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Algebraic and Geometric Surgery

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For Frank Auerbach

PREFACE

Surgery theory is a general method for the classification of manifolds. The theory was developed over the last 40 years and is now the standard tool in the study of manifolds of dimension $\geqslant 5$, starting with the Kervaire-Milnor classification of exotic differentiable structures on spheres. Unfortunately, surgery theory is quite difficult to learn, since it involves such a wide variety of algebraic and geometric techniques. Where to start?

This book aims to be an entry point to surgery theory for a reader who already has some background in topology. Familiarity with a book such as Bredon [10] or Hatcher [31] is helpful but not essential. The prerequisites of both the algebraic and the geometric parts of surgery are presented here, and enough machinery is developed to prove the main result of the theory : the *surgery exact sequence* computing the structure set of a differentiable manifold M of dimension $\geqslant 5$ in terms of the topological K-theory of vector bundles over M and the algebraic L-theory of quadratic forms over the fundamental group ring $\mathbb{Z}[\pi_1(M)]$. The surgery exact sequence is stated in Chapter 1, and finally proved in Chapter 13. Along the way, there are basic treatments of Morse theory, embeddings and immersions, handlebodies, Steenrod squares, Poincaré duality, vector bundles, cobordism, transversality, Whitehead torsion, the h- and s-Cobordism Theorems, algebraic and geometric intersections of submanifolds, the Whitney trick, Poincaré complexes, spherical fibrations, quadratic forms and formations, exotic spheres, as well as the surgery obstruction groups $L_*(\mathbb{Z}[\pi])$.

This text introduces surgery, concentrating on the basic mechanics and working out some fundamental concrete examples. It is definitely not an encyclopaedia of surgery theory and its applications. Many results and applications are not covered, including such important items as Novikov's theorem on the topological invariance of the rational Pontrjagin classes, surgery on piecewise linear and topological manifolds, the algebraic calculations of the L-groups for finite groups, the geometric calculations of the L-groups for infinite groups, the Novikov and Borel conjectures, surgery on submanifolds, splitting theorems, controlled topology, knots and links, group actions, stratified sets, index theory, $\ldots$. In other words, there is a vast research literature on surgery theory, to which this book is only an introduction.

The books of Browder [14], Novikov [65] and Wall [92] are by pioneers of surgery theory, and are recommended to any serious student of the subject. However, note that [14] only deals with the simply-connected case, that only a relatively small part of [65] deals with surgery, and that the monumental [92] is notoriously difficult for beginners, probably even with the commentary I had the privilege to add to the second edition. The papers collected in Ferry, Ranicki, and Rosenberg [24], Cappell, Ranicki, and Rosenberg [17] and Farrell and Lück [23] give a flavour of current research and include many surveys of topics in surgery theory. In addition, the books of Kosinski [42], Madsen and Milgram [45], Ranicki [70, 71, 74] and Weinberger [94] provide accounts of various aspects of surgery theory.

On the afternoon of my first day as a graduate student in Cambridge, in October, 1970 my official supervisor Frank Adams suggested that I work on surgery theory. This is still surprising to me, since he was a heavy duty homotopy theorist. In the morning he had indeed suggested three topics in homotopy theory, but I was distinctly unenthusiastic. Then at tea-time he proposed that I look at the recent work of Novikov [64] on surgery theory and hamiltonian physics. Novikov himself had not been permitted by the Soviet authorities to attend the Nice ICM in September, but Frank had attended the lecture delivered on Novikov's behalf by Mishchenko. The mathematics and the circumstances of the lecture definitely sparked my interest. However, as he was not himself a surgeon, Frank suggested that I actually work with Andrew Casson. Andrew explained that he did not have a Ph.D. himself and was therefore not formally qualified to be a supervisor of a Ph.D. student, though he would be willing to answer questions. He went on to say that in any case, this was the wrong time to start work on high-dimensional surgery theory! There had just been major breakthroughs in the field, and what was left to do was going to be hard. This brought out a stubborn streak in me, and I have been working on high-dimensional surgery theory ever since.

It is worth remarking here that surgery theory started in 1963 with the classification by Kervaire and Milnor [38] of the exotic spheres, which are the differentiable manifolds which are homeomorphic but not diffeomorphic to the standard sphere. Students are still advised to read this classic paper, exactly as I was advised to do by Andrew Casson in 1970.

This book grew out of a joint lecture course with Jim Milgram at Göttingen in 1987. I am grateful to the Leverhulme Trust for the more recent (2001/2002) Fellowship during which I completed the book. I am grateful to Markus Banagl, Jeremy Brookman (who deserves special thanks for designing many of the diagrams), Diarmuid Crowley, Jonathan Kelner, Dirk Schuetz, Des Sheiham, Joerg Sixt, Chris Stark, Ida Thompson, and Shmuel Weinberger for various suggestions.

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