

Errata for Algebraic L -theory and topological manifolds
by Andrew Ranicki
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This list contains corrections of misprints/errors in the book, and some additional material and references.

Please let me know of any further misprints/errors by e-mail to a.ranicki@ed.ac.uk
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- p. 6 l. 18 interpreted
- p. 7 l. 17 universal
- p. 15 l. -3 $\nu_M - (f^{-1})^* \nu_N$
- p. 33 l. 16 $\delta\phi_0 f^*$ should be $\delta\phi_0$
- p. 34 l. 2 $\delta\psi_0 f^*$ should be $\delta\psi_0$
- p. 38 l. 2 “An algebraic normal complex is . . .”
- p. 40 l. 12 In the definition of $(f, b)_{\%}$ should have

$$(\phi, \chi) \rightarrow (f_{\%}(\phi), \widehat{f}_{\%}(\chi) + \widehat{f}_{\phi_0}^{\%}(S^n b))$$

- p. 40 l. -8 In Definition 2.4 (ii) the second term in the second equation should be $\widehat{\delta\phi_0}(S^n \delta\gamma)$ and not $(\delta\phi_0, \phi_0)_{\%}(S^n \delta\gamma)$.
- p. 43 l. -2 Need “ n -dimensional (symmetric, quadratic) Poincaré pairs”
- p. 44 l. 16 Should have “ $\delta\phi_s = \gamma_{-n+s}$ ”.
- p. 44 l. -2 “ n -dimensional (normal, symmetric) pair”
- p. 45 l. 7 “sends the chain bundle $S^{n+1} \delta\gamma \in \widehat{Q}^{n+1}(D^{n+1-*})$ ”
- p. 45 l. 17 “defined in 2.6 (i)” instead of 2.8 (i)
- p. 45 l. 20 “ $(0 \rightarrow C, ((\phi, \gamma, \chi), 0))$ ” instead of $(C \rightarrow 0, (0, (1 + T)))$
- p. 64 l. -5 Replace

$$\begin{cases} \mathbb{B}(\mathbb{A})^*[K] = \mathbb{B}(\mathbb{A}^*[K]) \longrightarrow \mathbb{B}(\mathbb{A})_*(K) = \mathbb{B}(\mathbb{A}_*(K)) ; C \longrightarrow C^*[K] \\ \mathbb{B}(\mathbb{A})_*[K] = \mathbb{B}(\mathbb{A}_*[K]) \longrightarrow \mathbb{B}(\mathbb{A})^*(K) = \mathbb{B}(\mathbb{A}^*(K)) ; C \longrightarrow C_*[K] \end{cases}$$

by

$$\begin{cases} \mathbb{B}(\mathbb{A}^*[K]) \longrightarrow \mathbb{B}(\mathbb{A}_*(K)) ; C \longrightarrow C^*[K] \\ \mathbb{B}(\mathbb{A}_*[K]) \longrightarrow \mathbb{B}(\mathbb{A}^*(K)) ; C \longrightarrow C_*[K] \end{cases}$$

- p. 51 l. 82 Warning: the choice of terminology here is rather poor. A chain complex C is “ \mathbb{C} -contractible” if it is in \mathbb{C} , so in general C is not contractible !
- p. 66 l. -1 “Proposition 2.7”
- p. 72 l. -5 d''
- p. 75 l. 12 $d_{TM}(\tau, \sigma) = d_{TM(\sigma)}$ if $\sigma = \tau$
- p. 75 l. 14 replace $\tau = \delta_i \sigma$ by $\sigma = \partial_i \tau$
- p. 80 l. -1 symmetric Poincaré
- p. 85 l. -3 $M \otimes_{[R,K]} N = M \otimes_{\mathbb{A}[R,K]} N$ and $M \otimes_{(R,K)} N = M \otimes_{\mathbb{A}(R,K)} N$ are R -module chain complexes
- p. 93 l. -1 It should be noted that for a connected oriented n -dimensional pseudomanifold K the composite

$$H^n(K \times K) \xrightarrow{\Delta^*} H^n(K) \xrightarrow{[K] \cap -} H_0(K) = \mathbb{Z}$$

sends any element $x \in H^n(K \times K)$ with $\langle x, \Delta([K]) \rangle = 1 \in \mathbb{Z}$ to the Euler characteristic of K

$$[K] \cap \Delta^*(x) = \chi(K) \in \mathbb{Z} .$$

If K admits a geometric Thom class $U \in H^n(K \times K, K \times K \setminus \Delta)$ then U has image the Euler number of the homology tangent bundle τ_K of K

$$\Delta^* j^*(U) = \chi(\tau_K) \in H^n(K) = \mathbb{Z} ,$$

and $x = j^*(U) \in H^n(K \times K)$ is such that $\langle x, \Delta([K]) \rangle = 1 \in \mathbb{Z}$. Thus if K is a homology manifold

$$\chi(K) = \chi(\tau_K) \in H^n(K) = H_0(K) = \mathbb{Z} .$$

(For a differentiable manifold K this is proved in Milnor and Stasheff [111, pp. 124-130]).

- p. 99 l. 10 $\phi(x)(x) \in \widehat{H}^0(\mathbb{Z}_2; R) \subseteq \widehat{H}^0(\mathbb{Z}_2; R[\pi])$ ($x \in C^0$)
- p. 107 l. -1 $L^0(\mathbb{Z}[\mathbb{Z}_2])$
- p. 112 l. 2 $C(K) = \Delta(X')$
- p. 114 l. 2 i -connected
- p. 117 l. 3 sets
- p. 123 l. 4, p. 123 l. -3, p. 126 l. -5, p. 127 l. 6 $m + 1 = |J^{(0)}|$

p. 140 ll. 8,9 The Δ -sets $\mathbb{L}_n(\Lambda^*(K))$, $\mathbb{L}_n(\Lambda)^{K+}$ are not isomorphic, but they both have the realization $|\mathbb{L}_n(\Lambda)|^{|K+|}$, so they are homotopy equivalent. See *Multiplicative properties of Quinn spectra* by Gerd Laures and Jim McClure (<http://arxiv.org/abs/0907.2367>) for a proof.

p. 149 l. -3 $\mathbb{H} \cdot (K; \mathbb{L} \cdot (R))$

p. 168 l. 10 $\mathbb{S}_n \langle 0 \rangle (R, K) = \mathbb{S}_n(R, K) = \mathbb{S}_{n+4}(R, K)$ ($n \geq \max(\dim(K), 2)$)

p. 169 l. 3 $H_{n-1}(K; \mathbb{L} \cdot)$

p. 175 l. 13 Add: Let $\Omega^N(X, \nu)$ be the Kan Δ -set in which an n -simplex is an $(n - k)$ -dimensional normal space n -ad

$$(Y; \partial_1 Y, \dots, \partial_n Y; \nu_Y : Y \rightarrow BG(k), \rho : \Delta^n \rightarrow T(\nu_Y))$$

such that $\rho(\partial_i \Delta^n) \subset T(\nu_{\partial_i Y})$, with a normal map $(f, b) : (Y, \nu_Y) \rightarrow (X, \nu)$.
The map of Kan Δ -sets

$$\Omega^N(X, \nu) \rightarrow T(\nu_Y) ; (Y; \partial_1 Y, \dots, \partial_n Y; \nu_Y, \rho) \rightarrow \rho$$

induces the normal space Pontrjagin-Thom transversality isomorphisms $\Omega_n^N(X, \nu) \rightarrow \pi_n(T(\nu))$ with inverses

$$\pi_n(T(\nu_X)) \rightarrow \Omega_n^N(X, \nu) ; \rho_X \rightarrow ((X, \nu, \rho_X), 1)$$

and is thus a homotopy equivalence.

p. 185 l. 8 add 16.11 (x) Edwards ‘The topology of manifolds and cell-like maps’ (Proc. 1978 ICM Helsinki, 111–127 (1980)) showed that for $n \geq 5$ an n -dimensional combinatorial homology manifold K is a topological manifold if and only if the link of each simplex $\sigma \in K$ is simply-connected.

p. 193 l. -5 reducible

p. 199 l. -5 according to which

p. 200 l. 12 Corollary 18.6 (i) is only true after the 4-periodic stabilization of n : see Theorem B of I.Hamblen, *Surgery obstructions on closed manifolds and the inertia subgroup*, arXiv:0905.0104

p. 202 l. -2 closed manifold

p. 210 l. 4 Q^8 is obtained by coning off the boundary of a differentiable 8-dimensional manifold with boundary one of the 27 7-dimensional exotic spheres classified by Kervaire and Milnor [83], but it is not one of the original 7 examples of Milnor [108].

- p. 216 l. -9 The cohomology group $H^{-m}(B; \mathbb{L}')$ should be replaced by the cobordism group $L^m(B; \mathbb{Z})$ of homogeneous m -dimensional Poincaré cocycles over B defined in the appendix to Lück and Ranicki [100], with a forgetful map $L^m(B, \mathbb{Z}) \rightarrow H^{-m}(B; \mathbb{L}')$ and an assembly map $A : L^m(B, \mathbb{Z}) \rightarrow L^m(\pi_1(B), \mathbb{Z})$. For any PL fibration $F \rightarrow E \xrightarrow{p} B$ with base B a compact n -dimensional homology manifold and fibre F a compact m -dimensional homology manifold the Δ -map $B \rightarrow \mathbb{L}^{-m}(\mathbb{Z})$ sending each simplex $\tau \in B$ to the symmetric Poincaré fibre $\sigma^*(p^{-1}\tau)$ over \mathbb{Z} represents an element $(F, p)_{\mathbb{L}} \in L^m(B; \mathbb{Z})$. The composite

$$p!p^! : L_n(\mathbb{Z}[\pi_1(B)]) \longrightarrow L_{m+n}(\mathbb{Z}[\pi_1(E)]) \longrightarrow L_{m+n}(\mathbb{Z}[\pi_1(B)])$$

is shown in [100] to depend on the assembly

$$A((F, p)_{\mathbb{L}}) = \sigma^*(F, p) \in L^m(\pi_1(B), \mathbb{Z}) .$$

The total space E is a compact $(m+n)$ -dimensional manifold E such that the canonical \mathbb{L}' -homology fundamental class $[E]_{\mathbb{L}} \in H_{m+n}(E; \mathbb{L}')$ has image

$$p![E]_{\mathbb{L}} = [F, p]_{\mathbb{L}} \cap [B]_{\mathbb{L}} \in H_{m+n}(B; \mathbb{L}')$$

with $[F, p]_{\mathbb{L}} \in H^{-m}(B; \mathbb{L}')$ the image of $(F, p)_{\mathbb{L}}$.

- p. 220 l. -1 $I = \text{im}(K_0(\mathbb{Z})) = d\mathbb{Z} \subset K_0(M_d(\mathbb{Z}[\bar{\pi}])) = K_0(\mathbb{Z}[\bar{\pi}]) = \mathbb{Z} \oplus \tilde{K}_0(\mathbb{Z}[\bar{\pi}])$
- p. 234 l. -9 $\dim_K(A^+) = \dim_K(A^-) = d^2$
- p. 241 l. 11 $L_*(i_1^- : A \rightarrow A[\sqrt{a}]^-) \cong L_{*+1}(i_1^+ : A \rightarrow A[\sqrt{a}]^+)$
- p. 259 l. 6 *The image of the splitting obstruction*
- p. 276 l. -3 close brackets ($\times 2$)
- p. 283 l. -14 $B^{global}(C, \phi) - B^{local}(C, \phi) = (B\sigma^*(Y) - 1)/8$
- p. 297 l. -10 25.13 The construction of exotic homology manifolds by Bryant, Ferry, Mio and Weinberger is announced in ‘The topology of homology manifolds’ (Bull. A.M.S. 28, 324–328 (1993)).
- p. 325 l. -8 The definition of $\mathbb{A}[\pi]$ for an additive category \mathbb{A} and a group π is garbled. Let $\mathbb{A}[\pi]$ be the additive category with one object $M[\pi]$ for each object M in \mathbb{A} , and

$$\text{Hom}_{\mathbb{A}[\pi]}(M[\pi], N[\pi]) = \text{Hom}_{\mathbb{A}}(M, N)[\pi]$$

the additive group of formal linear combinations $\sum_{g \in \pi} f_g g$ with $f_g \in \text{Hom}_{\mathbb{A}}(M, N)$

such that $\{g \in \pi \mid f_g \neq 0\}$ is finite. An involution on \mathbb{A} is extended to an involution on $\mathbb{A}[\pi]$ by

$$* : \mathbb{A}[\pi] \rightarrow \mathbb{A}[\pi] ;$$

$$M[\pi] \rightarrow (M[\pi])^* = M^*[\pi] , f = \sum_{g \in \pi} f_g g \rightarrow f^* = \sum_{g \in \pi} (f_g)^* g^{-1} .$$

- p. 326 l. 12 The surgery obstruction $\sigma_*(f, b) \in L_n(\mathbb{C}_X(\mathbb{Z}[\pi]))$ of an n -dimensional X -bounded normal map (f, b) is the cobordism class of an n -dimensional quadratic Poincaré complex in $\mathbb{C}_X(\mathbb{Z}[\pi])$ which may be obtained either by considering the middle-dimensional form/formation remaining after surgery below the middle dimension as in Ferry and Pedersen [50], or else using the algebraic normal maps of 2.16. A normal map $(f, b): J \rightarrow K$ of X -bounded geometric Poincaré complexes induces an algebraic normal map $\sigma^*(J) \rightarrow \sigma^*(K)$ of symmetric Poincaré complexes in $\mathbb{C}_X(\mathbb{Z}[\pi])$, with a quadratic Poincaré kernel $\sigma_*(f, b)$ in $\mathbb{C}_X(\mathbb{Z}[\pi])$.
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