

Wave–flow interactions

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II. Case for support

Background

The fluids encountered in environmental and industrial applications are often endowed with restoring mechanisms which lead to the propagation of waves. Compressibility and sound waves, density stratification and internal-gravity waves, differential rotation and Rossby waves are just a few examples of such restoring mechanisms and associated waves. When they propagate in a fluid, waves can interact with the fluid motion in a variety of ways. Crucially, it is a two-way interaction: fluid motion can generate waves, but waves can also force fluid motion. The generation of sound by turbulence, and the generation of along-shore currents by surface-wave breaking provide familiar examples for this. Because waves and fluid flows often have very different spatial and temporal scales, and because they involve very different physical processes (large-distance propagation vs. turbulent cascades, for example), the modelling of wave–flow interactions often poses a considerable technical and conceptual challenge. The purpose of this Network is to address these challenges by bringing together a number of UK researchers with expertise in different aspects of the problem: mathematical modelling, numerical simulations, and laboratory experiments. The group will be strengthened by the inclusion of a few associate members from overseas who, apart from bringing in their unique expertise, will ensure the international visibility of the Network.

The creation of the Network appears particularly timely: research on wave–flow interactions is currently buoyant in the UK, with several groups working on different aspects of the topic, especially in the context of geophysical (stratified, rotating) fluids. Their contributions span a whole spectrum of mathematical methods, but so far the interactions have been limited. This is a reflection of a dangerous trend: research nowadays has become increasingly compartmentalised. Particular applications have become nearly the sole foci of studies, with less attention being paid to the underlying mathematical affinity of the physical processes. The Network aims at reversing this trend by facilitating collaboration between leading research groups. The complementarity of approaches among these groups is ideal for fruitful collaborations over a wide range of problems.

At the practical level the main aim of the Network, then, is to foster collaborative work between its members, especially by providing opportunities for face-to-face contact. To this end, four meetings are planned, which will gather the entire Network membership. Smaller-scale workshops will also be organised, on a flexible basis, to promote hands-on collaborations. The meetings will also provide opportunities to invite researchers working on issues of wave–flow interactions in contexts not necessarily covered by the Network memberships. The interaction between quasi-Newtonian flow and gravitational waves in relativistic media, the interaction between Alfvén waves and MHD flows, and the interaction of quantum vortices with waves in superfluids are examples of such contexts. The relevance of particular mathematical techniques, such as multi-scale numerical methods and stochastic models, will also be explored through the invitation of a few selected speakers. This policy will foster cross-disciplinary interactions and, if productive, enable us to extend the Network membership.

Another key feature of the Network activities will be the close involvement of PhD students. Students supervised by members will attend the meetings which will purposely include introductory

lectures reviewing the main tools employed in studies of wave–flow interactions. The students participation, through oral and poster presentations, will also be encouraged. Thus the Network will provide a unique opportunity for a number of PhD students to broaden the field of their expertise, and to interact, at an early stage, with a scientific community far more extensive than their own research environment.

Network membership

Initially, the Network will have 16 members, based in 13 institutions. Their names, positions, and research expertises are given in Appendix A. More details on their profiles, including relevant publications, can be found on their web pages, also listed. The Network will of course remain open to new members throughout its duration. In particular, PhD students supervised by members will be involved in the Network activities. In addition to the UK-based members, overseas associate members will participate in some activities. To date we have identified 10 associate members, from 5 different countries. Their names and affiliations are listed in Appendix A.

Research projects

Several research projects, which will start collaborative work, have been identified. These are described below to provide a sample of the problems which will be considered by the Network. (The initials refer to the Network members involved in the first instance.) We will, however, strongly encourage the development of new research themes, particularly those at the interface between the current interests of the Network members.

Wave–flow identification (MC, DGD, AM, PLR, IR, RKS, JV, DW). For both conceptual and practical purposes, it is essential to be able to separate all dynamical fields describing fluids into wave and flow (or vortex) components. At a linear level, this is simple: the flow is identified as the zero-frequency mode, whilst the waves have non-zero frequency. At a non-linear level, the wave–flow identification generally relies on time-scale separation. Working asymptotically with the associated parameter provides a means (through the definition of a slow manifold and of a projection) of isolating slow and fast components of the dynamics, then identified as (balanced, vortex) flow and waves, respectively. This procedure is well understood for finite-dimensional systems [8], for which one can establish the approximate invariance of slow manifolds to exponential accuracy. For infinite-dimensional systems such as fluids, however, the problem remains largely open. This is a subject of current research [12], which will be pursued through the Network.

The asymptotic approach has the merit of providing a sound mathematical basis to the wave–flow separation problem. Yet, from the practical viewpoint, it suffers from two drawbacks. The first is complexity: asymptotic procedures lead to a rapid increase in the number of terms involved with the order of accuracy, and even in idealised contexts it is hardly feasible to compute numerically slow manifolds accurate to more than the first few orders. The second drawback is the limitation to a formal time-scale separation when it is well documented that waves and balanced flow can remain well separated even in the absence of such separation. Research carried out by Network members will seek to remedy these disadvantages by developing practical definitions separating flows from waves which remain simple enough for practical implementation. Optimal Potential Vorticity (OPV) balance developed recently

by Dritschel and co-workers [16] is one such definition. The Network will make it possible to compare this with other approaches, test them in numerical models and experimental data [7, 10], and explore their applicability to a range of wave–flow problems. Work of a more theoretical nature will also be carried out to examine the conditions of existence of balanced solutions and relate their breakdown to wave generation.

Spontaneous wave generation (MC, DGD, RG, GE, AM, PLR, IR, SN, RKS, DW). Fluid flows, even when suitably initialised, or balanced, do generate waves. This is best understood for sound waves: the asymptotic theory of Lighthill’s radiation describes the resonant generation of large-scale (hence slow) sound waves by vortical flow motion. Similar mechanisms cause the generation of gravitational waves by mass motion in general relativity, and the generation of sound waves in Bose–Einstein condensates and superfluids. An asymptotic theory of Lighthill-like wave radiation can be developed to estimate not only the wave emission, but also their feedback on the flow; this leads to reduced models which describe the dynamics in terms of the flow variables only, whilst accounting implicitly for the dissipative effect of wave radiation [4]. Models of this kind are used extensively in the general-relativity context [2], but their potential for fluid dynamics remains largely unexplored. The members of the Network will be ideally placed to assess this potential and develop applications.

Lighthill radiation is relevant to non-dispersive waves and more generally to waves which are slow in the limit of large wavelengths. In some applications, the dispersion properties and flow regime lead to a frequency gap between slow flow and fast waves of all wavelengths. An example of this is provided by gravity waves in the presence of strong rotation. In this case, the development of an asymptotic theory is more delicate because the wave generation is expected to be exponentially weak [12]. The techniques of asymptotics beyond all orders, whose applications to partial differential equations are just emerging, need therefore to be employed. To date, this has only been done for toy models but it needs to be extended to more complex models [15, 14]. Collaborative work in this direction will be carried out by the Network members.

In many flows, the time-scale separation between waves and flows fails to exist in some regions of the flow. Recent numerical simulations demonstrate that significant wave generation does occur in these circumstances. These simulations require sophisticated numerical techniques (for time integration and for diagnosis) as well as major computing power, and only recently have reliable predications been brought within reach. Building on this progress, the Network members will be able to examine wave generation in complex flows. A long-term aim, here, is to answer the many open questions regarding wave generation by turbulent flows. This includes the scaling of the amplitude of this wave generation with the time-scale separation parameter, and the quantification of the dissipative effect of wave radiation on the flow. One of the roles of the Network will be to facilitate the free exchange of, and coordination of, numerical algorithms, software and data. This will enable direct comparisons between methods and ease the implementation of new ideas.

Wave–mean flow interaction theory (SB, CJC, RG, GE, MEM, PLR, SN, VIS). In many applications, waves are distinguished from the fluid flow not because of their high frequency, as described above, but because of their small spatial scale. The flow can then be identified as a mean flow through some averaging procedure. The nonlinear interactions between small-scale waves and large-scale mean flows that appear in this case are important in many fluid

dynamical situations and also in other physical sciences such as condensed matter physics. For example, the angular momentum transport mediated by small-scale inertia–gravity waves in the atmosphere is well known to contribute significantly to the global circulation of the atmosphere and therefore to the dynamics of our climate. Similar comments apply to the vertical mixing induced by breaking internal waves in the deep ocean because this mixing is intimately connected with the global ocean circulation. Nevertheless, these waves are far too small in spatial scale to be resolved by even the most powerful supercomputers and hence their impact on the resolved flow in these models needs to be ‘parameterized’ based on a combination of observations, theory, and numerical simulation. This is the general area of wave–mean interaction theory.

However, the relevant theoretical and numerical issues are reasonably well understood only in the context of small-amplitude waves propagating on a symmetric mean flow, i.e. a flow that does not depend on one spatial coordinate. The situation is much less understood in the more general context of *asymmetric* mean flows such as meandering jets in the atmosphere and in the ocean. There are theoretical suggestions and early numerical indications [1, 3, 9, 6, 5] that the dispersive wave dynamics as well as the nonlinear impact of the waves onto the mean flow are both significantly altered in this more general, and more realistic, setting. This should be of fundamental importance for the design of improved parametrization schemes to be used in numerical models.

The participants of this Network are well placed to make significant progress on the fundamental theoretical and numerical issues that arise in this area. Examples include targeted studies of wave propagation through a complex straining flow, the nonlinear interactions of the waves with the mean flow, and the development of stochastic parameterizations. Of particular interest is the role of wave breaking, which in the standard setting of symmetric mean flows is necessary for significant wave-induced mean-flow changes but appears to be much less relevant in the more general setting. This is a topic of contemporary research.

Water-wave–current interaction (SB, CJC, RG, SN, VIS, VV). Studies of the interactions between water waves and currents have a long history, but progress has been painfully slow and many fundamental aspects of these interactions remain poorly understood. We focus here on just two problems with the greatest potential for practical applications.

(i) Because of a sharp fall in the tolerance of loss of life at sea, the recent years have seen a surge in studies concerned with anomalously large waves in the ocean, called *freak* or *rogue* waves. The present progress in the mathematical description of freak waves is almost entirely confined to situations without currents, even though the experimental evidence indicates that the freak waves are much more frequent in the presence of shear currents. Identifying and modelling the specific mechanisms responsible for the anomalous amplification of particular wave crests is the challenge we will address. In the absence of currents, the prime mechanism leading to the formation of anomalous waves was found to be the modulational instability of narrow-band wavetrains. The study of the various regimes of the modulational instabilities of slightly non-potential nonlinear waves propagating on currents with vertical shear therefore acquires the highest priority. The present understanding of these regimes is patchy [11], and what is needed now is a detailed analysis of the existing asymptotic models from the perspective of freak-wave formation, as well as the development of new more sophisticated models supported by numerical simulations and, where possible, laboratory experiments.

(ii) To mitigate the devastation caused by tsunamis, one of the first priorities is to create adequate early warning systems. A very attractive solution to this challenge would be a space borne system able to detect tsunami waves in the open ocean. During the Sumatra tsunami, it was discovered that a tsunami wave in the open ocean can create variations of surface roughness detectable from space [13]. Understanding this mathematically requires to study the problem of interaction between capillary waves and the very gentle current caused by the tsunami. We will investigate which mechanisms might cause the observed variations in the characteristics of capillary ripples, and develop mathematical model to describe these variations for a wide range of weather conditions.

Activities and dissemination

The main events organised by the Network will be series of meetings: over three years, 4 main meetings and 3 small-scale workshops are planned. The main meetings, running over three days, will provide the opportunity for the whole membership to gather, present the results achieved, and plan further collaborative work. External speakers will be invited to describe either research related to wave–flow interactions in areas not covered by the Network membership, or novel techniques (especially numerical) with potential applicability to the study of wave–flow interactions. The first meeting will take place at the start of the Network; the other three will take place at the end of each year. The four workshops will be three-day meetings for smaller groups of Network members devoted to one or two the specific research projects. These workshops are designed for hands-on collaborative work, and adequate facilities (computing facilities in particular) will be made available.

The results obtained in the framework of the Network will be posted on a dedicated website, maintained by the PI. This website will allow easy exchange of data, software, papers, notes etc.; it will include a forum, accessible to Network members only, for efficient communication. The website will enhance the visibility of the Network in the broader scientific community. The usual means of scientific communication (papers, talks) will be of course also be used by the Network members.

Benefits

A general benefit of the Network will be to build a cohesive community of researchers currently addressing independently different aspects of the wave–flow problem. This will bring to light the commonalities of their work, stimulate new collaborations and, more broadly, energise the UK research in this area. The close involvement of several high-profile overseas researcher will ensure the international impact of the Network.

The Network will also play an important role in the scientific development of the postgraduate students who will take part, by exposing them to the breadth of problems and techniques involved, and giving them the opportunity to interact with researchers from outside their university environment.

The research projects described in this proposal will lead to some more specific benefits. For instance, our findings are likely to influence ways of parametrising the mean effect of gravity waves on vortical motion in the atmosphere and oceans. Present large-scale models can neither focus the necessary numerical resolution nor apply the proper mathematical framework to determine this effect with any confidence. But beyond this type of application-specific impact, we anticipate the development of new mathematical techniques with broader applicability. The problem of wave–

flow interaction is mathematically subtle and hence highly challenging. It requires the development of new analytical and numerical techniques, which may find widespread application in related problems, e.g. in MHD, etc. Theoretically, balance is deeply connected with the structure of PDEs, for instance their elliptic and/or hyperbolic character, their “memory” of earlier flow states, and potentially even their existence and uniqueness.

Resources

The main request is for the funding of the 4 planned 3-day meetings of the Network. For each meeting, we estimate the cost of travel and subsistence for 20 UK participants at £7800; the attendance of 5 overseas participant will also be supported, at an estimated cost of £3700. It is expected that most of the associate overseas member will cover their attendance costs. Thus, we request £46000 to hold the 4 meetings. The cost of 3 workshops planned for subsets of the Network involved in specific collaborative projects is estimated at £11700. A tentative timetable for the meetings and workshops is indicated in the attached workplan. The first meeting will be held in Edinburgh and organised by the PI; the subsequent ones will be held in the institutions of some of the other Network members.

The PI will manage the Network, develop and maintain a dedicated web page, and co-ordinate the main activities. Funds are requested to cover his salary costs for a modest 150 hours over the 3 years of the project. Salary costs for 50 hours of the CoI’s time are also requested for help with the Network coordination.

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Appendix A: Network membership

Name	Position	Research interests	Home page
S Belcher	Professor, University of Reading	Wave-turbulence interactions, air-sea interactions	www.met.rdg.ac.uk/~bl_met/people/stephen.html
C J Chapman	Professor of Applied Mathematics, University of Keele	High-speed flows, acoustic waves	www.keele.ac.uk/depts/ma/people/cjc/
M Cullen	Professor, Met Office	Atmospheric dynamics, data assimilation, non-linear PDEs	
D G Dritschel	Professor of Applied Mathematics, University of St Andrews	Geophysical wave and vortex dynamics, theory and computation	www.vortex.mcs.st-andrews.ac.uk/~dgd/
R Grimshaw	Professor of Applied Mathematics, University of Loughborough	Nonlinear waves in fluids; solitary waves; wave-mean flow interactions; vortex dynamics	www-staff.lboro.ac.uk/%7Emarhjg/
G Esler	Lecturer in Applied Mathematics	Geophysical fluid dynamics, middle atmosphere dynamics, flow over topography	www.atm.damp.cam.ac.uk/people/jge1000/
M E McIntyre	Professor of Applied Mathematics, University of Cambridge	Atmospheric dynamics	www.atm.damp.cam.ac.uk/people/mem/
A R Mohebalhojeh	NERC Advanced Research Fellow, University of St Andrews	Geophysical wave and vortex dynamics, theory and computation	http://www.vortex.mcs.st-and.ac.uk/~arm
S Nazarenko	Reader in Mathematics, University of Warwick	Turbulence, waves and vortices	www.maths.warwick.ac.uk/snazar/
P L Read	Professor of Atmospheric, Oceanic & Planetary Physics, University of Oxford	Geophysical fluid dynamics, baroclinic and barotropic instabilities, geostrophic turbulence	www.atm.ox.ac.uk/user/read/
I Roulstone	Professor of Applied Mathematics, University of Surrey	Hamiltonian fluid mechanics, atmospheric dynamics	www.maths.surrey.ac.uk/personal/st/I.Roulstone/
R K Scott	Lecturer in Applied Mathematics, University of St Andrews	Geophysical wave and vortex dynamics, theory and computation	www.vortex.mcs.st-and.ac.uk/~rks
V I Shrira	Professor of Applied Mathematics, University of Keele	Nonlinear waves, fluid mechanics, geophysical fluid dynamics	www.keele.ac.uk/depts/ma/people/vs.html
J Vanneste	Reader in Applied Mathematics, University of Edinburgh	Fluid mechanics, asymptotic methods.	www.maths.ed.ac.uk/~vanneste
V Vlasenko	Reader, School of Earth, Ocean & Environmental Sciences	Nonlinear waves, wave/flow/topography interactions, theory and computation	www.plymouth.ac.uk/pages/dynamic.asp?page=staffdetails&id=vvlasenko
D Wirosoetisno	Lecturer, University of Durham	Fluid dynamics, partial differential equations, applied analysis	http://maths.dur.ac.uk/~dma0dw/

Overseas associate members: P Bartello (McGill, Canada), O Bokhove, (Twente, The Netherlands), O Bühler (Courant Institute, USA), W Dewar (FSU, USA), T Dunkerton (Northwest Research Associates, USA), T Haine (Johns Hopkins, USA), G Caulliez (IRPHE, France), L Maas (NIOZ, The Netherlands, R Plougonven (LMD, France), I Yavneh (Technion, Israel), V Zeitlin (LMD, France).