CASTLE

361 feet above sea level

PALACE

119 feet above sea level

1 mile = 1760 yards = 5280 feet

The **Royal Mile** is built on the 'tail' of a glacier which swept past the Castle Rock, gouging out deep valleys either side. As we walk from the Castle to the Palace, the loss in height is 361 − 119 = [   ] ft.

The slope of the Royal Mile is

242 in 5280 = 1 in [   ]

which is nearly 5%.

MDCCXXII = [   ] AD

⭐ Look at the left hand gate post as you leave the Esplanade. Can you see this date? This is the date the **plaque** was made. Mark it on the time lines. Work out in Roman numerals the date 1591, the year many women condemned as 'witches' were burned.
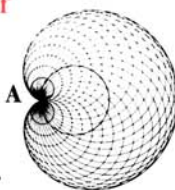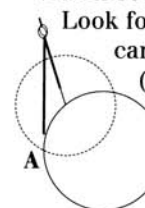
1591 = [   ]

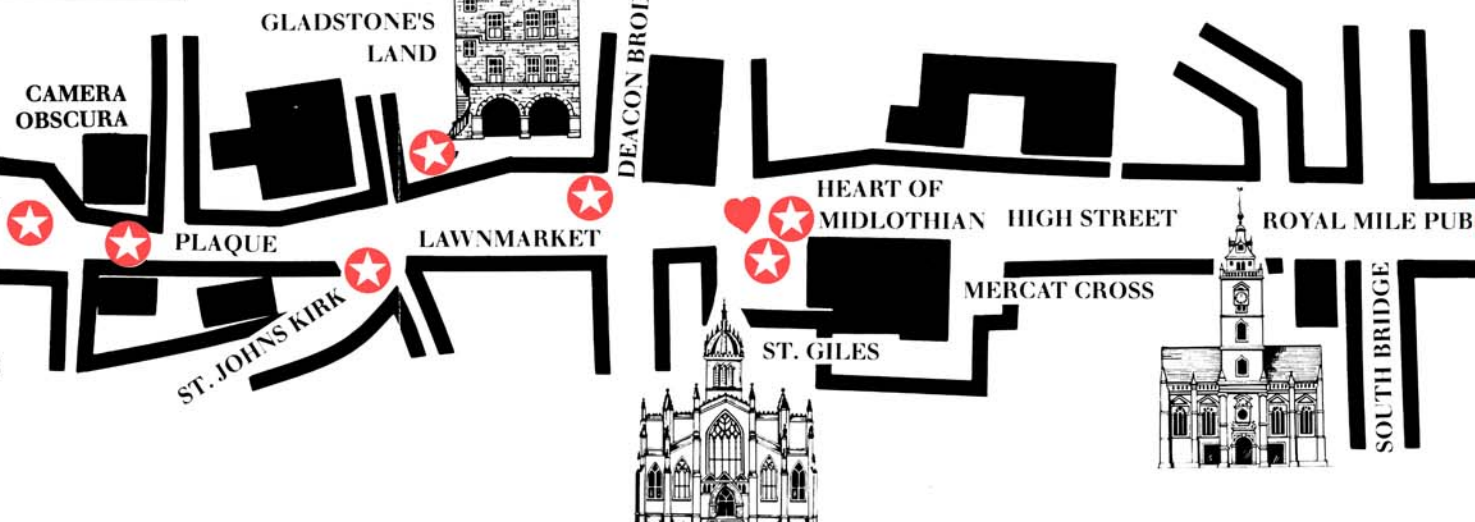⭐ When was **Gladstone's Land** built? Mark the date on the time lines.

⭐ At **Deacon Brodie's Tavern**, look out for two dimensional and three dimensional thistles. What do you think of the markings? Find out about Deacon Brodie. Mark the date of his execution on the time lines.

⭐ At the **Heart of Midlothian** stood the old tolbooth where citizens paid taxes. It was also a prison and many were executed outside including Deacon Brodie. Sir Walter Scott gave this grim place its name.

Look for the heart in the cobbles. You can draw a mathematical heart (cardioid) like this : draw a circle and mark a point A on it, now draw lots of other circles with centres on the original circle and passing through A.

GLADSTONE'S LAND

CAMERA OBSCURA

DEACON BRODIE'S

PLAQUE

LAWNMARKET

HEART OF MIDLOTHIAN

HIGH STREET

ROYAL MILE PUB

ST. JOHNS KIRK

MERCAT CROSS

ST. GILES

SOUTH BRIDGE

CASTLE ESPLANADE

⭐ Standing on the **Esplanade** you can see further out to sea than you can on the shore at Portobello. The distance in miles to the horizon is 1.225 × √(height above sea level, in feet). The height of the Esplanade is 361 feet.

Explanade
361 ft
Sea level
horizon
5960 miles
Centre of earth
Diagram not to scale
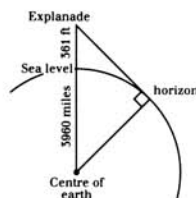
How far can you see? √361 = [   ]

20 × 1.225 = 24.5

1 × 1.225 = 1.225

19 × 1.225 = [   ]

Looking to the North East you might see Sanchar Point at Elie on the horizon. It is nearly 24 miles away.

⭐ Outside **St. John's Kirk** look around for some decorations to add to your tally chart.

Estimate the height of the **St. John's Kirk** spire by the 'Indian' method. Walk away from the church, bend down and look between your legs. The distance from the base of the tower is approximately the height of the spire when you can **just** see its top.
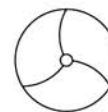
The height of the steeple is the same as the height of the Esplanade above Holyrood Palace.

⭐ Look at the East Window in **St Giles' Cathedral**. How many different symmetries can you identify?

1 line of reflection symmetry
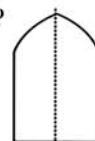
Rotational symmetry of order 3

Both line & rotational symmetry

Visit the Chepman Aisle, on the left of the organ. What is special about the number on the floor? Look at the patterns that form the mosaic floor. How many circles, hearts and thistles can you spot on the iron gates? In the window, look for the Napier coat of arms.

Look carefully at the organ case. It is constructed from interlocking squares. The square has four symmetries. The number four is associated with the four gospels, the pleasing musical interval 1:4, the four elements (air, water, earth and fire), the tetrahedron which has four corners and four faces. What other things occur in fours? In the Moray Aisle, to the right of the organ, look for two types of spirals!

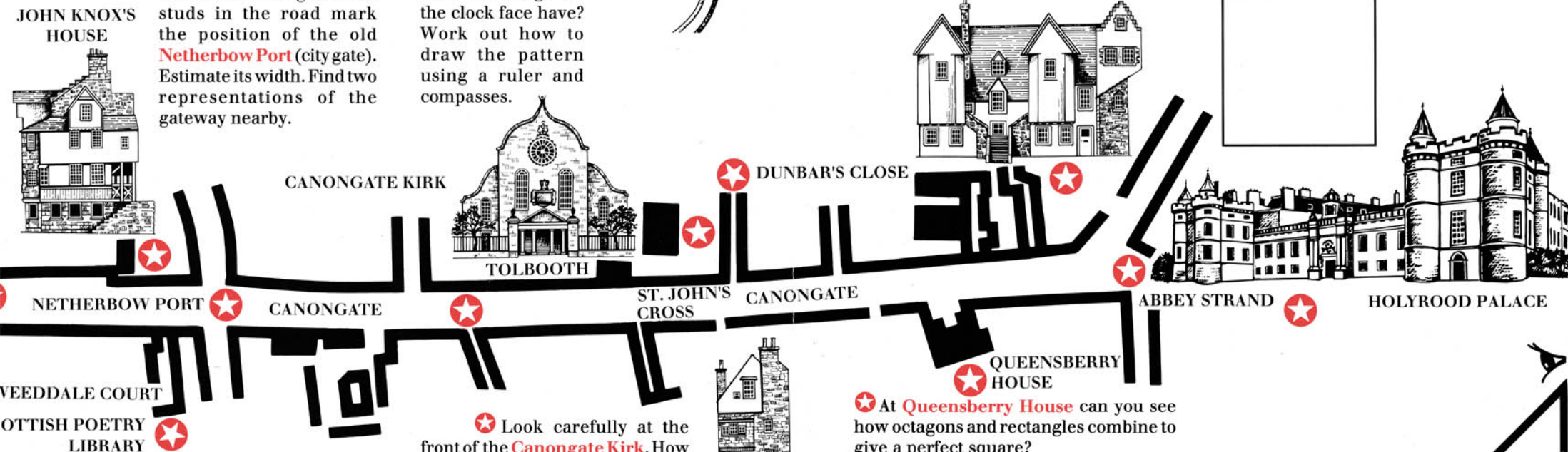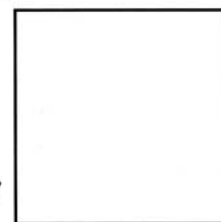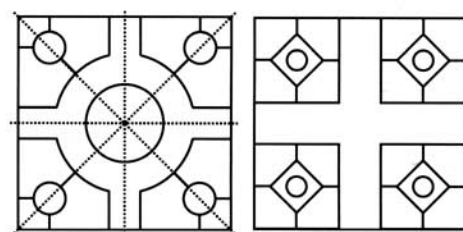⭐ How many panes of glass does the lantern outside the **Royal Mile public house** have? Look for the stars!

⭐ On **John Knox's house**, can you see Moses pointing to a Sun? He is reminding passers-by of God in Greek, Latin and English. What is the time by the sundial?

⭐ At the traffic lights brass studs in the road mark the position of the old **Netherbow Port** (city gate). Estimate its width. Find two representations of the gateway nearby.

JOHN KNOX'S HOUSE

⭐ Look at the clock on the **Canongate Tolbooth**. There are two dates 1884 and 1128. Look up 1128 on the time line. How many line and rotational symmetries does the design on the clock face have? Work out how to draw the pattern using a ruler and compasses.
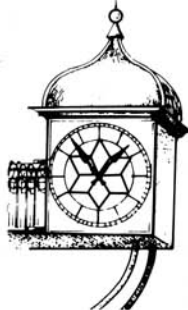
⭐ **Dunbar's Close Knot Garden** Seventeenth century gardeners liked to use geometrical patterns outlined in clipped hedges. The spaces in between were filled with flowers or coloured stones. How many lines of symmetry do the designs in Dunbar's Close have? Design your own Knot garden.
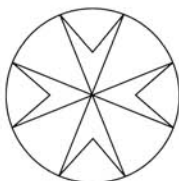
CANONGATE KIRK

DUNBAR'S CLOSE

TOLBOOTH

NETHERBOW PORT   **CANONGATE**

ST. JOHN'S CROSS   **CANONGATE**

ABBEY STRAND   **HOLYROOD PALACE**

QUEENSBERRY HOUSE

TWEEDDALE COURT

SCOTTISH POETRY LIBRARY

'Dreaming of apples on a wall,
And dreaming often, dear,
I dreamed that, if I counted all,
- How many would appear?'

A Brainteaser by Lewis Carroll (... of ten...)

⭐ Visit **Tweeddale Court**. Look at the sign for the **Scottish Poetry Library**. How many circles does it have? How many semicircles? How many areas is the sign divided into by the circles and semicircles?

⭐ Look for the **St John's cross** marked in the road. How many symmetries does it have?

⭐ Look carefully at the front of the **Canongate Kirk**. How many panes of glass are there in the circular window?

Work out
$1 + 1 \times 2 + 1 \times 2 \times 3 + 1 \times 2 \times 3 \times 4$.

Like 7 and 21, 33 is a Lucky Number. Now look at the porch over the front door from the centre of the gateway. By holding up your golden rectangle (tear off flap) between your eye and the porch, see that it has golden proportions.

Look at the window designs on the east side of the church. Work out how they could be constructed. Estimate the number of panes of glass for the whole building!
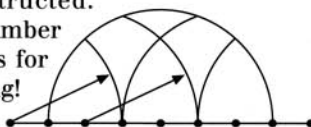
⭐ At **Queensberry House** can you see how octagons and rectangles combine to give a perfect square?

⭐ Coaches to London left from **White Horse Close**. It took four days to ride to London. The distance is 416 miles. What was the average speed of the horseman?
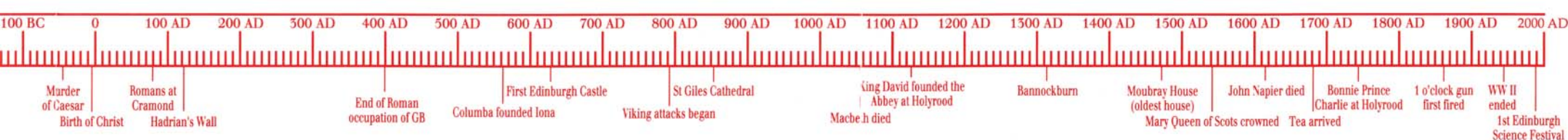
[   ] mph

⭐ Look for parts of the old gateway built into the wall on **Abbey Strand**. It is one mile from here to the drawbridge at the Castle.

⭐ **Holyrood Palace** was begun in 1498 by James IV. Mark the date on the time line.
Estimate the height of the cross on the crown above the doorway using the 'triangle method': Hold the triangle up and look along the sight line. Stand at the spot where the sight line points straight to the cross.
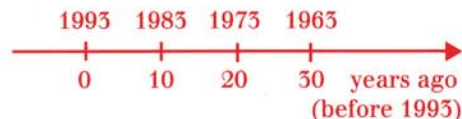
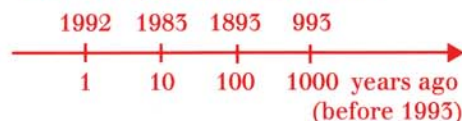Distance from door = height of palace

= [   ] paces.

Sight line

Vertical

Murder of Caesar · Romans at Cramond · End of Roman occupation of GB · First Edinburgh Castle · St Giles Cathedral · King David founded the Abbey at Holyrood · Bannockburn · Moubray House (oldest house) · John Napier died · Bonnie Prince Charlie at Holyrood · 1 o'clock gun first fired · WW II ended

Birth of Christ · Hadrian's Wall · Columba founded Iona · Viking attacks began · Macbeth died · Mary Queen of Scots crowned · Tea arrived · 1st Edinburgh Science Festival

TIME LINES

This is an arithmetic time line:

1993 1983 1973 1963

0 10 20 30 years ago
(before 1993)

The equal spaces between the points represent adding 10 years.

This is a logarithmic time line:

1992 1983 1893 993

1 10 100 1000 years ago
(before 1993)

It can represent very very large numbers more easily. Here the equal spaces between the points represent multiplying by 10. Working out the positions of the powers of 10 (100, 1000, 10000 etc) on the logarithmic number line is easy. The positions of other numbers like 2, 6.5, 20 were first worked out by John Napier.

Mark the year of your birth on each time line.

This mathematical trail was devised by the **Edinburgh Centre for Mathematical Education.**

The pamphlet was designed by AVS at the **University of Edinburgh.**

Building illustrations by Dorothy Stewart.

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JOHN NAPIER
BARON OF MERCHISTON
1550-1617

Many notable mathematicians have lived and worked in Edinburgh. James Gregory designed a reflecting telescope in the early 18th century. A little later Colin Maclaurin provided the mathematical basis for the management of pension funds. In this century Edmund Whittaker, Alexander Aitken and Arthur Erdelyi made outstanding contributions to mathematical research. The most famous Edinburgh mathematician is probably John Napier. His discovery of logarithms and his invention of a computational aid called Napier's Bones greatly reduced the labour and difficulty of the complex numerical calculations needed for the advancement of science and technology from the beginning of the seventeenth century up to the middle of the twentieth century.

DESIGNS

Regular shapes like hexagons with sides of equal lengths are pleasing to the eye and so are often used in decorations. Shapes with line or rotational symmetries also have eye appeal.
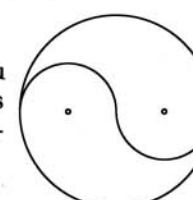
To construct a **hexagon**...

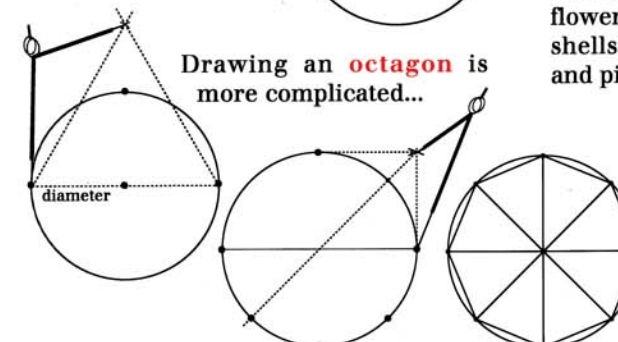It has 6 line symmetries and rotational symmetry of order 6.
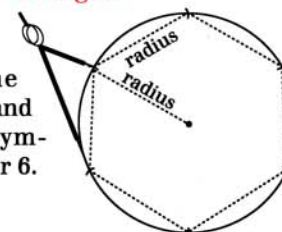
The **hexagon** can be used to construct a **trefoil**, which has 3 line symmetries and rotational symmetry of order 3.
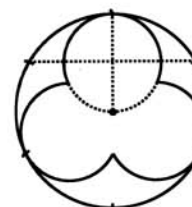
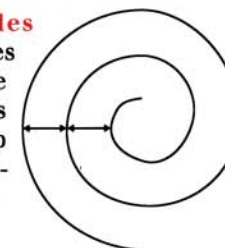Using compasses you can construct shapes which just have rotational symmetries.
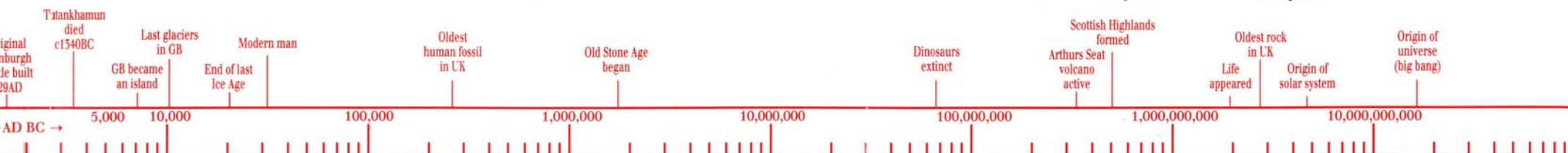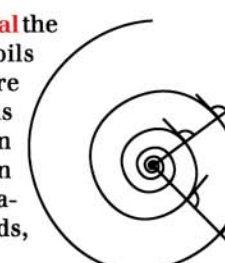
Drawing an **octagon** is more complicated...

diameter

Spirals have been used as decoration since at least 25000 BC.

Here are two types of spiral:

On an **Archimedes Spiral** the distances between successive coils are fixed. This is like a coiled up rope or an old fashioned watch spring.

On a **Descartes Spiral** the angles between coils and radial lines are fixed. These spirals occur naturally on thistle heads, sunflowers, fir cones, seashells, flower heads, and pineapples.

Tutankhamun died c1340BC · Last glaciers in GB · Modern man · Oldest human fossil in UK · Old Stone Age began · Dinosaurs extinct · Scottish Highlands formed · Oldest rock in UK · Origin of universe (big bang)

Original Edinburgh Castle built 29AD · GB became an island · End of last Ice Age · Arthurs Seat volcano active · Life appeared · Origin of solar system

GOLDEN RATIO $\frac{1+\sqrt{5}}{2} = 1.618033988\ldots$

In the regular pentagram, the ratio of the length AB to the length BC is the Golden Ratio.
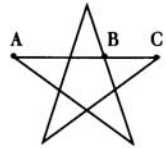
A rectangle whose sides have lengths in golden ratio is pleasing to the eye and architects have used it in designing buildings.

This flap has the shape of a golden rectangle. If we cut off a square from a golden rectangle, the remaining rect-
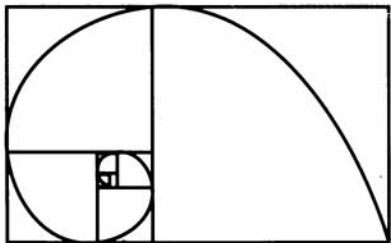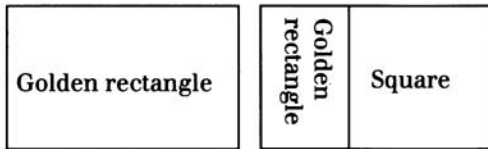
Golden rectangle	Golden rectangle	Square

angle is another golden rectangle. This can be repeated again and again, each time yielding a (smaller) golden rectangle.
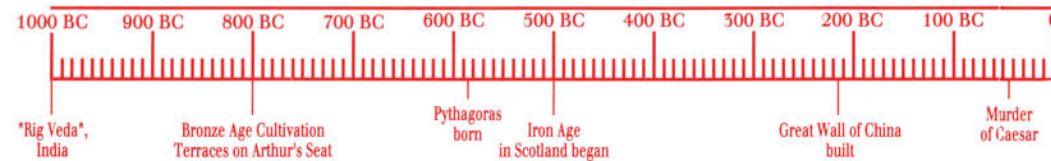
The picture shows a spiral through the corners of a succession of golden rectangles. It is a Descartes spiral and occurs naturally on fir cones and thistle heads (best seen by viewing them from below). Check any thistle decorations you find to see if they have the curved lines of the spirals on them.

ARITHMETIC TIME LINE

1000 BC	900 BC	800 BC	700 BC	600 BC	500 BC	400 BC	300 BC	200 BC	100 BC	

"Rig Veda", India — Bronze Age Cultivation Terraces on Arthur's Seat — Pythagoras born — Iron Age in Scotland began — Great Wall of China built — Murder of Caesar

DECORATIONS

Many buildings are decorated with mathematical shapes like spirals or octagons. Some incorporate cones or pyramids as part of the structure. How many shapes can you spot as you walk along the trail? Make a tally like this ⦀ ⫿

			Total
Hexagons			
Octagons			
Stars			
Trefoils			
Quatrefoils			
Hearts			
Spirals			
Pyramids			
Cones			
Thistles			

copus

LOGARITHMIC TIME LINE

YEARS AGO (BEFORE 1993)

Gulf war 1991AD — Scandic Crown hotel built 1990AD — 1st moonlanding 1969AD — WW II ended 1945AD — WW I began 1914AD — Camera Obscura 1855AD — Union of parliaments 1707AD — Union of Crowns 1605AD — King David founded Holyrood Abbey 1128AD — Original Edinburgh Castle built 629AD — Tutankhamun died c1340BC

1	2	3	4	5	6	10	20	30	100	200	500	1,000	← AD BC →	5