2 706 Ψ	W_2 943 Δ	L_2 1103 Γ	A_2 1326 O	Y_2 2 Λ	Y_2 3 Δ	V_2 6 Λ	W_2 14 Δ	S_2 22 Δ	Y_2 32 Ψ	X_2 58 X	X_2 177 Ψ
X_2 81 X	N_2 123 Δ	N_2 126 Λ	X_2 142 ω	X_2 143 Ψ	X_2 145 ω	U_2 171 Ψ	V_2 219 X	N_2 184 Δ	V_2 218 Ψ	V_2 236 ϕ	X_2 295 X	T_2 300 Δ
X_2 332 Ψ	V_2 341 X	U_2 364 Ψ	M_2 946 \equiv	Z_2 968 ϕ	Y_2 392 Δ	L_2 416 Ψ	L_2 420 Ψ	L_2 423 Ψ	U_2 425 Ψ	N_2 489 Γ	U_2 510 σ	V_2 518 X
V_2 519 X	M_2 521 X	M_2 524 X	U_2 614 ϕ	N_2 686 X	T_2 700 Ψ	U_2 707 Ψ	M_2 1110 X	D_2 1208 ν	K_2 1252 Π	B_2 1290 ρ	C_2 1297 Π	T_2 1345 ν
U_2 1576 ρ	W_2 25 Γ	W_2 33 X	U_2 59 Γ	U_2 157 Λ	E_2 187 Γ	T_2 281 Δ	L_2 247 Ψ	L_2 301 Δ	V_2 335 ϕ	L_2 671 ν	L_2 1149 ν	L_2 1147 ν
B_2 1179 ϕ	C_2 1217 ϕ	L_2 1277 O	U_2 1278 O	A_2 1303 Γ	X_2 1393 σ	K_2 1400 X	X_2 153 Ψ	X_2 38 Ψ	V_2 39 Ψ	Y_2 13 X	X_2 68 Ψ	N_2 199 Δ
V_2 323 Ψ	V_2 62 Ψ	L_2 418 Ψ	N_2 426 Ψ	X_2 428 Ψ	T_2 461 X	Y_2 527 X	V_2 744 Π	D_2 1111 X	L_2 1108 σ	M_2 1167 Π	V_2 1169 Π	U_2 1174 Π
D_2 1194 σ	C_2 1227 ρ	T_2 1332 σ	T_2 1336 O	Z_2 1457 ρ	C_2 1461 Γ	T_2 462 X	Y_2 859 ν	K_2 901 Δ	L_2 1117 ρ	B_2 781 Ψ	M_2 7 Ψ	L_2 139 Δ
L_2 344 Ψ	M_2 324 Ψ	L_2 242 X	L_2 627 σ	X_2 674 ν	N_2 678 ν	B_2 778 ϕ	L_2 895 Δ	C_2 995 Δ	B_2 941 Γ	L_2 1135 σ	G_2 1383 σ	G_2 1390 X
L_2 1440 ρ	V_2 65 X	F_2 20 X	S_2 18 Ψ	V_2 57 X	N_2 203 Λ	T_2 403 T	D_2 458 X	E_2 464 X	E_2 466 X	L_2 549 ρ	N_2 775 Π	D_2 828 ϕ
D_2 830 ϕ	D_2 832 Ψ	D_2 928 Δ	X_2 936 Π	U_2 151 σ	L_2 241 X	X_2 550 ρ	X_2 853 σ	X_2 871 X	B_2 782 Ψ	X_2 877 ρ	G_2 885 Δ	E_2 119 ϕ
T_2 60 T	U_2 69 T	N_2 174 X	E_2 204 X	L_2 226 T	L_2 243 Δ	D_2 400 T	L_2 512 σ	N_2 513 σ	L_2 623 σ	L_2 625 σ	N_2 628 σ	N_2 632 σ
V_2 746 Π	L_2 768 Π	Y_2 841 σ	W_2 19 σ	G_2 164 X	X_2 1039 Λ	N_2 34 T	D_2 405 T	V_2 406 T	M_2 9 ϕ	N_2 71 σ	L_2 Δ	E_2 202 X
M_2 227 ϕ	L_2 248 X	V_2 37 T	N_2 70 T	D_2 296 Δ	B_2 1428 S							

5. The manner of using these plates to identify a given elevenfold knot can be seen from the following example. Having drawn at random the figure in the margin, it is to be noticed that it is a *reduced, non-composite* form of eleven crossings. Mark the parts of the leading partition, and write down LISTING'S type symbol—



$$\begin{cases} 6^2 3^2 2^2 \\ 5 4^2 3 2^3 \end{cases}$$

As the leading partition has six parts, the knot belongs to Class VI. Write now a graph of the leading partition showing how the parts are arranged :—



The two 6-gons have six connections, a 3-gon and 2-gon, a 2-gon, a 3-gon, and two single crossings, which may be represented, in order, by *a, b, c, d, d*, and these letters have six circular arrangements as follows :—

abcdd bedad
acbdd acdbd
cabdd abdc

But for each of these arrangements *a* may have three forms since *c* is asymmetrical, as follows :—

—2—3—
 —3—3—
 =3—2—

There are, therefore, eighteen distinct forms of this knot. A glance at Plate II. shows that it is the last of the six eighteen-form knots there given—No. 353.