## Report

of

## the Experts

Darboux, Appel, Poincaré

## INTRODUCTION

Notions on the Probability of Causes

The system of Mr Bertillon, together with the other systems submitted to our examination, have the claim to be an application of probability theory : we have thus conducted before commencing the detailed study, to research for what conditions this calculation can be legitimately applied to the questions of this nature. The first attempts done by Mr Bertillon for the evaluation of the probabilities had been completely unfortunate.

In his memo presented at the Court of Cassation in 1899 he used a reasoning entirely inaccurate, which he then repeated in front of the Court Martial of Renne.

Having established four coincidences over the 26 beginnings and endings of the redoubled polysyllables, he wonders what conclusion one can draw from them. He evaluates the probability of an isolated coincidence as 0.2 and he concludes from that the probability of 4 coincidences is $(0.2)^{4}=0.0016$.

But the most superficial examination shows that it is the probability for there to be 4 coincidences over 4: that of 4 coincidences over 26 is 0.7 , that is to say 400 times larger.

When this error was pointed out, he responded that there had been in reality more than 4 coincidences and that the probability of each of them was smaller than 0.2 ; the reasoning none the less false, since it leads the author to a result 400 times weaker than that would give a correct calculation done with the same data.

Mr Bertillon, we believe, gave up; but the very story of his error shows us the necessity of properly establishing the fundamental principles to apply.

If one takes certain coincidences as evidence, and that one shows that there had been à priori few chances for those to happen, have we the right to conclude that they cannot be the effect by chance?

If the number 25 came out at the lottery, this would be an event of which the probability à priori were very weak, since the tickets were quite many; but that does not imply that the draw was not honest, for anyway some number or another comes out.

Thus this is not to be reasoned that way; it is not the calculation of the probability of this or that coincidence which you chose precisely because you noticed it; what is to be introduced, is the probability of a certain coincidence among those which you will count in your favour if it occurs.

Suppose that there were 1,000 letters in the "slip", with the differences in abscissas and ordinals, that makes 999,000 numbers; if one finds 10,000 coincidences, is one to be astonished? The probability to be sought, it would be 10,000 for over 999,000 numbers, which after 10 years of search, may appear remarkable to a spirit as attentive as Mr Bertillon; it is almost a certainty.

If one reproduced a million of documents, there would not be one in which one would find the same particularities, that is true, but there would be 900,000 in which one would find other particularities that you would not judge less remarkable.

We have said enough to make understood the necessity of a more solid reasoning base. It is what the founders of the probability theory sought, for the questions of this genre, but we cannot explain it without entering into some technical details.

They have distinguished the probability of effects and the probability of causes. As example of probability of effects, one chooses from an ordinary urn containing 90 white balls and 10 black balls; if one draws arbitrarily a ball from this urn, what is the probability for this ball be black; it is
evidently $1 / 10$.
The problems of probability of causes are far more complicated, but far more interesting.
Let us suppose for example two urns of identical exterior; we know that the one containing 90 white balls and 10 black balls, et the other, on the contrary 90 black balls and 10 white balls. We draw arbitrarily one ball from one of the urns, without knowing from which, and we notice that is is white. What is the probability for it to be in the first urn which we drew?

In this new problem, the effect is known, one noticed that the ball drawn was white; but the cause is unknown, one do not know from which urn one made the draw.

The problem that we concern here is of the same nature: the effect is known, it is the indicated coincidences, on the "slip", and it is the cause (forgery or natural writing) that is to be determined.

It is thus the said formulae of probability of causes that is convenient to apply. But the application of these formulae requires some precautions.

In the example cited above, the probability in search is $9 / 10$, but it is because we suppose that there is à priori no reason for one to be fallen on one of the urns, rather than on the other. But, the things would have been much different if we had 11 urns of which 10 composed like the first and only one like the second. A priori the probability for one to fall on an urn which the white dominates would have been already large; and the results should have been notably modified.

To be able to calculate, after an established event, the probability of a cause, we thus should have many data:
$1^{\circ}$ It should be known what was $\grave{a}$ priori, before the event, the probability of this cause.
$2^{0}$ It should be known then for each possible cause, what would be the probability of the noticed event. (It is thus in the example cited, it is necessary to know the composition of urns).

Now this probability à priori, in the questions like those we are concerned, is uniquely formed of the moral elements which escape absolutely from the calculation, and if, as we just saw, we cannot calculate anything without knowing it, all calculation becomes impossible.

And Auguste Conte was quite right to have said that the application of the probability theory to the moral sciences was the scandal of mathematics.

To want to eliminate the moral elements and to replace them by numbers, is dangerous and futile.

In a word, the probability theory is not, as people appear to believe, a better science that dispense the scientist of good sense.

This is why it is necessary to abstain absolutely from applying the calculation to the moral things; if we do it here, it means we are constrained.

It is the moral elements that should depend the judgement, we have not mentioned it here; but it is evident that if the author of the "slip" had wanted to fake, he should have chosen a simple system that would be sure to be remarked by the experts and on which no challenging would have been possible.

It suffices, to turn down the Bertillon system, to observe that it does not satisfy this condition.
One may say that it is not our rôle to examine the question form this viewpoint. We should give numbers, but we will be able to make it only under the following form.

As it is impossible to know the probability à priori, we will not be able to say such coincidence prove that the relation of the probability of the forgery to the inverse probability to such value. We would be only able to say, by the finding of this coincidence, this report becomes many times larger than before the finding.

Even, after we being thus restrained, there remain many traps to avoid. One is never sure to have done a complete enumeration of the possible causes, and it is as LAPLACE was carried away in a memorable error on the subject of the probable direction of the rotation of planets.

Here that enumeration is almost impossible, because it is necessary to search all the possible manners of fake a document. And if we are artificially restricted to two causes, the chance and the particular mode of forgery imagined by Mr Bertillon, an important difficulty still remains.

We should, as we said, know the composition of the two urns. Thus we know one of them, that corresponds to the natural script, its composition is determined by the law of chance, but we do not know the other, we do not know what is the probability for a coincidence of nature given to produce, to suppose that the author of the "slip" were employed by the Bertillon system.

Unable to determine it, we will still admit in the calculation that follows the hypothesis the most favourable to the Bertillon system .

This quick discussion has shown us how fragile all these logical elaboration which one would wish to make the honour of a man depend on; and something more is needed, the very multiplicity of these systems would provide us a clear demonstration of this fragility.

We have two systems in presence, that of Mr Bertillon and that of Mr Corps, they are absolutely incompatible, the procedures employed to treat are not the same, after these two inventors, no more than the motives that have been used in the forgery.

And while Mr Corps like Mr Bertillon piles up the coincidences, and tries hard, by the arguments equally conclusive, to show that they cannot be by chance.

But if the coincidences of Mr Bertillon are not by chance, it is that the traitor made at good use of the system imagined by this expert; and then it is Mr Corps who is wrong.

And if those of Mr Corps who does not give up for nothing for the first places, cannot be by chance, it is that Mr Corps is right and that Mr Bertillon was wrong.

Unless the mode of reasoning itself is not deceitful it will be our conclusion.
To refute Mr Corps et Mr Bertillon at the same time, it suffices thus to opposer them one against the other. It would suffice much better to oppose Mr Bertillon against himself. And, in fact, there are two Bertillon systems on which we will return in detail, but which one can rapidly realise in consulting an anonymous brochure entitled "the slip" by a former pupil of the École
Polytechnique (Paris Hardy 1904) and that we will call the green brochure for short. This brochure is accompanied of an atlas where one will remark two boards, the board 6 and the board 9 that represent the two Bertillon systems.

If then the coincidences of the board 9 were real, as they could not be by chance, they would prove that the "slip" was not calculated on the size of the board 6 , that is to say the word of interest.

If this were a scientific work, we would have stopped there; we would judge it useless to examine the details of a system which the very principle cannot sustain the examination: but the Court has entrusted us a mission which we have to accomplish right to the end.

## Reconstruction of the "slip"

We must observe first that the measures that served as basis to the systems had not been taken from the original "slip", but on a artificial document, which one will refer to under the name "reconstructed slip" and that we are going to explain the origin.

Immediately after the seizure of the "slip", Mr Tomps made two snapshots of it which we will call under the name of Tomps snapshots; these were the document which, beside the original itself, offered the closest as they were the oldest; but they were not able to serve Mr Bertillon because the watermarks of the paper did note appear there.

Mr Bertillon then made new snapshots by contact, and they are the snapshots which are reproduced on the boards 1 et 2 of the green brochure and which are the origin of the reconstruction.

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Conclusion
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The absurdity of Bertillon system is so evident that one will explain with difficulty the length of this discussion. One would risk of not understanding the necessity, if one did not recall the story of the affair.

When the system was for the first time known to public, when one learned that "the slip was not the work of the natural force only" there was a long burst of laughter. One has not forgotten the recitation of this epic seat, where a certain redan, struck by the battery of long S's, defended himself heroically, until at last the attacker, intimidated by the blotting paper, moved back in front of the initials.

Those who pushed further the examination discovered things no less stunning. In the middle of an incomprehensible jumble, they found the findings of that kind.

There are two barely perceptible points on the word intérêt of which the vertical distance represent precisely at the scale of 80,000 -th normal contour interval of the level chart of the Headquarters. Can this be by chance?

Thus this miserable, on the point of betraying his country, had only one idea: to reproduce, in imitating the handwriting of his brother, the exact contour intervals of the level.

But at a certain moment clever men understood which part one could draw from this precious and inexhaustible mine of ambiguities. They knew that the laughers grow tired and that the believers do not grow tired; they knew that the public do not pay attention to the value of the arguments, but to the tones of the sophists. And they began to support Mr Bertillon of their sharp and reiterated affirmations.

Well, he had a great advantage; the obscurity of his system defended him against the criticism, as well as the cuttlefish surrounds itself with a cloud of ink to escape from its enemies.

His system changed constantly, and by that he was able to extend the discussion, et that could make illusion to the public who had for a long time gave up to understand.

There is no inventor of the squaring of the circle who is not ready to extend the resistance indefinitely, the moment when one accepts to debate with him.

The mission that we were charged obliged us to examine the system as if it were serious. This examination brought us to the following results :

The reconstitution of the "slip" is wrong, the errors are generally of $1 / 2 \mathrm{~m} / \mathrm{m}$ and can go up to 1 or $2 \mathrm{~m} / \mathrm{m}$. It is this wrong reconstruction that Mr Bertillon uniquely used.

Their boards are the result of a complicated treatment imposed to the primitive document and form there that began to be altered. It has undergone a series of photographic enlargements and reductions and even tracings, reproductions, re-tracings, cuttings, collages, opaques, whitewashing and retouches.

One employed two entirely different reticules and one passes from the one to the other according the needs of the cause.

We have shown, by the application of the rules of probability theory, that the indicated coincidences, concerning the horizontal location of the polysyllables can be very well explained by chance and can prove nothing but the document has been forged. These coincidences in fact are only proximities.

They were true that they could not by only themselves prove that this forged document was done by Dreyfus. To arrive at this result, one invokes a letter seized at his residence, where the word intérêt was located. Thus it is necessary to find a relation of cause to effect between this word and this distance of $5 \mathrm{~m} / \mathrm{m}$ that one says to find so frequently on the "slip". Such is the origin of the
countless rantings that we discussed in our chapter on the word intérêt. Let us content with saying that these measures are only roughly approximate.

The location of the initials has been obtained thanks to 41 bits of a boost .
The locations of he non-initials does not exceed the properly calculated probable number.
The arguments, that Mr Bertillon drew by the application of the words of the "slip" ones on the others or on those of the letter of the blotting paper, have been reduced to their real value, which is null.

As for the application over the words of the pieces of comparison our examination proves that Mr Bertillon made a completely arbitrary choice from the very numerous words that he had at hand. It is then, either he had the desire of proving the guilt, or, which we prefer to believe, that he committed a serious method error.

We asked Mr Bertillon to reproduce in front of us, as he had offered at the Court of Cassation and as he had done at Rennes, the size together with the front and the back of the "slip"; he refused that, alleging that the memory and the hand would be lacking.

Thus there is only a mnemonic exercise.
We will not insist over the location of the vertical lines where we have seen only a theorem of arithmetic incontestable as well as naive.

The "slip" not having $207 \mathrm{~m} / \mathrm{m}$ as affirmed by Mr Bertillon, all the relations between the lines, the edges, and the notch fall themselves.

We have established in an incontestable manner, for all non biased minds, that the notch of the "slip", as well as those of the letter of the blotting paper, had been made afterwards the seizure of the piece. This demonstration only itself would suffice to collapse all the system.

The composite photography presents that the vague appearances where the auto-suggestion permits to see all one wants. The proof, is that Mr the Commandant Corps had believed all from the beginning see there appear not the word intérêt, but the framework of his grid.

Major Corps intervened spontaneously in this affair, so he did not have at his disposition all the methods of work that Mr Bertillon had had; he served meanwhile in the reconstitution of Mr Bertillon which, as we have seen, is false. On almost all the points his system disagree with that of Mr Bertillon, which constitutes an argument at the same time against the two systems.

What we said suffices for making comprehend the spirit of the "method" of Mr Bertillon. He himself has summarised it as a word: "when one searches, one always discovers".

When a coincidence is noticed, it is a damning evidence; if there is none, it is a more damning evidence, because that proves that the script tried to divert the suspicions.

One will not be surprised by the results he has obtained by this method. The naivety with which he revealed the secrets brings up to believe his bona fide.

To sum up, all these systems are absolutely deprived of any scientific value:
$1^{\circ}$ Because the application of probability theory to these matters is not legitimate;
$2^{\circ}$ Because the reconstitution of the memorandum is false;
$3^{\circ}$ Because the rules of probability theory have not been correctly applied.
In a word, because the authors have wrongly reasoned on the false documents.
Signed : Poincaré, Darboux, Appell.

