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★**The Hauptvermutung book.**

A collection of papers of the topology of manifolds.

K-Monographs in Mathematics, 1.

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The Hauptvermutung, short for "die Hauptvermutung der kombinatorischen Topologie", which means "main conjecture of combinatorial topology" in German, was formulated in 1908 by E. Steinitz and H. Tietze in two separate articles. H. Kneser in his 1925 textbook on the topology of manifolds, and P. Alexandroff and H. Hopf in their 1935 textbook on topology, gave statements of this conjecture. The conjecture states that the combinatorial topology of a simplicial complex is determined by the topology of the corresponding polyhedron which is its geometric realization. One of the applications for this conjecture was to be the topological invariance of simplicial homology; however, this was proven by S. Eilenberg and N. Steenrod in the 1940's without the use of the Hauptvermutung. In 2 dimensions the Hauptvermutung was known in 1908 by the classification of surfaces. For polyhedra of dimension ≤ 2 it was proven by Ch. Papakyriakopoulos in 1943 and by E. Moise for 3-manifolds in 1953. In 1961 J. Milnor showed that the Hauptvermutung was false for polyhedra, using Reidemeister torsion and some results of B. Mazur and J. Stallings on open manifolds.

The development of surgery theory for simply connected manifolds from 1956 to 1965 by Milnor, M. Kervaire, W. Browder, S. Novikov, C. T. C. Wall, Sullivan, R. Lashof and M. Rothenberg led to a better understanding of smoothing theory, in particular the relationship between differentiable and PL structures. In 1965 Novikov proved the topological invariance of rational Pontryagin classes. This theorem, which led to the solution of the Hauptvermutung, has since been proven analytically by Sullivan and N. Teleman (1983) using the existence of Lipschitz structures on manifolds proven by Sullivan in 1977, semi-analytically using bounded operator surgery by E. Pedersen, J. Roe and S. Weinberger (1995) and bounded geometry surgery by the reviewer (1996), and using controlled topology by S. Ferry and Weinberger (1995). The Hauptvermutung for manifolds was considered by R. Kirby and L. Siebenmann, who disproved it in 1969.

The papers in *The Hauptvermutung book*, aside from the 1996 article by Ranicki entitled "On the Hauptvermutung" and a Coda by Rourke written in 1972, were all written during the period 1966–1968. These papers are all relevant to the development of surgery on simply connected manifolds, and fall between the book by Browder [*Surgery on simply-connected manifolds*, Springer, New York, 1972; MR0358813 (50 #11272)], based on Princeton lecture notes to a course given in 1968, and Wall's monograph [*Surgery on compact manifolds*, Academic Press, London, 1970; MR0431216 (55 #4217)]. The article "Generalisations and applications of block bundles" by Casson is his dissertation, written in 1967, and deals with a notion introduced by Rourke and B. Sanderson around 1966 which is a substitute in the PL category for the normal bundle of a differentiable manifold. Casson introduces a classifying space for them and compares them to spherical fibrations. The fiber of the comparison map is called G/PL and has the property, which Casson proves here, that it is 8-fold periodic, and $\Omega^4(G/\mathrm{PL})$ is homotopy equivalent to $\Omega^8(G/\mathrm{PL})$. This statement is in fact off by a factor of $\mathbf{Z}$, as in topological K-theory, where $\Omega^8(B\mathrm{O}) = \mathbf{Z} \times B\mathrm{O}$. This same error occurs on p. 273 of the book of Kirby and Siebenmann [*Foundational essays on topological manifolds, smoothings, and triangulations*, Ann. of Math. Stud., 88, Princeton Univ. Press, Princeton, N.J., 1977; MR0645390 (58 #31082)]. Ranicki's book [*Algebraic L-theory and topological manifolds*, Cambridge Univ. Press, Cambridge, 1992; MR1211640 (94i:57051)] contains the correct statement on p. 285. F. Quinn's 1983 attempt to solve the resolution conjecture for homology manifolds posed by R. Edwards in his 1978 Helsinki ICM talk ignored this extra $\mathbf{Z}$, which S. Cappell pointed out in 1987 contained a transversality obstruction, which was then realized in 1992 by J. Bryant, Ferry, W. Mio and Weinberger. Casson's article also contains a result on the Hauptvermutung, namely that it is true for M closed, simply connected, with $\dim M \geq 5$ and $H^3(M; Z_2) = 0$.

Sullivan's article "Triangulating and smoothing homotopy equivalences and homeomorphisms. Geometric Topology Seminar Notes" consists of notes from a 1966 seminar at Princeton University on the Hauptvermutung. He shows how to classify the PL manifolds homotopy equivalent to a complex projective space $\mathbf{C}\mathrm{P}^n$, $n > 2$. Sullivan sets up an obstruction theory for making a homotopy equivalence into a PL homeomorphism, and then calculates the obstructions using bordism of singular manifolds. This leads to what Sullivan calls the "characteristic variety theorem", which enables one to calculate $[M\colon F/\mathrm{PL}]$, where BF is the classifying space for spherical fibrations. Sullivan applies this calculation to the Hauptvermutung, and obtains the result that the subgroup of homotopy triangulations of a manifold M^n with $n \geq 5$ generated by homeomorphisms has Z_2-dimension not greater than [2-torsion $H_3(M;\mathbf{Z})$]. The final section deals with smoothing triangulations.

"The Princeton notes on the Hauptvermutung" by Armstrong, Rourke and Cooke fall into three sections: the first, by Armstrong, gives the Lashof-Rothenberg proof of the Hauptvermutung for 4-connected manifolds. This proof dates from around 1966, and uses a splitting theorem of Novikov. The second, by Rourke, discusses the work of Casson and Sullivan, from 1966–1967, on G/PL and its relation to surgery. The third, by Cooke, proves the Hauptvermutung using the methods from Rourke's article under the assumptions that (i) M is either closed of dimension at least 5, or bounded of dimension at least 6; (ii) each component of both M and ∂M is simply connected; (iii) $H^3(M;Z_2)=0$.

The Coda by Rourke, written in 1972, discusses the connection between the work of Kirby and Siebenmann on topological manifolds and the calculation of the fiber of $G/\mathrm{PL} \to \Omega^{4n}(G/\mathrm{PL})$ in the articles by Rourke and Cooke.

Ranicki's article "On the Hauptvermutung", besides giving an overview of work on the conjecture by Milnor (1961) and Kirby-Siebenmann (1969), from the point of view of Wall's L-theory (or non-simply connected surgery), describes the work on the triangulation of homology manifolds during the 1970's by Edwards, M. Cohen, H. Sato, Sullivan, D. Galewski, R. Stern and T. Matumoto.

{For detailed discussions of the individual papers in this collection see the following eight reviews [MR1434101 (98c:57025); MR1434102 (98c:57026);
MR1434103 (98c:57027);
MR1434104 (98c:57028);
MR1434105 (98c:57029);
MR1434106 (98c:57030);
MR1434107 (98c:57031);
MR1434108 (98c:57032)].}

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