

Errata for Algebraic L -theory and topological manifolds
by Andrew Ranicki
Cambridge Tracts in Mathematics 102, CUP (1992)

This list contains corrections of misprints/errors in the book, and some additional material and references.

Please let me know of any further misprints/errors by e-mail to a.ranicki@ed.ac.uk
A.A.R. 19.1.2011

- p. 6 l. 18 interpreted
- p. 7 l. 17 universal
- p. 15 l. -3 $\nu_M - (f^{-1})^* \nu_N$
- p. 33 l. 16 $\delta\phi_0 f^*$ should be $\delta\phi_0$
- p. 34 l. 2 $\delta\psi_0 f^*$ should be $\delta\psi_0$
- p. 38 l. 2 “An algebraic normal complex is . . .”
- p. 40 l. 12 In the definition of $(f, b)_{\%}$ should have

$$(\phi, \chi) \rightarrow (f_{\%}(\phi), \widehat{f}_{\%}(\chi) + \widehat{f}_{\phi_0}{}_{\%}(S^n b))$$

- p. 40 l. -8 In Definition 2.4 (ii) the second term in the second equation should be $\widehat{\delta\phi_0}(S^n \delta\gamma)$ and not $(\delta\phi_0, \phi_0)_{\%}(S^n \delta\gamma)$.
- p. 43 l. -2 Need “ n -dimensional (symmetric, quadratic) Poincaré pairs”
- p. 44 l. 16 Should have “ $\delta\phi_s = \gamma_{-n+s}$ ”.
- p. 44 l. -2 “ n -dimensional (normal, symmetric) pair”
- p. 45 l. 7 “sends the chain bundle $S^{n+1} \delta\gamma \in \widehat{Q}^{n+1}(D^{n+1-*})$ ”
- p. 45 l. 17 “defined in 2.6 (i)” instead of 2.8 (i)
- p. 45 l. 20 “ $(0 \rightarrow C, ((\phi, \gamma, \chi), 0))$ ” instead of $(C \rightarrow 0, (0, (1 + T)))$
- p. 64 l. -5 Replace

$$\begin{cases} \mathbb{B}(\mathbb{A})^*[K] = \mathbb{B}(\mathbb{A}^*[K]) \longrightarrow \mathbb{B}(\mathbb{A})_*(K) = \mathbb{B}(\mathbb{A}_*(K)) ; C \longrightarrow C^*[K] \\ \mathbb{B}(\mathbb{A})_*[K] = \mathbb{B}(\mathbb{A}_*[K]) \longrightarrow \mathbb{B}(\mathbb{A})^*(K) = \mathbb{B}(\mathbb{A}^*(K)) ; C \longrightarrow C_*[K] \end{cases}$$

by

$$\begin{cases} \mathbb{B}(\mathbb{A}^*[K]) \longrightarrow \mathbb{B}(\mathbb{A}_*(K)) ; C \longrightarrow C^*[K] \\ \mathbb{B}(\mathbb{A}_*[K]) \longrightarrow \mathbb{B}(\mathbb{A}^*(K)) ; C \longrightarrow C_*[K] \end{cases}$$

- p. 51 l. 82 Warning: the choice of terminology here is rather poor. A chain complex C is “ \mathbb{C} -contractible” if it is in \mathbb{C} , so in general C is not contractible !
- p. 64 l. 4 Remove ”additive” from ”additive functors”.
- p. 66 l. -1 “Proposition 2.7”
- p. 72 l. -5 d''
- p. 75 l. 12 $d_{TM}(\tau, \sigma) = d_{TM(\sigma)}$ if $\sigma = \tau$
- p. 75 l. 14 replace $\tau = \delta_i \sigma$ by $\sigma = \partial_i \tau$
- p. 80 l. -1 symmetric Poincaré
- p. 85 l. -3 $M \otimes_{[R,K]} N = M \otimes_{\mathbb{A}[R,K]} N$ and $M \otimes_{(R,K)} N = M \otimes_{\mathbb{A}(R,K)} N$ are R -module chain complexes
- p. 93 l. -1 It should be noted that for a connected oriented n -dimensional pseudomanifold K the composite

$$H^n(K \times K) \xrightarrow{\Delta^*} H^n(K) \xrightarrow{[K] \cap -} H_0(K) = \mathbb{Z}$$

sends any element $x \in H^n(K \times K)$ with $\langle x, \Delta([K]) \rangle = 1 \in \mathbb{Z}$ to the Euler characteristic of K

$$[K] \cap \Delta^*(x) = \chi(K) \in \mathbb{Z} .$$

If K admits a geometric Thom class $U \in H^n(K \times K, K \times K \setminus \Delta)$ then U has image the Euler number of the homology tangent bundle τ_K of K

$$\Delta^* j^*(U) = \chi(\tau_K) \in H^n(K) = \mathbb{Z} ,$$

and $x = j^*(U) \in H^n(K \times K)$ is such that $\langle x, \Delta([K]) \rangle = 1 \in \mathbb{Z}$. Thus if K is a homology manifold

$$\chi(K) = \chi(\tau_K) \in H^n(K) = H_0(K) = \mathbb{Z} .$$

(For a differentiable manifold K this is proved in Milnor and Stasheff [111, pp. 124-130]).

- p. 99 l. 10 $\phi(x)(x) \in \widehat{H}^0(\mathbb{Z}_2; R) \subseteq \widehat{H}^0(\mathbb{Z}_2; R[\pi])$ ($x \in C^0$)
- p. 103 l. -1 Use the Pontrjagin-Thom isomorphism to represent

$$\rho_X \in \pi_{n+k}^s(T(\nu_X)) = \pi_{n+k}^s(D(\nu_X), S(\nu_X)) = \Omega_{n+k}^{fr}(D(\nu_X), S(\nu_X))$$

by a map $(W, \partial W) \rightarrow (D(\nu_X), S(\nu_X))$ from a framed $(n+k)$ -dimensional manifold. The composite

$$(W, \partial W) \rightarrow (D(\nu_X), S(\nu_X)) \rightarrow X \rightarrow K'$$

can be approximated by a simplicial map, and by the homotopy extension property of a Hurewicz fibration it may be arranged for this simplicial map to also factor in this way. The inverse images of the dual cells $D(\sigma, K) \subset K'$ ($\sigma \in K$) define a K -dissection $\{(W[\sigma], \partial W[\sigma]) \mid \sigma \in K\}$ of $(W, \partial W)$ with each $W(\sigma)$ an $(n + k - |\sigma|)$ -dimensional framed manifold. Thus each $(X[\sigma], \partial X[\sigma])$ is an $(n - |\sigma|)$ -dimensional normal pair, justifying the statement that the n -dimensional normal complex $\hat{\sigma}^*(X)$ is defined in $\mathbb{A}(R, K)$.

p. 104 l. 5 symmetric (not normal, in general)

p. 107 l. -1 $L^0(\mathbb{Z}[\mathbb{Z}_2])$

p. 112 l. 2 $C(K) = \Delta(X')$

p. 114 l. 2 i -connected

p. 117 l. 3 sets

p. 123 l. 4, p. 123 l. -3, p. 126 l. -5, p. 127 l. 6 $m + 1 = |J^{(0)}|$

p. 140 ll. 8,9 The Δ -sets $\mathbb{L}_n(\Lambda^*(K))$, $\mathbb{L}_n(\Lambda)^{K+}$ are not isomorphic, but they both have the realization $|\mathbb{L}_n(\Lambda)|^{K+}$, so they are homotopy equivalent. See *Multiplicative properties of Quinn spectra* by Gerd Laures and Jim McClure/Hurewicz (<http://arxiv.org/abs/0907.2367>) for a proof.

p. 149 l. -3 $\mathbb{H} \cdot (K; \mathbb{L} \cdot (R))$

p. 168 l. 10 $\mathbb{S}_n \langle 0 \rangle (R, K) = \mathbb{S}_n(R, K) = \mathbb{S}_{n+4}(R, K)$ ($n \geq \max(\dim(K), 2)$)

p. 169 l. 3 $H_{n-1}(K; \mathbb{L} \cdot)$

p. 175 l. 13 Add: Let $\Omega^N(X, \nu)$ be the Kan Δ -set in which an n -simplex is an $(n - k)$ -dimensional normal space n -ad

$$(Y; \partial_1 Y, \dots, \partial_n Y; \nu_Y : Y \rightarrow BG(k), \rho : \Delta^n \rightarrow T(\nu_Y))$$

such that $\rho(\partial_i \Delta^n) \subset T(\nu_{\partial_i Y})$, with a normal map $(f, b) : (Y, \nu_Y) \rightarrow (X, \nu)$.
The map of Kan Δ -sets

$$\Omega^N(X, \nu) \rightarrow T(\nu) ; (Y; \partial_1 Y, \dots, \partial_n Y; \nu_Y, \rho) \rightarrow T(b)\rho$$

induces the normal space Pontrjagin-Thom transversality isomorphisms $\Omega_n^N(X, \nu) \rightarrow \pi_n(T(\nu))$ with inverses

$$\pi_n(T(\nu)) \rightarrow \Omega_n^N(X, \nu) ; \rho_X \rightarrow ((X, \nu, \rho_X), 1)$$

and is thus a homotopy equivalence.

p. 185 l. 8 add 16.11 (x) Edwards 'The topology of manifolds and cell-like maps' (Proc. 1978 ICM Helsinki, 111–127 (1980)) showed that for $n \geq 5$ an n -dimensional combinatorial homology manifold K is a topological manifold if and only if the link of each simplex $\sigma \in K$ is simply-connected.

- p. 193 l. -5 reducible
- p. 199 l. -5 according to which
- p. 200 l. 12 Corollary 18.6 (i) is only true after the 4-periodic stabilization of n : see Theorem B of I.Hambleton, *Surgery obstructions on closed manifolds and the inertia subgroup*, arXiv:0905.0104
- p. 202 l. -2 closed manifold
- p. 210 l. 4 Q^8 is obtained by coning off the boundary of a differentiable 8-dimensional manifold with boundary one of the 27 7-dimensional exotic spheres classified by Kervaire and Milnor [83], but it is not one of the original 7 examples of Milnor [108].
- p. 216 l. -9 The cohomology group $H^{-m}(B; \mathbb{L}^\cdot)$ should be replaced by the cobordism group $L^m(B; \mathbb{Z})$ of homogeneous m -dimensional Poincaré cocycles over B defined in the appendix to Lück and Ranicki [100], with a forgetful map $L^m(B, \mathbb{Z}) \rightarrow H^{-m}(B; \mathbb{L}^\cdot)$ and an assembly map $A : L^m(B, \mathbb{Z}) \rightarrow L^m(\pi_1(B), \mathbb{Z})$. For any PL fibration $F \rightarrow E \xrightarrow{p} B$ with base B a compact n -dimensional homology manifold and fibre F a compact m -dimensional homology manifold the Δ -map $B \rightarrow \mathbb{L}^{-m}(\mathbb{Z})$ sending each simplex $\tau \in B$ to the symmetric Poincaré fibre $\sigma^*(p^{-1}\tau)$ over \mathbb{Z} represents an element $(F, p)_{\mathbb{L}} \in L^m(B; \mathbb{Z})$. The composite

$$p_! p^! : L_n(\mathbb{Z}[\pi_1(B)]) \longrightarrow L_{m+n}(\mathbb{Z}[\pi_1(E)]) \longrightarrow L_{m+n}(\mathbb{Z}[\pi_1(B)])$$

is shown in [100] to depend on the assembly

$$A((F, p)_{\mathbb{L}}) = \sigma^*(F, p) \in L^m(\pi_1(B), \mathbb{Z}) .$$

The total space E is a compact $(m+n)$ -dimensional manifold E such that the canonical \mathbb{L}^\cdot -homology fundamental class $[E]_{\mathbb{L}} \in H_{m+n}(E; \mathbb{L}^\cdot)$ has image

$$p_![E]_{\mathbb{L}} = [F, p]_{\mathbb{L}} \cap [B]_{\mathbb{L}} \in H_{m+n}(B; \mathbb{L}^\cdot)$$

with $[F, p]_{\mathbb{L}} \in H^{-m}(B; \mathbb{L}^\cdot)$ the image of $(F, p)_{\mathbb{L}}$.

- p. 220 l. -1 $I = \text{im}(K_0(\mathbb{Z})) = d\mathbb{Z} \subset K_0(M_d(\mathbb{Z}[\bar{\pi}])) = K_0(\mathbb{Z}[\bar{\pi}]) = \mathbb{Z} \oplus \tilde{K}_0(\mathbb{Z}[\bar{\pi}])$
- p. 234 l. -9 $\dim_K(A^+) = \dim_K(A^-) = d^2$
- p. 241 l. 11 $L_*(i_1^- : A \rightarrow A[\sqrt{a}]^-) \cong L_{*+1}(i_1^+ : A \rightarrow A[\sqrt{a}]^+)$
- p. 259 l. 6 *The image of the splitting obstruction*
- p. 273 l. 18 π is finitely presented.
- p. 276 l. -3 close brackets ($\times 2$)
- p. 283 l. -14 $B^{global}(C, \phi) - B^{local}(C, \phi) = (B\sigma^*(Y) - 1)/8$

p. 297 l. –10 25.13 The construction of exotic homology manifolds by Bryant, Ferry, Mio and Weinberger is announced in ‘The topology of homology manifolds’ (Bull. A.M.S. 28, 324–328 (1993)).

p. 325 l. –8 The definition of $\mathbb{A}[\pi]$ for an additive category \mathbb{A} and a group π is garbled. Let $\mathbb{A}[\pi]$ be the additive category with one object $M[\pi]$ for each object M in \mathbb{A} , and

$$\mathrm{Hom}_{\mathbb{A}[\pi]}(M[\pi], N[\pi]) = \mathrm{Hom}_{\mathbb{A}}(M, N)[\pi]$$

the additive group of formal linear combinations $\sum_{g \in \pi} f_g g$ with $f_g \in \mathrm{Hom}_{\mathbb{A}}(M, N)$

such that $\{g \in \pi \mid f_g \neq 0\}$ is finite. An involution on \mathbb{A} is extended to an involution on $\mathbb{A}[\pi]$ by

$$* : \mathbb{A}[\pi] \rightarrow \mathbb{A}[\pi] ;$$

$$M[\pi] \rightarrow (M[\pi])^* = M^*[\pi], f = \sum_{g \in \pi} f_g g \rightarrow f^* = \sum_{g \in \pi} (f_g)^* g^{-1} .$$

p. 326 l. 12 The surgery obstruction $\sigma_*(f, b) \in L_n(\mathbb{C}_X(\mathbb{Z}[\pi]))$ of an n -dimensional X -bounded normal map (f, b) is the cobordism class of an n -dimensional quadratic Poincaré complex in $\mathbb{C}_X(\mathbb{Z}[\pi])$ which may be obtained either by considering the middle-dimensional form/formation remaining after surgery below the middle dimension as in Ferry and Pedersen [50], or else using the algebraic normal maps of 2.16. A normal map $(f, b): J \rightarrow K$ of X -bounded geometric Poincaré complexes induces an algebraic normal map $\sigma^*(J) \rightarrow \sigma^*(K)$ of symmetric Poincaré complexes in $\mathbb{C}_X(\mathbb{Z}[\pi])$, with a quadratic Poincaré kernel $\sigma_*(f, b)$ in $\mathbb{C}_X(\mathbb{Z}[\pi])$.

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