Chuck Weibel

On the work of Daniel Quillen (1940-2011)

Daniel Gray Quillen (June 27 1940 – April 30 2011)

Born in New Jersey, went to Harvard '61, stayed for PhD '64
   23 when he got his PhD, advisor R. Bott, on linear PDEs
   10 year track job at MIT
1st 3 yrs he had no significant publications

1967: Homotopical Algebra (SLN #43)
- Notion of a weak equivalence
- Notion of a cofibration, pairs up with a fibration
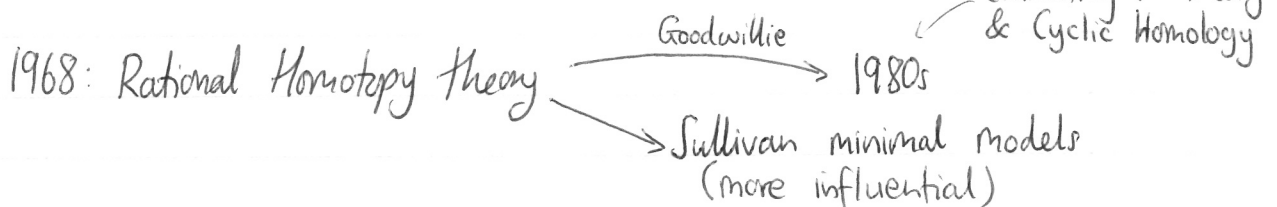
'68-'69 he was in Paris
- André-Quillen cohomology — had an enormous impact in deformation theory of commutative rings
- K_0 of algebraic varieties (influenced by Grothendieck)
- Étale homotopy theory $\longrightarrow$ Adams conjecture for BU

Frank Adams 1963 $BO \xrightarrow{\psi^2-1} BO \longrightarrow BF[\frac{1}{q}]$
              should be nullhomotopic

Friedlander 1970 thesis

1968: Rational Homotopy theory $\xrightarrow{\text{Goodwillie}}$ 1980s — Connecting k-theory & Cyclic Homology
          $\searrow$ Sullivan minimal models
               (more influential)

1969-70 he was at the IAS
    $MU^* \cong$ Lazard ring

$F(u,v) = u+v+a_{11}uv+\sum a_{ij} u^i v^j$

Formal group law $F(u,v) \in V[[t]]$
$F(u,v) = F(v,u)$, $F(u,0) = u$ ~~mod~~
Associativity, $f(0,v) = v$
Universal construction
$\mathbb{Z}[a_{ij}]/\text{above conditions} = \mathbb{Z}[X_1, X_2, \ldots]$
$|X_i| = 2i$

(1

$\rightarrow$ Algebraic cobordism (L.M.) $\leftarrow$ Levine & Morel

Representation theory (mod p)
  $H^*(G, \mathbb{Z}/p)$ graded ring, Proj $\longrightarrow$ projective variety.
  this leads to the notion of support varieties.

Nice ICM 1970 $\leftarrow$ definition $K_n(R) = \pi_n BGL_n(R)^+$

1971 - His 2nd attack on the Adams Conjecture

$$BU \xrightarrow{\psi^q - 1} BU \xrightarrow{J} BJ[\tfrac{1}{q}]$$

$BGL_n(\mathbb{F}_q)$ Brauer
                        induction

he solved the Adams Conjecture this way.

1970-1978: K-theory community used + construction & homology
            of $GL(R)$

1971: "On the group completion of a simplicial monoid" preprint
  published 1994 memoirs AMS (appendix to FM)
  In this he tied the + construction to the "Segal machine"
      Small categories are just special simplicial sets
      Symmetric monoidal category $S \times S \longrightarrow S$ (Boardman - Vogt)
      Example: S = free modules + automorphisms
            $\sim \coprod GL_n(R)$
      we call this $\wedge$ a "group completion" in topology
            construction

1972: Annals paper $H_* GL_n(\mathbb{F}_q)$
  He showed $BGL(\mathbb{F}_q)^+$ is the h.fibre of $BU \xrightarrow{\psi^q - 1} BU$
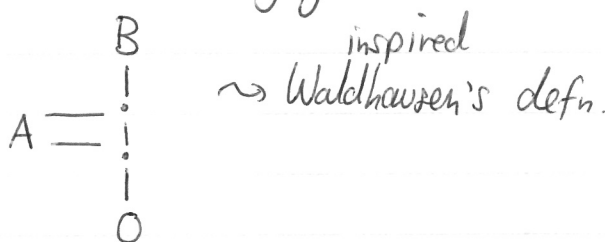 $\approx$ calculation of $K_*(\mathbb{F}_q)$ and $K_*(\overline{\mathbb{F}_q})$

July 1972: He wrote a letter to Milnor on $\text{Im}(J)$, π_*^s and $k_*(\mathbb{Z})$.
   He mailed a manuscript to Segal (for Oxford symposium)
   "landed on a friendly but alien planet" – Bass

August 1972: Arrived in Battelle, Washington
   full manuscript "Higher Algebraic K-theory – I"

Part I: Categories as top. spaces, functors as maps.
   theorem A and theorem B
Part II: The Q construction of an exact category.
   subquotients

$$A = \begin{array}{c} B \\ \vdots \\ \vdots \\ O \end{array} \quad \rightsquigarrow \quad \begin{array}{c} \text{inspired} \\ \text{Waldhausen's defn.} \end{array}$$

Part III: Fundamental theorems generalised to Higher Algebraic K-theory

Part IV: K-theory of varieties.

   Gersten Conjecture
   Bloch formula
   Severi-Brauer Varieties
      $\rightarrow$ 1983 $\underset{\text{S.}}{}$ Suslin Merkurjev

Aftermath – Finite generation of K_* $\begin{array}{l} K_*\mathbb{Z} \qquad\qquad 1972 \\ K_*(\text{curves}/\mathbb{F}_q) \;\; 1980 \end{array}$

   Open – for R finitely generated is $K_n(R)$ fin. gen? – Bass
   Question

Higher Algebraic K-theory II (Grayson 1976)

1974 · Vancouver ICM
   · MIT course – Reps and K-theory
      $\searrow$ λ-operations (Hiller 1981)

1976: Serre's conjecture

1978: he won the Fields Prize

Alain Connes — HC   (also Feigen Tsygan)

Loday - Quillen $H_*(\mathfrak{gl}_n R) \sim HC_{*-1}(R)$ primitive generators.

 Superconnection

1989 - 97: Seven papers in series J. Cuntz "bivariant HC HP"