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Selected Matches for: Anywhere=(Sheiham)

MR2375818 (2009c:57042)
Levine, Jerome(1-BRND)
Concordance of boundary links. (English summary)
J. Knot Theory Ramifications 16 (2007), no. 9,
1111--1120.
57Q45 (57M25 57Q60)

Citations
From References: 3
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This paper proves the concordance invariance of certain Atiyah-Patodi-Singer ρ-invariants of links. The importance of this result can be understood in the context of on-going efforts to understand links up to concordance.

Roughly speaking, high-dimensional links with m components can be classified by invoking the Cappell-Shaneson homology surgery theory with the Vogel homology localization of the wedge of m circles as the target space, due to work of J.-Y. Le Dimet [Mém. Soc. Math. France (N.S.) No. 32 (1988), {\rm ii}+92 pp.; MR0971415 (90e:57046)]. Nonetheless, the structure of link concordance classes is still largely unknown, because of the lack of understanding of the Vogel homology localization and related algebraic and homotopy theoretic problems.

On the other hand, if one thinks of smaller classes of links (and appropriate notions of concordance), classifications are more accessible. The most important case is boundary concordance of boundary links, which were classified by S. E. Cappell and J. L. Shaneson [Comment. Math. Helv. **55** (1980), no. 1, 20--49; MR0569244 (81j:57011)] in terms of homology surgery obstructions, by K. H. Ko [Trans. Amer. Math. Soc. **299** (1987), no. 2, 657--681; MR0869227 (88h:57018)] and W. Mio [Math. Proc. Cambridge Philos. Soc. **101** (1987), no. 2, 259--266; MR0870597 (88e:57023)] in terms of Seifert matrices, and by J. Duval [C. R. Acad. Sci. Paris Sér. I Math. **299** (1984), no. 18, 935--938; MR0774672 (86c:57020)] in terms of Blanchfield forms. In 2001, D. Sheiham gave a full computation of the structure of boundary concordance group of boundary links [Mem. Amer. Math. Soc. **165** (2003), no. 784, x+110 pp.; MR1997849 (2005f:57038)].

It is known that the concordance classification of links is not reduced to the boundary concordance classification of boundary links; T. D. Cochran and K. E. Orr proved that not all links are concordant to boundary links [Ann. of Math. (2) **138** (1993), no. 3, 519--554; MR1247992 (95c:57042)]. But, for boundary links, whether concordance and boundary concordance are the same still remains as one of the most important open problems. In this regard, one is naturally led to investigate concordance \text{(non-)}invariance of boundary concordance invariants.

In the paper under review, the author considers the boundary link concordance invariants defined in his earlier work [Comment. Math. Helv. **69** (1994), no. 1, 82--119; MR1259607 (95a:57009)]. To avoid technical complications, he states results for m-component boundary links $L\subset S^n$ endowed with a choice of a homomorphism $\pi_1(S^n-L) \to F$ sending meridians to generators, where F denotes the free group of rank m. Such a link is called an F-link. Suppose

$n=2q+1$. Let $R_k(F)$ be the space of k-dimensional unitary representations of F, which is a real algebraic variety. For a given F-link L, the author defines a function $\rho_L\colon R_k(F) \to \Bbb{R}$, which is the Atiyah-Patodi-Singer ρ-invariant of the surgery manifold M_L of L. He also defines a subvariety $D_L\subset R_k(F)$ of representations θ such that $H_q(M_L;\Bbb{C}^k)$ with coefficients given by θ is nontrivial.

The main result is the following (ordinary) concordance invariance of ρ_L: If L_0 and L_1 are concordant (or give concordant disk links), then ρ_{L_0} and ρ_{L_1} are equal on $R_k(F)-(D_{L_0} \cup D_{L_1})$. For a boundary slice link L, $\rho_L=0$ on $R_k(F)-D_L$.

In [op. cit.], Sheiham proved that signature invariants obtained from certain quiver representations determine the boundary concordance classes modulo torsion. From the result of the paper under review, it follows that if all the Sheiham signatures are captured by the ρ-invariants, then boundary concordance is equal to ordinary concordance modulo torsion. Including this, some interesting open problems are listed in the last section of this paper.

Reviewed by Jae Choon Cha

References

1. M. F. Atiyah, V. K. Patodi and I. M. Singer, Spectral asymmetry and Riemannian Geometry. I, *Math. Proc. Cambridge Phil. Soc.* **77** (1975) 43--69 MR0397797 (53 #1655a)
2. M. F. Atiyah, V. K. Patodi and I. M. Singer, Spectral asymmetry and Riemannian Geometry. II, *Math. Proc. Cambridge Phil. Soc.* **78** (1975) 405--32. MR0397798 (53 #1655b)
3. M. F. Atiyah, V. K. Patodi and I. M. Singer, Spectral asymmetry and Riemannian Geometry. III, *Math. Proc. Cambridge Phil. Soc.* **79** (1976) 71--99. MR0397799 (53 #1655c)
4. S. Cappell and J. L. Shaneson, Link cobordism, *Comment. Math. Helv.* **55** (1980) 20--49. MR0569244 (81j:57011)
5. P. M. Cohn, *Free Rings and Their Relations* (Academic Press, 1971). MR0371938 (51 #8155)
6. J. Cha and K. Ko, Signature invariants from links and irregular covers and non-abelian covers, *Proc. Cambridge Phil. Soc.* **127**(1) (1999) 67--81. MR1692511 (2000d:57014)
7. M. Farber, Noncommutative rational functions and boundary links, *Math. Ann.* **293** (1992) 543--568. MR1170526 (94m:68137)
8. S. Friedl, Link concordance, boundary link concordance and eta invariants, math.GT/0306149.
9. S. Garoufalidis and J. Levine, Analytic invariants of boundary links, *J. Knot Theory Ramifications* **11** (2002) 283--294. MR1905685 (2003f:57012)
10. M. Gutierrez, Links of codimension two, *Revista Colombiana de Matematicas*, Monografias de Matemática, Vol. 10 (1974). MR0415628 (54 #3709)
11. M. Gutierrez, Boundary links and an unlinking theorem, *Trans. Amer. Math. Soc.* **171** (1972) 491--99. MR0310902 (46 #10000)
12. N. Habegger and X. S. Lin, On link concordance and Milnor's $\bar{µ}$-invariants, *Bull. London Math. Soc.* (1998) 419--428. MR1620841 (2000e:57023)
13. K. H. Ko, Seifert matrices and boundary link cobordism, *Trans. Amer. Math.*

Soc. **299** (1987) 657--681. MR0869227 (88h:57018)
14. J. Y. Le Dimet, Cobordisme d'enlacement de disques, *Bull. Soc. Math. France* **116** (1988).
15. J. Levine, Polynomial invariants of knots of codimension two, *Ann. Math.* **84** (1966) 537--554. MR0200922 (34 #808)
16. J. Levine, Link invariants via the eta invariant, *Comment. Math. Helv.* **69** (1994) 82--119. MR1259607 (95a:57009)
17. J. Levine, Link concordance and algebraic closure of groups. II, *Invent. Math.* **96** (1989) 571--592. MR0996555 (91g:57007)
18. W. Mio, On boundary link cobordism, *Math. Proc. Cambridge Phil. Soc.* **101** (1987) 259--266. MR0870597 (88e:57023)
19. D. Sheiham, Invariants of boundary link cobordism, math.AT/0110249.
20. L. Smolinsky, Invariants of link cobordism, in *Proceedings of the 1987 Georgia Topology Conference: Topol. Appl.* **32**(2) (1989) 161--168. MR1007988 (90h:57028)
21. P. Vogel, Representations of link groups, in *KNOTS 90: Proceedings of Osaka Knot Theory Conference* (1990) (W. de Gruyter, Berlin), pp. 381--388. MR1177434 (93i:57032)

This list reflects references listed in the original paper as accurately as possible with no attempt to correct error.

MR2286030 (2007k:57006)
Cimasoni, David(1-CA)
Slicing Bing doubles. (English summary)
Algebr. Geom. Topol. 6 (2006), 2395--2415.
57M25 (57M27)

Citations
From References: 5
From Reviews: 1

A knot $K\subset S^3$ is called slice if it bounds a smoothly embedded disk $D\subset D^4$. Furthermore, a link is called slice if the components bound disjointly embedded disks in D^4. One of the most interesting questions in the study of slice links is the question of how certain topological constructions affect the sliceness of a link. In this paper the author studies the effect of Bing doubling on the sliceness of a knot. The Bing double $B(K)$ of a knot K is the result of wrapping the trivial link with two components in a certain nontrivial way around K. Given a slice knot K it is well-known that the Bing double $B(K)$ is a slice link. It is a natural question to ask whether the converse holds, i.e. whether $B(K)$ slice implies that K is slice. Some evidence for an affirmative answer had been given independently by P. Teichner and S. Harvey in their study of the L^2-von Neumann ρ-invariant of $B(K)$.

In the paper under review the author first shows that the Bing double of a knot K is a boundary link, i.e. the components bound disjoint Seifert surfaces in S^3. In the case of a boundary link there is a stronger notion of sliceness: a boundary link is called boundary slice if it is slice and if the union of the Seifert surfaces and the slice disks bound disjoint embedded 3-manifolds in D^4. It is not known whether a slice boundary link is necessarily boundary slice.

The main theorem of this paper is the following:

Theorem. If the Bing double $B(K)$ is boundary slice, then the knot K is algebraically slice.

Here a knot K is called algebraically slice if its Seifert matrix has a metabolizer.

This theorem is remarkable as it is the first theorem of its kind to completely capture the information on the algebraic concordance class of K. The proof of this theorem relies heavily on D. Sheiham's study of boundary slice concordance [Mem. Amer. Math. Soc. **165** (2003), no. 784, x+110 pp.; MR1997849 (2005f:57038)].

Cimasoni's theorem has since been strengthened by J. C. Cha, C. Livingston and D. Ruberman [``Algebraic and Heegaard-Floer invariants of knots with slice Bing doubles'', Math. Proc. Cambridge Philos. Soc., to appear]; in fact, they showed that in the theorem it is enough to assume that $B(K)$ is slice.

Finally, in the last section the author gives further evidence to the conjecture that a knot is slice if and only if its Bing double is slice by studying the Rasmussen-Beliakova-Wehrli invariant of links.

Reviewed by Stefan K. Friedl

References

1. **A Beliakova, S Wehrli**, *Categorification of the colored Jones polynomial and Rasmussen invariant of links* arXiv:math.QA/0510382 MR2462446
2. **R H Bing**, *A homeomorphism between the* 3- *sphere and the sum of two solid horned spheres*, Ann. of Math. (2) 56 (1952) 354--362 MR0049549 MR0049549 (14,192d)
3. **R Budney**, *JSJ-decompositions of knot and link complements in the 3-sphere* arXiv:math.GT/0506523
4. **J C Cha, K H Ko**, *Signature invariants of links from irregular covers and non-abelian covers*, Math. Proc. Cambridge Philos. Soc. 127 (1999) 67--81 MR1692511 MR1692511 (2000d:57014)
5. **D Cimasoni, V Florens**, *Generalized Seifert surfaces and signatures of colored links* arXiv:math.GT/0505185 MR2357695 (2009b:57009)
6. **D Cooper**, *The universal abelian cover of a link*, from: "Low-dimensional topology (Bangor, 1979)", London Math. Soc. Lecture Note Ser. 48, Cambridge Univ. Press, Cambridge (1982) 51--66 MR662427 MR0662427 (83g:57002)
7. **M H Freedman, F Quinn**, *Topology of 4-manifolds*, Princeton Mathematical Series 39, Princeton University Press MR1201584 MR1201584 (94b:57021)
8. **S Friedl**, *Link concordance, boundary link concordance and eta-invariants*, Math. Proc. Cambridge Philos. Soc. 138 (2005) 437--460 MR2138572 MR2138572 (2006c:57020)
9. **S Harvey**, *Homology cobordism invariants and the Cochran-Orr-Teichner filtration of the link concordance group* arXiv:math.GT/0609378 MR2390349 (2009d:57045)
10. **A Hatcher**, *Basic Topology of 3-Manifolds* Available at math.cornell.edu/$\sim$hatcher/
11. **A Kawauchi**, *On the Alexander polynomials of cobordant links*, Osaka J. Math. 15 (1978) 151--159 MR0488022 MR0488022 (58 #7599)
12. **K H Ko**, *private communication*
13. **K H Ko**, *Seifert matrices and boundary link cobordisms*, Trans. Amer. Math. Soc. 299 (1987) 657--681 MR869227 MR0869227 (88h:57018)
14. **C Livingston**, *Computations of the Ozsváth-Szabó knot concordance invariant*, Geom. Topol. 8 (2004) 735--742 MR2057779 MR2057779 (2005d:57019)
15. **C Livingston, S Naik**, *Ozsváth-Szabó and Rasmussen invariants of doubled knots*, Algebr. Geom. Topol. 6 (2006) 651--657 MR2240910 MR2240910 (2007d:57023)

16. **K Murasugi**, *On the Arf invariant of links*, Math. Proc. Cambridge Philos. Soc. 95 (1984) 61--69 MR727081 MR0727081 (85j:57007)
17. **J Rasmussen**, *Khovanov homology and the slice genus* arXiv:math.GT/0402131
18. **L Rudolph**, *Quasipositivity as an obstruction to sliceness*, Bull. Amer. Math. Soc. (N.S.) 29 (1993) 51--59 MR1193540 MR1193540 (94d:57028)
19. **L Rudolph**, *An obstruction to sliceness via contact geometry and "classical" gauge theory*, Invent. Math. 119 (1995) 155--163 MR1309974 MR1309974 (95k:57013)
20. **D Sheiham**, *Invariants of boundary link cobordism*, Mem. Amer. Math. Soc. 165 (2003) x+110 MR1997849 MR1997849 (2005f:57038)
21. **A Shumakovitch**, *Rasmussen invariant, Slice-Bennequin inequality, and sliceness of knots* arXiv:math.GT/0411643 MR2384833 (2008m:57034)
22. **P Teichner**, *private communication*

This list reflects references listed in the original paper as accurately as possible with no attempt to correct error.

MR2284050 (2008g:57026)

Ranicki, Andrew(4-EDIN-SM); Sheiham, Desmond
Blanchfield and Seifert algebra in high-dimensional boundary link theory. I. Algebraic K-theory. (English summary)
Geom. Topol. 10 (2006), 1761--1853 (electronic).
57Q45 (19D35 20E05)

Citations
From References: 2
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Let L be a boundary link of m copies of S^n in S^{n+2}. The free cover $\tilde E$ of the exterior E of the link is tesselated by copies of E cut open along a collection V of disjoint Seifert surfaces of the components of L [cf. J. A. Hillman, *Algebraic invariants of links*, World Sci. Publ., River Edge, NJ, 2002; MR1932169 (2003k:57014); N. Sato, Trans. Amer. Math. Soc. **264** (1981), no. 2, 499--505; MR0603777 (82j:57020)]. In particular, the chain complex $C_*(\tilde E)$ is a quotient of ${C_*(S^{n+2}\sbs V)[F_m]}$ with kernel isomorphic to $C_*(V)[F_m]$. Here F_m is the free group on $t_1,\dots,t_m$, and the inclusion of the kernel is given by $(f_1^+-t_1f_1^-,\dots,f_m^+ -t_mf_m^-)$, where $f_i^\pm$ are induced by the two pushoffs of the ith component of V into $S^{n+2}\sbs V$. The authors show that not only $C_*(\tilde E)$, but every chain complex of $A[F_m]$-modules, where A is an associative ring with 1, admits a presentation of this type (Theorem A). The case $m=1$ was proved by F. Waldhausen [in *Algebraic K-theory, II: ``Classical'' algebraic K-theory and connections with arithmetic (Proc. Conf., Battelle Memorial Inst., Seattle, Wash., 1972)*, 155--179, Lecture Notes in Math., 342, Springer, Berlin, (1973); MR0370576 (51 #6803)], and the authors note that the general case can also be deduced from his results.

This ``algebraic transversality'' is used to obtain decompositions of the algebraic K-groups $K_i(A[F_m])$ (Theorem D(i)) and of the negative K-groups of the Cohn localization $\Sigma^{-1}A[F_m]$ (Theorem E). The latter decomposition extends to K_1 and, if $\Sigma^{-1}A[F_m]$ is stably flat, also to the higher K-groups. The stable flatness condition holds when A is a PID and is not known to fail in general. In the case $m=1$ these results specialize to the Bass-Heller-Swan decomposition of $K_1(A[t^{\pm 1}])$ and its analogue for the higher K-groups, and to a splitting theorem of the first author [*High-dimensional knot theory*, Springer, New York, 1998; MR1713074 (2000i:57044)]. The case $A=\Bbb Z$, in which $K_i(\Bbb Z[F_m])$ has been long known to be the direct sum of $K_i(\Bbb Z)$ and m copies of $K_{i-1}(\Bbb Z)$, is discussed in Remark 5.17 at the very end of the paper.

The new decompositions involve the algebraic K-groups of categories of Seifert and Blanchfield modules, which are abstract versions of the Seifert and Blanchfield modules of m-component boundary links. A ``Seifert module'' is a triple consisting of an A-module P, its endomorphism, and a system of idempotents expressing P as an m-fold direct sum. A Blanchfield module (previously called an F_m-link module by the second author [in *Non-commutative localization in algebra and topology*, 143--219, Cambridge Univ. Press, Cambridge, 2006; MR2222485 (2007h:57035)]) is an $A[F_m]$-module M such that the A-module homomorphism $M\oplus\dots\oplus M\to M$ defined by $(a_1,\dots,a_m)\mapsto (t_1-1)a_1+\dots+(t_m-1)a_m$ is an isomorphism. In the present paper, the authors observe that M is Blanchfield if and only if $\roman{Tor}^{A[F_m]}_*(A,M)=0$. The ``algebraic transversality'' is used to establish a long exact sequence relating the algebraic K-groups of the categories of Blanchfield and Seifert modules (Theorem D(ii)), which is new even in the knot case $m=1$.

Reviewed by Sergey A. Melikhov

References

1. **P Ara, W Dicks,** *Universal localizations embedded in power-series rings,* Forum Mathematicum (to appear) cf. MR2313846 (2008e:16023)
2. **P Balmer, M Schlichting,** *Idempotent completion of triangulated categories,* J. Algebra 236 (2001) 819--834 MR1813503 MR1813503 (2002a:18013)
3. **H Bass, A Heller, R G Swan,** *The Whitehead group of a polynomial extension,* Inst. Hautes Études Sci. Publ. Math. (1964) 61--79 MR0174605 MR0174605 (30 #4806)
4. **H Bass,** *Projective modules over free groups are free,* J. Algebra 1 (1964) 367--373 MR0178032 MR0178032 (31 #2290)
5. **H Bass,** *Algebraic* K--*theory,* W A Benjamin (1968) MR0249491 Downloadable from: http://www.math.uni-bielefeld.de/$\sim$rehmann/DML/ MR0249491 (40 #2736)
6. **A J Berrick, M E Keating,** *The* K--*theory of triangular matrix rings,* from: "Applications of algebraic K-theory to algebraic geometry and number theory, Part I, II (Boulder, Colo., 1983)", Contemp. Math. 55, Amer. Math. Soc., Providence, RI (1986) 69--74 MR862629 MR0862629 (88a:18013)
7. **F Borceux,** *Handbook of categorical algebra. 1,* Encyclopedia of Mathematics and its Applications 50, Cambridge University Press, Cambridge (1994) MR1291599 MR1291599 (96g:18001a)
8. **S E Cappell, J L Shaneson,** *The codimension two placement problem and homology equivalent manifolds,* Ann. of Math. (2) 99 (1974) 277--348 MR0339216 MR0339216 (49 #3978)
9. **S E Cappell, J L Shaneson,** *Link cobordism,* Comment. Math. Helv. 55 (1980) 20--49 MR569244 MR0569244 (81j:57011)
10. **P M Cohn,** *Free rings and their relations,* Academic Press (1971) MR0371938 MR0371938 (51 #8155)
11. **W Dicks, E D Sontag,** *Sylvester domains,* J. Pure Appl. Algebra 13 (1978) 243--275 MR509164 MR0509164 (80j:16014)
12. **J Duval,** *Forme de Blanchfield et cobordisme d'entrelacs bords,* Comment. Math. Helv. 61 (1986) 617--635 MR870709 MR0870709 (88a:57037)

13. **M Farber,** *Hermitian forms on link modules,* Comment. Math. Helv. 66 (1991) 189--236 MR1107839 MR1107839 (92j:57014)
14. **M Farber, P Vogel,** *The Cohn localization of the free group ring,* Math. Proc. Cambridge Philos. Soc. 111 (1992) 433--443 MR1151322 MR1151322 (93b:16049)
15. **M Farber,** *Noncommutative rational functions and boundary links,* Math. Ann. 293 (1992) 543--568 MR1170526 MR1170526 (94m:68137)
16. **M Farber,** *Stable-homotopy and homology invariants of boundary links,* Trans. Amer. Math. Soc. 334 (1992) 455--477 MR1079054 MR1079054 (93a:57023)
17. **P Gabriel, M Zisman,** *Calculus of fractions and homotopy theory,* Ergebnisse series 35, Springer, New York (1967) MR0210125 MR0210125 (35 #1019)
18. **P Gabriel,** *Des catégories abéliennes,* Bull. Soc. Math. France 90 (1962) 323--448 MR0232821 MR0232821 (38 #1144)
19. **S M Gersten,** *On class groups of free products,* Ann. of Math. (2) 87 (1968) 392--398 MR0224655 MR0224655 (37 #254)
20. **H Gillet,** *Riemann--Roch theorems for higher algebraic* K--*theory,* Adv. in Math. 40 (1981) 203--289 MR624666 MR0624666 (83m:14013)
21. **D R Grayson,** *Exact sequences in algebraic* K--*theory,* Illinois J. Math. 31 (1987) 598--617 MR909785 MR0909785 (89c:18011)
22. **M A Gutiérrez,** *Boundary links and an unlinking theorem,* Trans. Amer. Math. Soc. 171 (1972) 491--499 MR0310902 MR0310902 (46 #10000)
23. **A Haghany, K Varadarajan,** *Study of modules over formal triangular matrix rings,* J. Pure Appl. Algebra 147 (2000) 41--58 MR1744654 MR1744654 (2000k:16004)
24. **B Keller,** *Derived categories and their uses,* from: "Handbook of algebra, Vol. 1", North-Holland, Amsterdam (1996) 671--701 MR1421815 MR1421815 (98h:18013)
25. **J Levine,** *An algebraic classification of some knots of codimension two,* Comment. Math. Helv. 45 (1970) 185--198 MR0266226 MR0266226 (42 #1133)
26. **J Levine,** *Knot modules I,* Trans. Amer. Math. Soc. 229 (1977) 1--50 MR0461518 MR0461518 (57 #1503)
27. **C C Liang,** *An algebraic classification of some links of codimension two,* Proc. Amer. Math. Soc. 67 (1977) 147--151 MR0458439 MR0458439 (56 #16642)
28. **W Lück, A Ranicki,** *Chain homotopy projections,* J. Algebra 120 (1989) 361--391 MR989903 MR0989903 (90i:57013)
29. **J W Milnor,** *Infinite cyclic coverings,* from: "Conference on the Topology of Manifolds (Michigan State Univ., E. Lansing, 1967)", Prindle, Weber & Schmidt, Boston (1968) 115--133 MR0242163 MR0242163 (39 #3497)
30. **A Neeman, A Ranicki,** *Noncommutative localisation and chain complexes I: Algebraic* K--*and* K--*theory* arXiv:math.RA/0109118
31. **A Neeman, A Ranicki,** *Noncommutative localisation in algebraic* L--*theory I,* Geom. Topol. 8 (2004) 1385--1425 MR2119300 MR2119300 (2005k:19006)
32. **D Quillen,** *Higher algebraic* K--*theory I,* from: "Algebraic *K*---theory I: Higher *K*---theories (Proc. Conf., Battelle Memorial Inst., Seattle, Wash., 1972)", Springer, Berlin (1973) 85--147. Lecture Notes in Math., Vol. 341 MR0338129 MR0338129 (49 #2895)
33. **A Ranicki,** *High-dimensional knot theory,* Springer Monographs in Mathematics, Springer, New York (1998) MR1713074 MR1713074 (2000i:57044)
34. **A Ranicki,** *Blanchfield and Seifert algebra in high-dimensional knot theory,* Mosc. Math. J. 3 (2003) 1333--1367 MR2058802 MR2058802 (2005b:19008)
35. **A Ranicki,** *Algebraic and combinatorial codimension--1 transversality,* from:

"Proceedings of the Casson Fest", (C Gordon, R Y, editors), Geom. Topol. Monogr. 7 (2004) 145--180 MR2172482 MR2172482 (2006j:57064)

36. **A Ranicki,** *Noncommutative localization in topology,* from: "Non-commutative localization in algebra and topology", London Math. Soc. Lecture Note Ser. 330, Cambridge Univ. Press, Cambridge (2006) 81--102 MR2222483 MR2222483 (2007f:57064)
37. **N Sato,** *Free coverings and modules of boundary links,* Trans. Amer. Math. Soc. 264 (1981) 499--505 MR603777 MR0603777 (82j:57020)
38. **M Schlichting,** *Negative* K--*theory of derived categories,* Math. Z. 253 (2006) 97--134 MR2206639 MR2206639 (2006i:19003)
39. **A H Schofield,** *Representation of rings over skew fields,* London Mathematical Society Lecture Note Series 92, Cambridge University Press, Cambridge (1985) MR800853 MR0800853 (87c:16001)
40. **D Sheiham,** *Whitehead groups of localizations and the endomorphism class group,* J. Algebra 270 (2003) 261--280 MR2016661 MR2016661 (2004i:16014)
41. **D Sheiham,** *Invariants of boundary link cobordism II: The Blanchfield--Duval form,* from: "Non-commutative localization in algebra and topology (Edinburgh 2002)", LMS Lecture Note Ser. 330, Cambridge Univ. Press, Cambridge (2006) 143--219 MR2222485 MR2222485 (2007h:57035)
42. **D Sheiham,** *Universal localization of triangular matrix rings,* Proc. Amer. Math. Soc 134 (2006) 3465--3474 MR2240657 (2007d:16052)
43. **N Smythe,** *Boundary links,* from: "Wisconsin Topology Seminar", Ann. of Maths. Studies 60, Princeton UP (1966) 69--72
44. **J Stallings,** *Whitehead torsion of free products,* Ann. of Math. (2) 82 (1965) 354--363 MR0179270 MR0179270 (31 #3518)
45. **R W Thomason, T Trobaugh,** *Higher algebraic* K--*theory of schemes and of derived categories,* from: "The Grothendieck Festschrift, Vol. III", Progr. Math. 88, Birkhäuser, Boston, MA (1990) 247--435 MR1106918 MR1106918 (92f:19001)
46. **H F Trotter,** *Knot modules and Seifert matrices,* from: "Knot theory (Proc. Sem., Plans-sur-Bex, 1977)", Lecture Notes in Math. 685, Springer, Berlin (1978) 291--299 MR521739 MR0521739 (81c:57019)
47. **F Waldhausen,** *Whitehead groups of generalized free products,* from: "Algebraic *K*---theory II: "Classical" algebraic *K*---theory and connections with arithmetic (Proc. Conf., Battelle Memorial Inst., Seattle, Wash., 1972)", Springer, Berlin (1973) 155--179. Lecture Notes in Math., Vol. 342 MR0370576 MR0370576 (51 #6803)
48. **F Waldhausen,** *Algebraic* K--*theory of generalized free products, Part 1,* Ann. of Math. (2) 108 (1978) 135--204 MR0498807 MR0498807 (58 #16845a)
49. **F Waldhausen,** *Algebraic* K--*theory of generalized free products, Part* 2, Ann. of Math. (2) 108 (1978) 205--256 MR0498808 MR0498808 (58 #16845b)
50. **F Waldhausen,** *Algebraic* K--*theory of spaces,* from: "Algebraic and geometric topology (New Brunswick, N.J., 1983)", Lecture Notes in Math. 1126, Springer, Berlin (1985) 318--419 MR802796 MR0802796 (86m:18011)

This list reflects references listed in the original paper as accurately as possible with no attempt to correct error.

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Sheiham, Desmond

Universal localization of triangular matrix rings. (English summary)

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16S10 (16S50)

Citations
From References: 2
From Reviews: 0

Let R be a triangular matrix ring $\left(\smallmatrix A&M\\0&B\endsmallmatrix \right)$ where A and B are associative rings with 1 and M is an (A,B)-bimodule. Let P and Q be the finitely generated projective modules $\left(\smallmatrix A\\ 0\endsmallmatrix\right)$ and $\left(\smallmatrix M\\ B \endsmallmatrix\right)$, respectively. Given a morphism $\sigma\colon P \rightarrow Q$, this paper gives a description of the ``universal localization'' $R \rightarrow \sigma^{-1}R$ which is universal with respect to the property that $1\times\sigma\colon\sigma^{-1}R \otimes_RP\rightarrow\sigma^{-1}R \otimes_RQ$ is an isomorphism. A construction of the ``universal localization'' of an R-module is also given.

Reviewed by Mary H. Wright

References

1. P. Ara, M. A. Gonz'alez-Barroso, K. R. Goodearl, and E. Pardo. Fractional skew monoid rings. *Journal of Algebra*, 278(1):104--126, 2004. MR2068068 (2005f:16042)
2. D. J. Benson. *Representations and cohomology. I. Basic Representation Theory of finite groups and associative algebras*. Cambridge Studies in Advanced Mathematics, 30. Cambridge University Press, 1995. MR1110581 (92m:20005)
3. G. M. Bergman. Modules over coproducts of rings. *Transactions of the American Mathemat- ical Society*, 200:1--32, 1974. MR0357502 (50 #9970)
4. G. M. Bergman. Coproducts and some universal ring constructions. *Transactions of the American Mathematical Society*, 200:33--88, 1974. MR0357503 (50 #9971)
5. G. M. Bergman and W. Dicks. Universal derivations and universal ring constructions. *Pacific Journal of Mathematics*, 79(2):293--337, 1978. MR0531320 (81b:16024)
6. P. M. Cohn. Localization in general rings, a historical survey. Proceedings of the Conference on Noncommutative Localization in Algebra and Topology, ICMS, Edinburgh, 29-30 April, 2002, *London Mathematical Society Lecture Notes* 330, Cambridge University Press, 5--23, 2006. MR2222479 (2007f:16073)
7. P. M. Cohn. *Free Rings and their Relations*. London Mathematical Society Monographs, 2. Academic Press, London, 1971. MR0371938 (51 #8155)
8. P. M. Cohn. Rings of fractions. *American Mathematical Monthly*, 78:596--615, 1971. MR0285561 (44 #2779)
9. P. M. Cohn. *Free Rings and their Relations*. London Mathematical Society Monographs, 19. Academic Press, London, second edition, 1985. MR0800091 (87e:16006)
10. P.M. Cohn and W. Dicks. Localization in semifirs. II. *J. London Math.Soc. (2)*, 13(3):411-- 418, 1976. MR0424856 (54 #12814)
11. W. Dicks and E. Sontag. Sylvester domains. *J. Pure Appl. Algebra*, 13(3):243--275, 1978. MR0509164 (80j:16014)
12. M. Farber and P. Vogel. The Cohn localization of the free group ring. *Mathematical Proceed- ings of the Cambridge Philosophical Society*, 111(3):433--443, 1992. MR1151322 (93b:16049)
13. A. Haghany and K. Varadara jan. Study of formal triangular matrix rings. *Communications in Algebra*, 27(11):5507--5525, 1999. MR1713049 (2000g:16003)
14. A. Haghany and K. Varadara jan. Study of modules over formal triangular matrix rings. *Journal of Pure and Applied Algebra*, 147(1):41--58, 2000. MR1744654 (2000k:16004)

15. T. Y. Lam. *Lectures on Modules and Rings*. Number 189 in Graduate Texts in Mathematics. Springer, New York, 1999. MR1653294 (99i:16001)
16. A. A. Ranicki. Noncommutative localization in topology. Proceedings of the Conference on Noncommutative Localization in Algebra and Topology, ICMS, Edinburgh, 29-30 April, 2002. arXiv:math.AT/0303046, *London Mathematical Society Lecture Notes* 330, Cambridge Uni- versity Press, 81--102, 2006. MR2222483 (2007f:57064)
17. A. H. Schofield. *Representations of rings over skew fields*, Volume 92 of *London Mathematical Society Lecture Note Series*. Cambridge University Press, 1985. MR0800853 (87c:16001)

This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2222649 (2007f:55002)

Non-commutative localization in algebra and topology. Proceedings of the workshop held in Edinburgh, April 29--30, 2002. Edited by Andrew Ranicki. London Mathematical Society Lecture Note Series, 330. *Cambridge University Press, Cambridge,* 2006. xiv+313 pp. ISBN: 978-0-521-68160-5; 0-521-68160-X
55-06 (16-06 55P60)

Citations
From References: 0
From Reviews: 0

Contents: Andrew Ranicki, Dedicated to the memory of Desmond Sheiham (13th November 1974--25th March 2005) (vii--viii) MR2222477; Andrew Ranicki, Preface (ix) MR2222650; John A. Beachy, On flatness and the Ore condition (1--4) MR2222478; P. M. Cohn, Localization in general rings, a historical survey (5--23) MR2222479; William G. Dwyer, Noncommutative localization in homotopy theory (24--39) MR2222480; Peter A. Linnell, Noncommutative localization in group rings (40--59) MR2222481; Amnon Neeman, A non-commutative generalisation of Thomason's localisation theorem (60--80) MR2222482; Andrew Ranicki, Noncommutative localization in topology (81--102) MR2222483; Holger Reich, L^2-Betti numbers, isomorphism conjectures and noncommutative localization (103--142) MR2222484; Desmond Sheiham, Invariants of boundary link cobordism. II. The Blanchfield-Duval form (143--219) MR2222485; Zoran Škoda, Noncommutative localization in noncommutative geometry (220--313) MR2222486.

{Most of the papers are being reviewed individually.}

MR2222485 (2007h:57035)

Sheiham, Desmond

Invariants of boundary link cobordism. II. The Blanchfield-Duval form. (English summary)
Non-commutative localization in algebra and topology, 143--219, London Math. Soc. Lecture Note Ser., 330, *Cambridge Univ. Press, Cambridge,* 2006.
57Q45 (19G12)

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The Blanchfield-Duval form is used to define a complete set of invariants of boundary links ${S^n\times\{1,\dots,m\}}\hookrightarrow S^{n+2}$, $n\ge 2$, endowed with a choice of disjoint Seifert surfaces, up to concordance extending to disjoint cobordisms of the Seifert surfaces. Here $n=2q-1$ since in the even-dimensional case this equivalence relation is well known to have just one equivalence class.

In Part I [Mem. Amer. Math. Soc. **165** (2003), no. 784, x+110 pp.; MR1997849

(2005f:57038)], the author already defined such a complete set of invariants in terms of Seifert forms, which is ``convenient in explicit computations''. The objective of the present Part II is to redefine these invariants in ``more intrinsic'' terms, which are also ``more amenable to generalization from boundary links to arbitrary links''.

The choice of the cobordism class of Seifert surfaces is equivalent, by the Pontryagin construction, to a choice of a homomorphism $h\colon \pi_1(X)\to F_m$ sending meridians to generators, where X is the link exterior and F_m is the free group on m generators. The Poincaré duality in the cover X_ω of X corresponding to the kernel $\gamma_\omega\pi_1(X)$ of h gives rise to a version of the Blanchfield pairing.

The Blanchfield pairing is best understood and easiest to describe in the case of the free abelian cover $\tilde X$ [cf. J. A. Hillman, *Algebraic invariants of links*, World Sci. Publ., River Edge, NJ, 2002; MR1932169 (2003k:57014)]. Given two q-cycles x, y in $\tilde X$ such that $\alpha x=\partial u$ and $\beta y=\partial v$ for some nonzero $\alpha,\beta\in\Bbb Z[\Bbb Z^m]=\Bbb Z[t_1,\dots,t_m]$ and some $(q+1)$-chains u, v, the Blanchfield pairing $B(x,y)=\alpha^{-1} \sum_{g\in\Bbb Z^m} (u\cdot gy)g$ is a rational function in $t_1,\dots,t_m$, well-defined up to addition of finite polynomials.

The Blanchfield-Duval pairing on $H_q(X_\omega)$ is a noncommutative analogue of this construction. The polynomial ring $\Bbb Z[\Bbb Z^m]$ is replaced by $\Bbb Z[F_m]$, and its field of fractions $\Bbb Q(t_1,\dots,t_m)$ by the Cohn localization of $\Bbb Z[F_m]$, defined by formally adjoining inverses to all square matrices with entries from $\Bbb Z[F_m]$ that become invertible under the augmentation $\Bbb Z[F_m]\to\Bbb Z$. It was shown by Farber and Vogel [cf. M. S. Farber, Math. Ann. **293** (1992), no. 3, 543--568; MR1170526 (94m:68137)] that the Cohn localization of $\Bbb Q[F_m]$ can be identified with the subring of $\Bbb Q\langle\langle t_1,\dots,t_m\rangle\rangle$ consisting of those noncommutative power series that have the form $\sum f(T_{i_r}\dots T_{i_1} v) t_{i_1}\dots t_{i_r}$ for some vector $v\in V$, linear function $f\in V^*$ and transformations $T_1,\dots,T_m\in V\otimes V^*$ on a finite-dimensional $\Bbb Q$-vector space V.

In Section 2, the author discusses Blanchfield-Duval forms in more detail and defines their Witt groups. He works out complete invariants of these groups in Section 3. Sections 4--6 identify these invariants with those constructed in Part I.

Seifert forms have already been related to Blanchfield-Duval forms in the present context by Farber [op. cit.] and by T. D. Cochran and K. E. Orr [Topology **33** (1994), no. 3, 397--427; MR1286923 (95f:57041)]; see also [S. Garoufalidis and J. P. Levine, J. Knot Theory Ramifications **11** (2002), no. 3, 283--293; MR1905685 (2003f:57012)]. In the author's own words: ``Although the present paper is logically independent of [Farber's] work, we take up a number of his ideas in Sections 4 and 5, providing a systematic treatment in the language of Hermitian categories. Whereas Farber takes coefficients in a field or in $\Bbb Z$, in these sections we allow the coefficients to lie in an arbitrary associative ring A.''

Let us note that the relation of the Blanchfield-Duval form with the Seifert form is parallel to the connection between the Alexander polynomial and the Conway polynomial (as defined by Kauffman in terms of the Seifert form, see [D. Cimasoni, Comment. Math. Helv. **79** (2004), no. 1, 124--146; MR2031702 (2005d:57005)] along with the reviewer's preprint [``Colored finite type invariants and a multi-variable analogue of the Conway polynomial'', preprint, \url{arxiv.org/abs/math.GT /0312007}] for the multi-variable case), as well as to the connection [T. D. Cochran, Comment. Math. Helv. **60** (1985), no. 2, 291--311; MR0800009 (87f:57021)] between the Kojima-Yamasaki η-function and Cochran's derived invariants (as defined in terms of a Seifert form [see P. M. Gilmer and C. Livingston, Proc. Edinburgh Math. Soc. (2) **34** (1991), no. 3, 455--462; MR1131964 (93a:57024)]).

It is terribly sad to learn that Desmond Sheiham passed away, only a few years after completing his PhD---and 50 years after Richard Blanchfield suffered a similar fate. The investigations of the present paper are continued in [A. A. Ranicki and D. Sheiham, Geom. Topol. **10** (2006), 1761--1853 (electronic); MR2284050].

{For the entire collection see MR2222649 (2007f:55002).}

Reviewed by Sergey A. Melikhov

MR2222477

Ranicki, Andrew

Dedicated to the memory of Desmond Sheiham (13th November 1974--25th March 2005). *Non-commutative localization in algebra and topology,* vii--viii, London Math. Soc. Lecture Note Ser., 330, *Cambridge Univ. Press, Cambridge,* 2006.

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Citations
From References: 0
From Reviews: 1

{This item will not be reviewed individually.}

{For the entire collection see MR2222649.}

MR2016661 (2004i:16014)

Sheiham, Desmond(1-CAR)

Whitehead groups of localizations and the endomorphism class group. (English summary)

J. Algebra 270 (2003), no. 1, 261--280.

16E20 (19B99)

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From References: 5
From Reviews: 0

Let A and B be associative rings. A ring homomorphism $\epsilon\colon B\to A$ is said to be local in case a square matrix α with entries in B is invertible if $\epsilon (\alpha)$ is invertible. The homomorphism ϵ is said to be an augmentation if there is a ring homomorphism $j\colon A\to B$ such that $\epsilon j=1_A$. Every augmentation $\epsilon\colon B\to A$ induces a local augmentation $\epsilon_{\Sigma}\colon \Sigma ^{-1}B\to A $, where Σ is the set of all square matrices α over B such that $\epsilon (\alpha)$ is invertible, and $\Sigma ^{-1}B$ is the universal localization of B with respect to Σ.

The main result of the paper is as follows. Let $\epsilon\colon B\to A$ be a local augmentation. Then $K_1(B)\cong K_1(A)\oplus \epsilon^{-1}(1)/C$, where C is the subgroup of $\epsilon^{-1}(1)$ generated by the elements of the form $(1+ab)(1+ba)^{-1}$, where $a,b\in B$ and $\epsilon (ab)=\epsilon (ba)=0$. This result is applied to the computation of the Whitehead groups of (twisted) formal power series rings and the augmentation localizations of group rings and polynomial rings. Using a result of A. Ranicki [*High-dimensional knot theory*, Springer, New York, 1998; MR1713074 (2000i:57044) (Proposition 10.21)], the author obtains from his main result an isomorphism $\widetilde{\text{End}}_0(A)\cong \epsilon_{\Sigma}^{-1}(1)/C$, where $\epsilon _{\Sigma}\colon \Sigma ^{-1}A[x]\to A$ is the local augmentation induced by the augmentation $\epsilon \colon A[x]\to A$, $x\mapsto 0$, for any associative ring A.

Reviewed by Pere Ara

References

1. G. Almkvist, Endomorphisms of finitely generated projective modules over a commutative ring, Ark. Mat. 11 (1973) 263--301. MR0424786 (54 #12744)
2. G. Almkvist, The Grothendieck ring of the category of endomorphisms, J. Algebra 28 (1974) 375--388. MR0432738 (55 #5721)
3. G. Almkvist, K-theory of endomorphisms, J. Algebra 55 (2) (1978) 308--340. MR0523461 (80i:18018)
4. P. Ara, Finitely presented modules over Leavitt algebras, Preprint. cf. MR2048304 (2005b:16012)
5. P. Ara, K.R. Goodearl, E. Pardo, K_0 of purely infinite simple regular rings, *K*-Theory 26 (2002) 69--100. MR1918211 (2004e:19001)
6. H. Bass, Algebraic K-theory, Benjamin, New York, 1968. MR0249491 (40 #2736)
7. R. Bott, Morse theory indomitable, Inst. Hautes Études Sci. Publ. Math. 68 (1988) 99--114. MR1001450 (90f:58027)
8. P.M. Cohn, Free Rings and their Relations, in: London Math. Soc. Monogr., Vol. 2, Academic Press, London, 1971. MR0371938 (51 #8155)
9. P.M. Cohn, Free Rings and their Relations, 2nd edition, in: London Math. Soc. Monogr., Vol. 19, Academic Press, London, 1985. MR0800091 (87e:16006)
10. J. Dieudonné, Les déterminants sur un corps non commutatif, Bull. Soc. Math. Fr. 71 (1943) 27--45. MR0012273 (7,3a)
11. P.K. Draxl, Skew Fields, in: London Math. Soc. Lecture Note Ser., Vol. 81, Cambridge Univ. Press, Cambridge, 1983. MR0696937 (85a:16022)
12. D.R. Grayson, The ${\rm K}$-theory of endomorphisms, J. Algebra 48 (1977) 439--446. MR0480699 (58 #853)
13. R. Hartshorne, Algebraic Geometry, in: Grad. Texts Math., Vol. 52, Springer-Verlag, New York, 1977. MR0463157 (57 #3116)
14. W. Klingenberg, Die Struktur der linearen Gruppe über einem nichtkommutativen lokalen Ring, Arch. Math. 13 (1962) 73--81. MR0143817 (26 #1367)
15. T.Y. Lam, A First Course in Noncommutative Rings, 2nd edition, Springer-Verlag, New York, 2001. MR1838439 (2002c:16001)
16. P. Menal, J. Moncasi, K_1 of von Neumann regular rings, J. Pure Appl. Algebra (1984). MR0761635 (86i:18014)
17. J. Milnor, Morse Theory, in: Ann. Math. Stud., Vol. 51, Princeton Univ. Press, Princeton, NJ, 1963. MR1747528 (2001c:57002)
18. J. Milnor, Lectures on the h-Cobordism Theorem, Princeton Univ. Press, Princeton, NJ, 1965, Notes by L. Siebenmann and J. Sondow.
19. A. Neeman, A.A. Ranicki, Noncommutative localization and chain complexes I. Algebraic *K* - and *L*-theory, math.RA/0109118.
20. S.P. Novikov, Multivalued functions and functionals. An analogue of Morse theory, Dokl. Akad. Nauk SSSR 260 (1) (1981) 31--35; English translation: Soviet Math. Dokl. 24 (2) (1981) 222--226. MR0630459 (83a:58025)
21. S.P. Novikov, The Hamiltonian formalism and a multivalued analogue of Morse theory, Uspekhi Mat. Nauk 37 (5) (1982) 3--49; English translation: Russ.

Math. Surv. 37 (1982) 1--56. MR0676612 (84h:58032)
22. A. Pajitnov, A.A. Ranicki, The Whitehead group of the Novikov ring, *K*-theory 21 (4) (2000) 325--365, math.AT/0012031. MR1828181 (2002g:19001)
23. A.V. Pajitnov, On the Novikov complex for rational Morse forms, Ann. Fac. Sci. Toulouse IV Sér. Math. (2) (1995) 297--338. MR1344724 (97f:57033)
24. A.V. Pajitnov, Simple homotopy type of the Novikov complex, and the Lefschetz ζ-function of the gradient flow, Uspekhi Mat. Nauk 54 (1) (1999) 117--170; Translation: Russ. Math. Surv. 54 (1) (1999) 119--169, math.dg-ga/9706014. MR1706835 (2001m:57041)
25. A.V. Pajitnov, Closed orbits of gradient flows and logarithms of non-abelian Witt vectors, *K*-Theory 21 (4) (2000) 301--324, math.DG/99808010. MR1828180 (2002e:57048)
26. A.A. Ranicki, High-dimensional Knot Theory, Springer-Verlag, Berlin, 1998. MR1713074 (2000i:57044)
27. A.A. Ranicki, Circle valued Morse theory and Novikov homology, in: F.T. Farrell, L. Göttsche, W. Lück (Eds.), Topology of High-Dimensional Manifolds, Proceedings of the Summer School on High-Dimensional Manifold Topology, Trieste, 21 May---8 June 2001, in: Abdus Salam ICTP, Lecture Notes, Vol. 2, 2002, math.AT/0111317. MR1937024 (2003m:57069)
28. G. Révész, On the abelianized multiplicative group of a universal field of fractions, J. Pure Appl. Algebra 27 (3) (1983) 277--297. MR0688759 (84f:16004)
29. J. Rosenberg, Algebraic K-theory and its applications, in: Grad. Texts Math., Vol. 147, Springer-Verlag, New York, 1994. MR1282290 (95e:19001)
30. A.H. Schofield, Representations of Rings over Skew Fields, in: London Math. Soc. Lecture Note Ser., Vol. 92, Cambridge Univ. Press, Cambridge, 1985. MR0800853 (87c:16001)
31. D. Sheiham, Non-commutative characteristic polynomials and Cohn localization, J. London Math. Society (2) 64 (1) (2001) 13--28, math.RA/0104158. MR1840768 (2002g:16013)
32. J.R. Silvester, Introduction to Algebraic K-Theory, Chapman & Hall, London, 1981. MR0629979 (83f:18013)
33. V. Srinivas, Algebraic K-theory, in: Progr. Math. Vol. 90, Birkhäuser, Boston, 1996.

This list reflects references listed in the original paper as accurately as possible with no attempt to correct error.

MR1997849 (2005f:57038)
Sheiham, Desmond
Invariants of boundary link cobordism. (English summary)
Mem. Amer. Math. Soc. 165 (2003), no. 784, x+110 pp.
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Citations
From References: 4
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An algorithm to decide whether two given n-knots $S^n\subset S^{n+2}$, $n>1$, are concordant was given by J. Levine [Invent. Math. **8** (1969), 98--110; addendum, ibid., 355; MR0253348 (40 #6563)]. The very basic ingredient was the existence of a Seifert surface, i.e., a submanifold $V^{n+1}\subset S^{n+2}$ with $\partial V=S^n$. (Here all submanifolds can be assumed to be PL locally flat; originally they were smooth, but with possibly exotic smooth structures on S^n and $S^n\times I$.) In an attempt to generalize the Seifert surface approach to the m-component case, $m>1$, one is often content to obtain results for boundary n-links, whose components bound disjoint Seifert surfaces. Still, the generalization is usually not easy, amounting to a transition from $\bold Z$ to the nonabelian free group F_m. Indeed, collections of disjoint Seifert surfaces for an n-link $\coprod S^n\subset S^{n+2}$ up to disjoint cobordisms rel ∂ correspond via the Pontryagin construction to epimorphisms $\pi_1(S^{n+2}\coprod S^n)\to F_m$, taking meridians onto conjugates of the elements of some fixed basis $\{x_1, \dots,x_m\}$.

The main result of this memoir is an algorithm to decide whether two given boundary n-links, $n>1$, with given collections of disjoint Seifert surfaces can be joined by a concordance extending to disjoint cobordisms of the Seifert surfaces. The author's approach is based on the Seifert form and extends M. A. Kervaire's treatment of the one-component case [in *Manifolds--Amsterdam 1970 (Proc. Nuffic Summer School)*, 83--105, Lecture Notes in Math., 197, Springer, Berlin, 1971; MR0283786 (44 #1016)], which may be a valuable complement to the memoir for the geometrically-minded reader. In his recent preprint [D. Sheiham, ``Invariants of boundary link cobordism. II. The Blanchfield-Duval form'', preprint, arxiv.org/abs/math.AT/0404229] the author gives an alternative classification in terms of the Blanchfield-Duval form. The difference between the two forms is akin to that between the Conway and Alexander polynomials in classical knot theory.

For $n>1$ the equivalence classes of pairs (m-component boundary n-link, collection of disjoint Seifert surfaces) up to concordance, extending to disjoint cobordisms between the Seifert surfaces, form an abelian group C^n_m (addition is given by connected sum of representatives along any collection of meridians mapping onto $\{x_1,\dots,x_m\}$, and inverse by reflection of both domain and range). S. Cappell and J. Shaneson showed that $C^{2q}_m=0$ for all q and m, and Levine [op. cit.] proved that $$C^{2q-1}_1\simeq\bigoplus_{i=1}^\infty(\bold Z\oplus\bold Z/2\oplus\bold Z/4)$$ for each $q>1$. As a corollary to his algorithmic classification, Sheiham establishes that $$C^{2q-1}_m\simeq \bigoplus_{i=1}^\infty (\bold Z\oplus\bold Z/2\oplus\bold Z/4\oplus\bold Z/8)$$ for each $q>1$ and $m>1$.

His starting point is a description of C^{2q-1}_m due to K. Ko and W. Mio. They showed that C^{2q-1}_m is isomorphic for each $q>2$ to the Witt group of bilinear forms λ on finitely generated free abelian groups $M=\bigoplus_{i=1}^m M_i$ such that $\phi\coloneq \lambda+(-1)^q\lambda^t$ is non-singular and satisfies $\phi(M_i,M_j)=0$ for $i\ne j$. In particular, $C^n_m\simeq C^{n+4}_m$ for $n\ne 1,3$; as for C^3_m, it is an index 2^m subgroup in C^7_m. Geometrically, ϕ is the intersection form on the union of the Seifert surfaces, and the Seifert form $\lambda(x,y)$ is the linking number of x with the $+$-pushoff of y.

Ordinary Witt groups are defined for forms with some kind of symmetry, yet λ need not be symmetric. But there is some additional symmetry in ϕ: if M is regarded as a $\bold Z[s]$-module, with $s\colon M\to M$ acting by $\phi(sx,\cdot)=\lambda(x,\cdot)$, ϕ becomes $(-1)^q$-Hermitian (i.e., $\phi(x,y)=(-1)^q\overline{\phi(y,x)}$\,) with respect to the involution $\overline s=1-s$ on $\bold Z[s]$. To take into account the block structure of M for $m>1$, the idempotents $\pi_i\colon M\to M_i\to M$ are assembled formally with s into the noncommutative ring $$P_m\coloneq\bold Z\langle s,\pi_1,\dots,\pi_m\rangle/(\pi_i^2-\pi_i,\pi_i\pi_j,\pi_1+\dots+\pi_m-1).$$ Thus C^{2q-1}_m, $q>2$, becomes the Witt group $W^{(-1)^q}(P_m\text{-}\bold Z)$ of $(-1)^q$-Hermitian forms on modules over P_m, which are finitely generated projective over $\bold Z$. The ring P_m was introduced by M. S. Farber [Math. Ann. **293** (1992), no. 3, 543--568; MR1170526 (94m:68137)], and turns out to coincide with the path ring of the complete quiver on m vertices (with m^2 arrows). This interpretation is used to show that there are only finitely many finite-dimensional simple representations of P_m over $\bold Q$.

Tensoring by $\bold Q$ leads to an injection $$W^\pm(P_m\text{-}\bold Z)\to W^\pm(Q_m\text{-}\bold Q),$$ where Q_m denotes $P_m\otimes\bold Q$. The Hermitian Jordan-Hölder theorem identifies the Witt class of $(M\otimes\bold Q,\phi)$ with the direct sum of the Witt classes of its simple subfactors (N_i,ϕ_i). Each N_i is simple as a Q_m-module, so each ϕ_i is either non-singular or metabolic. Hence the Hermitian Morita equivalence yields an isomorphism $$W^\pm(Q_m\text{-}\bold Q)\simeq\bigoplus_T W^+({\rm End}_{Q_m}T),$$ where T runs over all isomorphism classes of simple Q_m-modules, finite-dimensional over $\bold Q$ and admitting a non-singular $\pm$-Hermitian form θ; the involution on ${\rm End}_{Q_m}T$ is defined by $\theta(\overline f(x),\cdot)= \theta(x,f(\cdot))$ for some choice of θ. (This isomorphism associates to the Witt class of each (N_i,ϕ_i) such that $N_i\simeq T$ the Witt class of $({\rm Hom}_{Q_m}(T,N_i),\phi^\theta_i)$, where ϕ^θ_i is defined by $\theta(\phi^\theta_i(\alpha,\beta)(x), \cdot)= \pm\phi_i(\beta(x),\alpha(\cdot))$.) By the Schur lemma, each $E\coloneq{\rm End}_{Q_m}T$ is a division ring.

In the case of knots T runs over $\bold Q[s]$-modules $\bold Q[s]/(p)$ where p is an irreducible polynomial such that $p(s)=\pm p(1-s)$, and E is again $\bold Q[s]/(p)$ with $\overline s=1-s$.

Algorithmic Witt classification of Hermitian forms over the finite-dimensional division $\bold Q$-algebra E is known. For each of a certain five classes of such algebras, a complete list of Witt invariants is given by some combination of: the dimension modulo 2, the signatures (i.e., at most $[E\colon\bold Q]$ integers), the discriminant, the Hasse--Witt invariant and the Lewis θ-invariant, which is only defined when all the other invariants vanish. In contrast to the other invariants, the θ-invariant is non-local, i.e., does not factor through tensoring by $\bold Q_p$'s. It is only needed for the class of quaternionic algebras with the ``non-standard'' involution $\overline i=-i$, $\overline j=j$. Being noncommutative, they do not occur as $\bold Q[s]/(p)$; nor do commutative algebras with trivial involution, which are the only class requiring the Hasse--Witt invariant. (To be precise, $\bold Q[s]/(s-\frac12)$ does occur among them, but is not induced from any $\bold Z[s]$-module.)

In contrast, the author shows that for each $m>1$ and each sign ($+$ or $-$), every finite-dimensional division $\bold Q$-algebra with involution occurs as ${\rm End}_{Q_m}(T)$ for infinitely many T's as above, induced from P_m-modules. Starting from $\bold Q(\sqrt{-7})$ with the trivial involution, he exhibits explicit 8×8 Seifert matrices of 2-component links which have order 8 in C^5_2 and C^7_2.

Reviewed by Sergey A. Melikhov

References

1. A. A. Albert, *Structure of Algebras*, Amer. Math. Soc. Colloq. Publ., vol. 24, American Mathematical Society, New York, 1939.

2. M. Auslander, I. Reiten, and S. O. Smalø, *Representation Theory of Artin Algebras*, Cambridge Stud. Adv. Math., 36, Cambridge University Press, 1995. MR1314422 (96c:16015)
3. H.-J. Bartels, *Invarianten hermitescher Formen über Schiefkörpern*, Math. Ann. **215** (1975), 269--288. MR0419353 (54 #7374)
4. H.-J. Bartels, *Zur Klassifikation Schiefhermitescher Formen über Zahlkörpern*, Math. Ann. **219** (1976), no. 1, 13--19. MR0387190 (52 #8035)
5. H. Bass, *Algebraic* K-*theory*, W. A. Benjamin Inc, New York-Amsterdam, 1968. MR0249491 (40 #2736)
6. D. J. Benson, *Representations and cohomology.I. Basic representation theory of finite groups and associative algebras*, Cambridge Stud. Adv. Math., 30, Cambridge University Press, 1995. MR1644252 (99f:20001a)
7. R. C. Blanchfield, *Intersection theory of manifolds with operators with applications to knot theory*, Ann. of Math. (2) **65** (1957), 340--356. MR0085512 (19,53a)
8. J. Bocknak, M. Coste, and M.-F. Roy, *Real Algebraic Geometry*, Springer, Berlin, 1998. MR1226238 (94b:14055)
9. N. Bourbaki, *Éléments de Mathématique, Book 2 Ch.8*, Hermann, Paris, 1958. MR0417224 (54 #5282)
10. S. E. Cappell and J. L. Shaneson, *Topological knots and knot cobordism*, Topology **12** (1973), 33--40. MR0321099 (47 #9632)
11. S. E. Cappell and J. L. Shaneson, *The codimension two placement problem, and homology equivalent manifolds*, Ann. of Math. (2) **99** (1974), 277--348. MR0339216 (49 #3978)
12. S. E. Cappell and J. L. Shaneson, *Link cobordism*, Comment. Math. Helv. **55** (1980), 20--49. MR0569244 (81j:57011)
13. A. J. Casson and C. McA. Gordon, *On slice knots in dimension three*, Algebraic and geometric topology, Proc. Sympos. Pure Math., no. XXXII, Part 2, American Mathematical Society, Providence RI, 1978, pp. 39--53. MR0520521 (81g:57003)
14. A. J. Casson and C. McA. Gordon, *Cobordism of classical knots*, À la recherche de la topologie perdue, Progr. Math., 62, Birkhäuser Boston, Boston, MA, 1986, pp. 181--199. MR0900252
15. T. D. Cochran and K. E. Orr, *Not all links are concordant to boundary links*, Bull. Amer. Math. Soc. (N.S.) **23** (1990), no. 1, 99--106. MR1031581 (91c:57012)
16. T. D. Cochran and K. E. Orr, *Not all links are concordant to boundary links*, Ann. of Math. (2) **138** (1993), 519--554. MR1247992 (95c:57042)
17. T. D. Cochran, K. E. Orr, and P. Teichner, *Knot concordance, Whitney towers and* L^2-*signatures*, e-print math.GT/9908117, 1999.
18. P. M. Cohn, *Skew Fields, Theory of General Division Rings*, Encyclopedia Math. Appl., 57, Cambridge University Press, 1995. MR1349108 (97d:12003)
19. C. W. Curtis and I. Reiner, *Methods of Representation Theory. Vol. I. With Applications to Finite Groups and Orders*, John Wiley & Sons, New York, 1981. MR0632548 (82i:20001)
20. P. K. Draxl, *Skew Fields*, London Math. Soc. Lecture Note Ser., 81, Cambridge University Press, 1983. MR0696937 (85a:16022)
21. D. Dubois and G. Efroymson, *A dimension theorem for real primes*, Canad. J. Math. **26** (1974), no. 1, 108--114. MR0337953 (49 #2722)
22. J. Duval, *Forme de Blanchfield et cobordisme d'entrelacs bords*, Comment. Math. Helv. **61** (1986), no. 4, 617--635. MR0870709 (88a:57037)
23. M. Farber, *The classification of simple knots*, Uspekhi Mat. Nauk. **38** (1983), no. 5, 59--106, Russian Math. Surveys 38:5 (1983) 63--117. MR0718824

(85g:57012)
24. M. Farber, *Hermitian forms on link modules*, Comment. Math. Helv. **66** (1991), no. 2, 189--236. MR1107839 (92j:57014)
25. M. Farber, *Noncommutative rational functions and boundary links*, Math.Ann. **293** (1992), no. 3, 543--568. MR1170526 (94m:68137)
26. M. Farber, *Stable-homotopy and homology invariants of boundary links*, Trans. Amer. Math. Soc. **334** (1992), no. 1, 455--477. MR1079054 (93a:57023)
27. M. Farber and P. Vogel, *The Cohn localization of the free group ring*, Math. Proc. Cambridge Philos. Soc. **111** (1992), no. 3, 433--443. MR1151322 (93b:16049)
28. E. Formanek, *Generating the ring of matrix invariants*, Ring theory (Antwerp 1985), Lecture Notes in Math., 1197, Springer, Berlin, 1986, pp. 73--82. MR0859385 (87i:16028)
29. R. H. Fox and J. Milnor, *Singularities of* 2- *spheres in* 4- *space and cobordism of knots*, Osaka J. Math. **3** (1966), 257--267. MR0211392 (35 #2273)
30. L. Fuchs, *Infinite Abelian Groups. Volume I*, Pure and Applied Mathematics, 36, Academic Press, New York and London, 1970. MR0255673 (41 #333)
31. P. Gilmer, *Slice knots in* S^3, Quart. J. Math. Oxford Ser. (2) **34** (1983), no. 135, 305--322. MR0711523 (85d:57004)
32. P. Gilmer, *Classical knot and link concordance*, Comment. Math. Helv. **68** (1993), no. 1, 1--19. MR1201199 (94c:57007)
33. M. A. Gutiérrez, *Boundary links and an unlinking theorem*, Trans. Amer. Math. Soc. **171** (1972), 491--499. MR0310902 (46 #10000)
34. I. Hambleton and I. Madsen, *On the computation of the projective surgery obstruction groups, K*-theory **7** (1993), no. 6, 537--574. MR1268592 (95d:19003)
35. J. A. Hillman, *Alexander ideals of links*, Lecture Notes in Math., 895, Springer, Berlin-New York, 1981. MR0653808 (84j:57004)
36. J. A. Hillman, *Algebraic Invariants of Links*, World Scientific Publishing, to appear. cf. MR1932169 (2003k:57014)
37. M. W. Hirsch, *Differential Topology*, Grad. Texts in Math., No. 33, Springer, 1976. MR0448362 (56 #6669)
38. C. Kearton, *Blanchfield duality and simple knots*, Trans. Amer. Math. Soc. **202** (1975), 141--160. MR0358796 (50 #11255)
39. C. Kearton, *Cobordism of knots and Blanchfield duality*, J. London Math. Soc. (2) **10** (1975), no. 4, 406--408. MR0385873 (52 #6732)
40. M. Kervaire and C. Weber, *A survey of multidimensional knots*, Proc. 1977 Plans Conf. Knot Theory, Lecture Notes in Math., 685, Springer, 1978, pp. 61--134. MR0521731 (80f:57009)
41. M. A. Kervaire, *Les noeuds de dimensions supérieures*, Bull. Soc. Math. France **93** (1965), 225--271. MR0189052 (32 #6479)
42. M. A. Kervaire, *Knot cobordism in codimension two*, Manifolds--Amsterdam 1970, Lecture Notes in Math., 197, Springer, Berlin, 1971, pp. 83--105. MR0283786 (44 #1016)
43. Paul Kirk and Charles Livingston, *Twisted Alexander invariants, Reidemeister torsion and Casson-Gordon invariants*, Topology **38** (1999), no. 3, 635--661. MR1670420 (2000c:57010)
44. M.-A. Knus, *Quadratic and Hermitian Forms over Rings*, Grundlehren der Mathematischen Wissenschaften, 294, Springer, Berlin, 1991. MR1096299 (92i:11039)
45. K. H. Ko, *Seifert matrices and boundary link cobordisms*, Trans. Amer. Math. Soc. **299** (1987), no. 2, 657--681. MR0869227 (88h:57018)
46. K. H. Ko, *A Seifert-matrix interpretation of Cappell and Shaneson's approach to*

link cobordisms, Math. Proc. Cambridge Philos. Soc. **106** (1989), 531--545. MR1010376 (90j:57017)

47. T. Y. Lam, *The Algebraic Theory of Quadratic Forms*, W.A.Benjamin, Reading, Massachusetts, 1973. MR0396410 (53 #277)
48. T. Y. Lam, *An introduction to real algebra*, Rocky Mountain J. Math. **14** (1984), no. 4, 767--814. MR0773114 (86g:12013)
49. W. Landherr, *Äquivalenz Hermitescher Formen über einem beliebigen algebraischen Zahlkörper*, Abh. Math. Sem. Univ. Hamburg **11** (1936), 245--248.
50. S. Lang, *Algebra*, 3rd ed., Addison-Wesley, 1993. MR0197234 (33 #5416)
51. L. Le Bruyn and C. Procesi, *Semisimple representations of quivers*, Trans. Amer. Math. Soc. **317** (1990), no. 2, 585--598. MR0958897 (90e:16048)
52. J.-Y. Le Dimet, *Cobordisme d'enlacements de disques*, Bull. Soc. Math. France **116** (1988), ii+92, Mémoire no. 32. MR0971415 (90e:57046)
53. C. F. Letsche, *An obstruction to slicing knots using the eta invariant*, Math. Proc. Cambridge Philos. Soc. **128** (2000), no. 2, 301--319. MR1735303 (2001b:57017)
54. J. Levine, *Unknotting spheres in codimension two*, Topology **4** (1965), 9--16. MR0179803 (31 #4045)
55. J. Levine, *Invariants of knot cobordism*, Invent. Math. **8** (1969), 98--110, Addendum, 8:355. MR0253348 (40 #6563)
56. J. Levine, *Knot cobordism groups in codimension two*, Comment. Math. Helv. **44** (1969), 229--244. MR0246314 (39 #7618)
57. J. Levine, *Knot modules I*, Trans. Amer. Math. Soc. **229** (1977), 1--50. MR0461518 (57 #1503)
58. J. Levine, *Signature invariants of homology bordism with applications to links*, Knots 90 (Osaka 1990), de Gruyter, Berlin, 1992, pp. 395--406. MR1177436 (94g:57026)
59. J. Levine, *Link invariants via the eta invariant*, Comment. Math. Helv. **69** (1994), no. 1, 82--119. MR1259607 (95a:57009)
60. J. Levine and K. E. Orr, *A survey of applications of surgery to knot and link theory*, Surveys on surgery theory, Vol 1, Ann. of Math. Stud., 145, Princeton University Press, Princeton, NJ, 2000, pp. 345--364. MR1747541 (2001e:57034)
61. D. W. Lewis, *A note on Hermitian and quadratic forms*, Bull. London Math. Soc. **11** (1979), no. 3, 265--267. MR0554390 (81h:10031)
62. D. W. Lewis, *The isometry classification of Hermitian forms over division algebras*, Linear Algebra Appl. **43** (1982), 245--272. MR0656449 (83e:15026)
63. D. W. Lewis, *New improved exact sequences of Witt groups*, J. Algebra **74** (1982), no. 1, 206--210. MR0644226 (83d:10025)
64. D. W. Lewis, *Quaternionic skew-Hermitian forms over a number field*, J. Algebra **74** (1982), no. 1, 232--240. MR0644229 (83c:10029)
65. W. B. R. Lickorish, *An introduction to knot theory*, Grad. Texts in Math., 175, Springer, New York, 1997. MR1472978 (98f:57015)
66. D. Luna, *Slices étales*, Bull. Soc. Math. France Mémoirs **33** (1973), 81--105. MR0342523 (49 #7269)
67. D. Luna, *Adhérances d'orbite et invariants*, Invent. Math. **29** (1975), no. 3, 231--238. MR0376704 (51 #12879)
68. J. W. Milnor, *Infinite cyclic coverings*, Conference on the Topology of Manifolds, Prindle, Weber & Schmidt, Boston, Mass, 1968, pp. 115--133. MR0242163 (39 #3497)
69. J. W. Milnor, *On isometries of inner product spaces*, Invent. Math. **8** (1969), 83--97. MR0249519 (40 #2764)

70. J. W. Milnor and D. Husemoller, *Symmetric Bilinear Forms*, Springer, 1973. MR0506372 (58 #22129)
71. W. Mio, *On boundary-link cobordism*, Math. Proc. Cambridge Philos. Soc. **101** (1987), 259--266. MR0870597 (88e:57023)
72. D. Mumford, J. Fogarty, and F. Kirwan, *Geometric Invariant Theory*, 3rd ed., Springer, Berlin, 1994. MR1304906 (95m:14012)
73. W. D. Neumann, *Equivariant Witt Rings*, Bonner Math. Schriften, 100, Universität Bonn, Mathematisches Institut, 1977. MR0494248 (80a:57010)
74. S. P. Novikov, *On manifolds with free Abelian fundamental groups and their applications*, Izvestiya Akademii Nauk SSSR. Seriya Matematicheskaya (1966), 207--246,; Amer. Math. Soc. Transl. Ser. 2, 71 (1968), 1--42. MR0196765 (33 #4951)
75. O. T. O'Meara, *Introduction to quadratic forms*, Die Grundlehren der mathematischen Wissenschaften, 117, Academic Press, New York; Springer, Berlin, 1963. MR0152507 (27 #2485)
76. W. Pardon, *Local surgery and applications to the theory of quadratic forms*, Bull. Amer. Math. Soc. **82** (1976), no. 1, 131--133. MR0397751 (53 #1609)
77. W. Pardon, *Local surgery and the exact sequence of a localization for Wall groups*, Mem. Amer. Math. Soc. **12** (1977), no. 196, iv+171. MR0458450 (56 #16653)
78. C. Procesi, *The invariant theory of* $n \times n$ *matrices*, Adv. Math. **19** (1976), 306--381. MR0419491 (54 #7512)
79. C. Procesi, *A formal inverse to the Cayley-Hamilton theorem*, J. Algebra **107** (1987), 63--74. MR0883869 (88b:16033)
80. H.-G. Quebbemann, W. Scharlau, and M. Schulte, *Quadratic and Hermitian forms in additive and abelian categories*, J. Algebra **59** (1979), no. 2, 264--289. MR0543249 (82d:18015)
81. A. A. Ranicki, *The algebraic theory of surgery I. Foundations*, Proc. London Math. Soc. (3) **40** (1980), no. 1, 87--192. MR0560997 (82f:57024a)
82. A. A. Ranicki, *The algebraic theory of surgery II. Applications to topology*, Proc. London Math. Soc. (3) **40** (1980), no. 2, 193--283. MR0566491 (82f:57024b)
83. A. A. Ranicki, *Exact Sequences in the Algebraic Theory of Surgery*, Princeton University Press, New Jersey; University of Tokyo Press, Tokyo, 1981. MR0620795 (82h:57027)
84. A. A. Ranicki, *High-dimensional Knot Theory*, Springer, Berlin, 1998. MR1713074 (2000i:57044)
85. V. Retakh, C. Reutenauer, and A. Vaintrob, *Noncommutative rational functions and Farber's invariants of boundary links*, Differential topology, infinite-dimensional Lie algebras, and applications, Amer. Math. Soc. Transl., Ser. 2, 194, American Mathematical Society, Providence, RI, 1999, pp. 237--246. MR1729366 (2000h:57024)
86. L. Ribes and P. Zalesskii, *Profinite Groups*, Ergeb. Math. Grenzgeb. (3), 40, Springer, Berlin, 2000. MR1775104 (2001k:20060)
87. D. Rolfsen, *Knots and Links*, Mathematics Lecture Series, 7, Publish or Perish, Inc, Berkeley, California, 1990, Corrected reprint of the 1976 original. MR1277811 (95c:57018)
88. J. Rosenberg, *Algebraic* K*-theory and its applications*, Grad. Texts in Math., 147, Springer, New York, 1994. MR1282290 (95e:19001)
89. N. Sato, *Algebraic invariants of boundary links*, Trans. Amer. Math. Soc. **265** (1981), no. 2, 359--374. MR0610954 (82j:57021)
90. N. Sato, *Free coverings and modules of boundary links*, Trans. Amer. Math. Soc. **264** (1981), no. 2, 499--505. MR0603777 (82j:57020)
91. N. Sato, *Alexander modules*, Proc. Amer. Math. Soc. **91** (1984), no. 1,

159--162. MR0735584 (85k:57021)
92. W. Scharlau, *Induction theorems and the structure of the Witt group*, Invent. Math. **11** (1970), 37--44. MR0292752 (45 #1834)
93. W. Scharlau, *Quadratic and Hermitian forms*, Grundlehren Math. Wiss., 270, Springer, Berlin, 1985. MR0770063 (86k:11022)
94. J.-P. Serre, *Linear Representations of Finite Groups*, Grad. Texts in Math., 42, Springer, New York-Heidelberg, 1977. MR0450380 (56 #8675)
95. J. R. Smith, *Complements of codimension-two submanifolds - III - cobordism theory*, Pacific J. Math. **94** (1981), no. 2, 423--484. MR0628594 (84a:57032)
96. N. Smythe, *Boundary links*, Topology Seminar, Wisconsin, 1965 (R.H. Bing and R.J. Bean, eds.), Ann. of Math. Stud., 60, Princeton University Press, Princeton, N.J., 1966, pp. 69--72. MR0202100 (34 #1974)
97. N. W. Stoltzfus, *Unraveling the integral knot concordance group*, Mem. Amer. Math. Soc., 192, vol. 12, Issue 1, American Mathematical Society, Providence, Rhode Island, 1977. MR0467764 (57 #7616)
98. A. G. Tristram, *Some cobordism invariants for links*, Proceedings of the Cambridge Philosophical Society **66** (1969), 251--264. MR0248854 (40 #2104)
99. P. Vogel, *Localisation in algebraic* L- *theory*, Proc. 1979 Siegen Topology Conf., Lecture Notes in Math., 788, Springer, 1980, pp. 482--495. MR0585677 (81k:18011)
100. P. Vogel, *On the obstruction group in homology surgery*, Publ. Math. I.H.E.S. **55** (1982), 165--206. MR0672183 (84e:57021)
101. C. T. C. Wall, *Surgery on Compact Manifolds*, 2nd ed., Math. Surveys Monogr., vol. 69, American Mathematical Society, 1999, (1st edition published 1970). MR1687388 (2000a:57089)

This list reflects references listed in the original paper as accurately as possible with no attempt to correct error.

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Non-commutative characteristic polynomials and Cohn localization. (English summary)

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For a noncommutative ring A the generalised characteristic polynomial of an endomorphism α of a finitely generated projective A-module P is defined to be the Whitehead torsion $[1 - \alpha X] \in K_{1}(A[[X]])$. The purpose of this paper is to give an example of A,P and α as above for which the generalised characteristic polynomial does not determine (P,α) up to extensions. This is in contrast to the situation when A is commutative, where the positive result was proved in [G. Almkvist, J. Algebra **28** (1974), 375--388; MR0432738 (55 #5721)]. The noncommutative pathology is explained in terms of the failure of a certain Cohn localisation of $A[X]$ to embed in $A[[X]]$. The latter localisation is that defined in terms of inverting matrices, as described in, e.g., Chapter 7 of [P. M. Cohn, *Free rings and their relations*, Second edition, Academic Press, London, 1985; MR0800091 (87e:16006)].

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References

1. {\sc G. Almkvist}, `Endomorphisms of finitely generated projective modules

over a commutative ring', *Ark. Mat.* 11 (1973) 263--301. MR0424786 (54 #12744)

2. {\sc G. Almkvist}, `The Grothendieck ring of the category of endomorphisms', *J. Algebra* 28 (1974) 375--388. MR0432738 (55 #5721)
3. {\sc J. Berstel} and {\sc C. Reutenauer}, *Rational series and their languages* (Springer, Berlin, 1988). MR0971022 (90e:68054)
4. {\sc P. M. Cohn}, `Algebra and language theory', *Bull. London Math. Soc.* 7 (1975) 1--29. MR0359409 (50 #11863)
5. {\sc P. M. Cohn}, *Free rings and their relations*, 2nd edn (Academic Press, London, 1985). MR0800091 (87e:16006)
6. {\sc M. Farber} and {\sc P. Vogel}, `The Cohn localization of the free group ring,' *Proc. Cambridge Philos. Soc.* 111 (1992) 433--443. MR1151322 (93b:16049)
7. {\sc D. R. Grayson}, `The K-theory of endomorphisms', *J. Algebra* 48 (1977) 439--446. MR0480699 (58 #853)
8. {\sc J. L. Kelley} and {\sc E. H. Spanier}, `Euler characteristics', *Pacific J. Math.* 26 (1968) 317--339. MR0260842 (41 #5463)
9. {\sc A. Pajitnov}, `Closed orbits of gradient flows and logarithms of non-abelian Witt vectors', K-*Theory*, to appear (e-print server, http://xxx.soton.ac.uk/math/9908010). cf. MR1828180 (2002e:57048)
10. {\sc A. Pajitnov}, `On the Novikov complex for rational Morse forms', *Ann. Fac. Sci. Toulouse IV* (1995) 297--338. MR1344724 (97f:57033)
11. {\sc A. Pajitnov} and {\sc A. A. Ranicki}, `The Whitehead group of the Novikov ring', K-*Theory*, to appear (math.AT/0012031). cf. MR1828181 (2002g:19001)
12. {\sc A. A. Ranicki}, *High-dimensional knot theory* (Springer, Berlin, 1998). MR1713074 (2000i:57044)
13. {\sc J. Rosenberg}, *Algebraic* K-*theory and its applications* (Springer, New York, 1994). MR1282290 (95e:19001)
14. {\sc A. H. Schofield}, *Representations of rings over skew fields*, London Mathematical Society Lecture Note Series 92 (Cambridge University Press, Cambridge, 1985). MR0800853 (87c:16001)
15. {\sc M. P. Schützenberger}, `On the definition of a family of automata', *Inform. and Control* 4 (1961) 245--270. MR0135680 (24 #B1725)
16. {\sc M. P. Schützenberger}, `On a theorem of R. Jungen', *Proc. Amer. Math. Soc.* 13 (1962) 885--890. MR0142781 (26 #350)

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