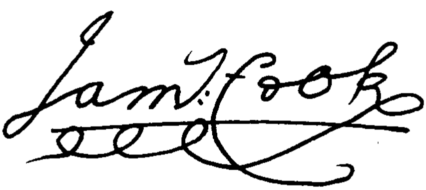
JAMES COOK MATHEMATICAL NOTES

Volume 4 Issue Number 40

June 1986



Editor and publisher: B. C. Rennie
Address as from later in 1986: 66, Hallett Road, Burnside, SA 5066, Australia.

The "James Cook Mathematical Notes" is published in three issues per year, in February, June and October. The subscription rate for one year (three issues) is (in Singapore dollars):

In Singapore (including postage) $20
Elsewhere (including air mail postage)$30

Subscribers in countries with no exchange control (such as Australia, the United Kingdom and the U.S.A.) may send cheques in their own currencies, the amount calculated at the current exchange rate. The rates at the time of writing (16th June, 1986) are

$1 (Singapore) = 64 cents (Australian)
= 30 pence (U.K.)
= 45 cents (U.S.A.)

All currencies are acceptable, please make cheques payable to B.C.Rennie.

As your JCMN is posted to you directly from the printer in Singapore it is not practicable to include reminders of subscriptions becoming due.

The Editor must offer apologies for the lateness of this issue; please send contributions for the October issue as soon as you can to the address above.

CONTENTS

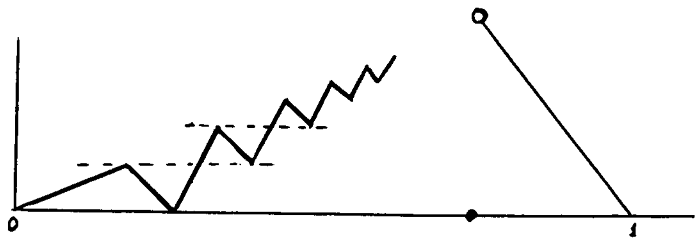
k - FOLD REAL FUNCTIONS

(JCMN 39, p.4174)

Jordan Tabov

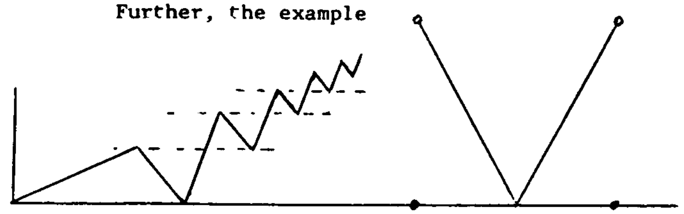
The article in the previous issue establishes that $1 \leq \nu(k) \leq \infty$ for $k = 4, 5, 6, \ldots$

The following example



constructed on the base of example I of the above paper (p.4179) shows that $\nu(4) \leq 1$; consequently $\nu(4) = 1$.

Further, the example



shows that $\nu(5) \leq 2$; consequently $1 \leq \nu(5) \leq 2$.

Now, it is clear that, in general,

$$1 \leq \nu(n) \leq \left[\frac{n-1}{2}\right] \quad \text{for } n > 2.$$

I am able also to prove that $\lambda(2) = \nu(2) = \infty$.

Here I will sketch the proof of $\lambda(2) = \infty$; the second result ($\nu(2) = \infty$) may be obtained by similar considerations.

Suppose that $f = f(x)$ is a 2-fold function on the interval $(0,1)$, which has exactly $n < \infty$ points of discontinuity, namely $a_1 < a_2 < \ldots < a_n$; denote by A the set of all a_i. According to the theorem 1 of the paper quoted above (JCMN p.4174) A is not empty.

We will show that there is a finite set of real numbers $D = \{0 = d_0 < d_1 < d_2 < \ldots < d_p = 1\}$ such that

(1) $f(x)$ is continuous and monotone on each (d_{i-1}, d_i), $i = 1,2,\ldots,p$;

(2) if the ranges R_i and R_j of f in (d_{i-1}, d_i) and (d_{j-1}, d_j) for some i and j have a common value, then $R_i \equiv R_j$.

<u>Lemma</u>. If f is continuous in $(a,b) \subsetneq (0,1)$, then f has (if any) no more than 2 extrema in (a,b).

The proof is quite standard.

Since A is finite, it follows from this lemma that the set

$$M = \{ m \in (0,1) \mid f \text{ has a local extremum at } m \}$$

is finite. Denote by B the set $A \cup M \cup \{0,1\}$; let $B = \{0 = b_0 < b_1 < \ldots < b_m = 1\}$. For any $i = 0,1,2,\ldots,m$ let

$$\lim_{\substack{x \to b_i \\ x < b_i}} f(x) = B'_i \quad , \quad \lim_{\substack{x \to b_i \\ x > b_i}} f(x) = B''_i$$

The set

$$C = \{ c \in (0,1) \mid f(c) = \text{either } B'_i \text{ or } B''_i \text{ for some } i \}$$

is finite, since B is finite and f is 2-fold.

Now we can define D as $B \smile C$. It is clear that D possesses the required properties (1) and (2).

Using the set D, we will come to a contradiction. First, since f is 2-fold, then the range R_i of f on an arbitrary interval (d_{i-1}, d_i) must coincide with the range R_j of f on exactly one of the other such intervals; therefore p must be even. However, f must attain on the set

$$\{d_1, d_2, \ldots, d_{p-1}\}$$

values different from the values of $\bigcup_{j=1}^{p} R_j$, and since f is 2-fold, $p-1$ must be even - a contradiction, since p and p-1 cannot be even simultaneously.

* * *

What about 2-fold complex functions?

BINOMIAL IDENTITY 18

Jamie Simpson

Show that if n is a positive integer

$$\sum_{j=1}^{n} \binom{n}{j} (-1)^j j^n = (-1)^n n!$$

TRIANGLE IDENTITIES: A PAIR OF ONE-PARAMETER FAMILIES

Andrew P. Guinand

Notations.

Let ABC be a triangle, O the circumcentre, H the orthocentre, N the nine-point centre, I the incentre and I_a the excentre opposite vertex A. Let s equal the semiperimeter, R the circumradius, r the inradius, and r_a the exradius about I_a. Use Σ and Π respectively, to indicate cyclic sums and products over the angles A, B, and C of the triangle.

In addition, let K be the mirror image of H in O, and let K_c be the point on the Euler line OH that divides OH in the ratio 1:2c.

Introduction

Two results in recent literature are:

(i) Blundon's Theorem .[2], [3]

At least one of the angles of the triangle has measure θ if $s = 2R\sin\theta + r\cot\frac{1}{2}\theta$.

(ii) The following identity arose in investigating the "acentric lacuna".[4]

$$(4R+r)^2 - IK^2 = 6R^2\Pi(1+\cos A).$$

The present note discusses a one-parameter family of identities associated with each of (i) and (ii) as follows:

(I) For any angle θ

$$s\sin\tfrac{1}{2}\theta - 2R\sin\theta\sin\tfrac{1}{2}\theta - r\cos\tfrac{1}{2}\theta = 4R\Pi\sin\tfrac{1}{2}(\theta - A).$$

(II) For any real c

$$\tfrac{1}{2}(1-c)(2c+1)^2 IK_c^{\,2} - \{2c(1-c)R - r\}^2$$
$$= 2R^2(1-2c)\Pi(c - \cos A).$$

We need the following standard trigonometric results [3].

$$r = R(\sum \cos A - 1) \tag{1}$$

$$s = R \sum \sin A \tag{2}$$

$$IN = \tfrac{1}{2}(R - 2r) \tag{3}$$

$$OI^2 = R(R - 2r) \tag{4}$$

$$ON^2 = \tfrac{1}{4}OH^2 = \tfrac{1}{4}R^2(1 - 8 \prod \cos A) \tag{5}$$

$$\sum \cos^2 A = 1 - 2 \prod \cos A \tag{6}$$

Proof of (I).

Writing $F(\theta) = s \sin \tfrac{1}{2}\theta - 2R \sin \theta \sin \tfrac{1}{2}\theta - r \cos \tfrac{1}{2}\theta$ and substituting (1) and (2), we get

$$\begin{aligned} F(\theta) &= R \left\{ \sin \tfrac{1}{2}\theta (\sum \sin A) - (\cos \tfrac{1}{2}\theta - \cos \tfrac{3}{2}\theta) - \cos \tfrac{1}{2}\theta (\sum \cos A - 1) \right\} \\ &= R \left\{ \cos \tfrac{3}{2}\theta - \sum (\cos \tfrac{1}{2}\theta \cos A - \sin \tfrac{1}{2}\theta \sin A) \right. \\ &= R \left\{ [\cos \tfrac{3}{2}\theta - \cos(\tfrac{1}{2}\theta + A)] - [\cos(\tfrac{1}{2}\theta + B) + \cos(\tfrac{1}{2}\theta + C)] \right\} \\ &= 2R \left\{ \sin(\theta + \tfrac{1}{2}A) \sin \tfrac{1}{2}(A - \theta) - \cos \tfrac{1}{2}(\theta + B + C) \cos \tfrac{1}{2}(B - C) \right\} \end{aligned}$$

But $\cos \tfrac{1}{2}(\theta + B + C) = \cos \tfrac{1}{2}(\theta + \pi - A) = -\sin \tfrac{1}{2}(\theta - A)$, and

$\sin(\theta + \tfrac{1}{2}A) = \sin(\theta + \tfrac{1}{2}\pi - \tfrac{1}{2}B - \tfrac{1}{2}C) = \cos(\tfrac{1}{2}B + \tfrac{1}{2}C - \theta)$.

Hence

$$\begin{aligned} F(\theta) &= -2R \sin \tfrac{1}{2}(\theta - A) \left\{ \cos(\tfrac{1}{2}B + \tfrac{1}{2}C - \theta) - \cos \tfrac{1}{2}(B - C) \right\} \\ &= 4R \sin \tfrac{1}{2}(\theta - A) \sin \tfrac{1}{2}(\theta - B) \sin \tfrac{1}{2}(\theta - C), \end{aligned}$$

as required.

Proof of (II).

The nine point centre N is the midpoint of OH, and K_c divides OH in the ratio 1:2c. Hence K_c divides ON in the ratio $2:2c - 1$, and so, by Stewart's theorem [1],

$$(2c - 1).OI^2 + 2.IN^2 = (2c + 1).IK_c^{\,2} + \left\{ 2(2c - 1)/(2c + 1) \right\}.ON^2.$$

Substituting (3), (4) and (5) and rearranging

$$\begin{aligned} &\tfrac{1}{2}(1 - c)(2c - 1)^2.IK_c^{\,2} - \left\{ 2c(1 - c)R - r \right\}^2 \\ &\quad = \tfrac{1}{2}(1 - c)(4c^2 - 1)R(R - 2r) + \tfrac{1}{4}(1 - c)(2c + 1)(R - 2r)^2 \\ &\quad\quad - \tfrac{1}{4}(1 - c)(2c - 1)(1 - \tfrac{1}{4}\prod \cos A)R^2 - \left\{ 2c(1 - c)R - r \right\}^2. \end{aligned} \tag{7}$$

Putting (1) for r in (7) and noting that, by (6),

$$\begin{aligned} (\sum \cos A)^2 &= (\sum \cos^2 A) + 2\sum \cos B \cos C \\ &= 1 - 2 \prod \cos A + 2 \sum \cos B \cos C \end{aligned}$$

we have, by some straightforward but tedious algebra, a reduction of (7) to

$$\begin{aligned} &2R^2(1 - 2c) \{ c^3 - c^2(\sum \cos A) + c(\sum \cos B \cos C) - \prod \cos A \\ &\quad = 2R^2(1 - 2c) \prod (c - \cos A), \end{aligned}$$

as required.

Analogues for ex-centres.

(Ia) $\quad (s - a) \sin \tfrac{1}{2}\theta + 2R \sin \theta \sin \tfrac{1}{2}\theta - r_a \cos \tfrac{1}{2}\theta$

$$= 4R \sin \tfrac{1}{2}(\theta - A) \cos \tfrac{1}{2}(\theta - B) \cos \tfrac{1}{2}(\theta - C).$$

(IIa) $\quad \tfrac{1}{2}(1 - c)(2c + 1)^2 I_a K_c^{\,2} - [2c(1 - c)R + r_a]^2$

$$= 2R^2(1 - 2c)(c - \cos A)(c + \cos B)(c + \cos C).$$

Proofs are similar to those of (I) and (II).

Comments.

Blundon's theorem (i) follows immediately from the product formula (I) on division by $\sin \tfrac{1}{2}\theta$.

Another consequence is the following : if $0 < \theta < \pi$ then $F(\theta)$ is positive or negative according as θ exceeds or is exceeded by an odd number of the angles A,B,C. (cf.[6])

If $c = \cos A$ then K_c is the point where the internal bisector of the angle meets the Euler line. This is seen by noting that both BAO and CAH equal $\tfrac{1}{2}\pi - C$, so AK_c also bisects the angle OAH. But angle bisectors divide the opposite side in the ratio of adjacent sides, so $OK_c : K_cH = OA : AH = R : 2R \cos A = 1 : 2c$, as required.

References

1 Altshiller-Court, N., College Geometry, Barnes and Noble, 1952, pp. 152-153.
2 Bankoff, L. and C.W.Trigg, A property of triangles, Math. Magazine 57 (1984) 294-296.
3 Blundon, W.J., Generalizations of a relation involving right angles, Am. Math. Monthly, 74 (1967) 566-567.
4 Guinand, A.P., Incentres and excentres viewed from the Euler line, Math. Magazine 58 (1985) 89-92.
5 Hobson, E.W., A Treatise on Plane Trigonometry, Cambridge (1939) ch 12.
6 Rennie, B.C., Crux Math., 11 (1985) 289, Problem 1088.

QUOTATION CORNER 20

Of Queensland's 103899 girls between the ages of 15 and 19, 2078 or 2 per cent receive supporting parent's benefits, while 1708 or 1 per cent of Victoria's 170755 teenage girls receive the benefit.

(From The Weekend Australian, 12-13 April 1986, page 22)

LARGE AND SMALL

An entire analytic function f satisfies:-
$f(x+iy) = O(\exp(-ax^2))$ for each y, with $a = a(y) > 0$ and $f(x+iy) = O(\exp(by^2))$ for all x with constant $b > 0$. Does it follow that $f(x+iy) = O(\exp(-ax^2+by^2))$ for some positive a and b as $x+iy \to \infty$? This problem is connected with a question asked by N.G.de Bruijn.

MAPPING THE SOLAR SYSTEM, JAMES COOK'S CONTRIBUTION

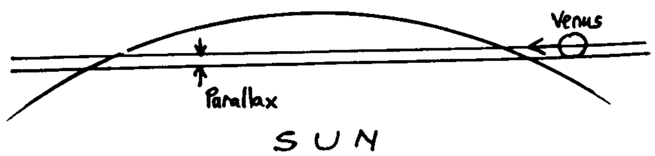
A. Brown

In an article in the Quarterly Journal of the Royal Astronomical Society (QJRAS 26 (1985) 289-294) R. D. Davies described the dedication of a cairn and plaque to commemorate a famous experiment of the Reverend Nevil Maskelyne, who was Astronomer Royal when James Cook was exploring the eastern coast of Australia. It will be remembered that Cook went to Tahiti to observe the 1769 transit of Venus, and his subsequent voyage around New Zealand and along the coast of Australia was more or less a spin-off from the astronomical purpose of the expedition. As Astronomer Royal, Maskelyne was closely involved, and indeed the observations of the transit were passed to him for analysis, since the observations in Tahiti were only part of an international effort to derive information from the transit. (See "The Transits of Venus" by Harry Woolf. The amount of effort that was put ino the transit observations in 1761 and 1769 is amazing, and the stories about the various expeditions make fascinating reading.)

The purpose of all this effort was to measure the distance between the Earth and the Sun, and thus establish the linear scale of the solar system. Kepler's third law allowed the major axis of any planetary orbit to be determined from the period, in terms of the Earth-Sun distance as unit. The only way of finding this distance was to find the parallax, that is the difference between the directions of the Sun from two points at a known distance apart, so that it was a problem of measuring an angle of only a few seconds of arc. Scientific instruments had been greatly improved in the eighteenth century, but measuring to a second of arc was still out of their reach. Edmund Halley (1656-1742) after whom the famous comet is named, had recommended determining the solar parallax by observing transits of Venus, for this involved measurement of times rather than angles.

The sketch shows how the angle of parallax is related to the distance apart of the two observers.

The distance SV from the Sun to Venus is (we now know) 67 million miles and VE from Venus to Earth is 26 million miles. Stations a thousand miles apart will give a parallax angle of 5 seconds.



The sketch above shows how the two observers see different paths of Venus across the Sun.

Earlier attempts to measure the distance from Earth to Sun had given rather uncertain results, and there was a sense of urgency about the 1769 transit, for the next transit of Venus was to be in 1874. Sad to say, the results from the 1769 observations were not as precise as had been hoped, although they helped to reduce the limits of uncertainty and to confirm that the Sun was further away from the Earth than had been believed for centuries.

To go back to Maskelyne, the plaque was set up in Perthshire to commemorate what is usually known as the Schiehallion experiment, in which Maskelyne measured the deflection of a plumbline due to the gravitational attraction of the mountain Schiehallion. This deflection is an obvious consequence of Newton's law of gravitation, and indeed Newton had considered it but calculated that the deflection would be too small to measure. Almost a hundred years later the situation had changed and, with better equipment available, Maskelyne realised that the effect was within reach. Schiehallion was selected because it had a smooth shape and was "tolerably detached from other hills". The surroundings were carefully surveyed and contour maps were drawn, the idea of contour mapping was invented for this purpose by Charles Hutton, the mathematician who did all the data reduction. In the summer of 1774 the latitude was determined astronomically at observing stations halfway up the mountain on the North and South sides, and the observed difference was compared with what it should have been from the survey. This gave the sum of the deflections as 11·6 seconds of arc with an error estimated at less than one second.

All this is set out in greater detail in the article by Davies. The connection with Cook is simple but important. To quote Davies directly - "Funds were made available from the residual grant by George III for the 1769 transit of Venus expedition made in the South Seas by the "Endeavour" with James Cook as captain."

George III had made a personal grant of £4000 towards Cook's expedition. The residue was used to provide a bust of the King and to finance the Schiehallion experiment. That is the story as I remember it, although I am sure that no modern scientist is going to believe that there could possibly have been a residue.

Essentially Maskelyne was measuring the mass of the Earth, say m, and at the same time obtaining a value for the gravitational constant G, for the product Gm was already known accurately from the local force of gravity and the size of the Earth, with confirmation from the Moon's orbit. The horizontal attraction of the mountain contains a factor G but the vertical force (Gm/R^2) also contains this factor, and so the deflection of the plumbline therefore gives m.

Maskelyne's experiment was the first determination of G, it was found to be about 25 per cent too high when Cavendish made his laboratory determination 23 years later. The error turned out not to be in the astronomy or the surveying over which so much care had been taken, but in the estimation of the rock density inside the mountain.

The Schiehallion experiment then links up with the transit of Venus, for when G and the distance of the Sun are known we can get the mass of the Sun from the planetary orbits. The contributions of Maskelyne and Cook were not the last words on these problems, but were important in their day.

FROM CAPTAIN COOK'S JOURNAL

(During the 3-month stay at Tahiti Cook kept his Journal according to Civil Time, with the days going from midnight to midnight)

Thursday, June 1st.- This day I sent Lieut. Gore in the Long boat to York Island with Dr. Monkhouse and Mr. Sporing (a Gentleman belonging to Mr. Banks) to Observe the Transit of Venus, Mr. Green having furnished them with Instruments for that purpose. Mr. Banks and some of the Natives of this Island went along with them.

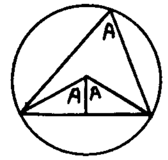
Friday, 2nd.- Very early this morning Lieut. Hicks, Mr. Clark, Mr. Pickersgill and Mr. Saunders went away in the Pinnace to the Eastward, with orders to fix upon some Convenient situation upon this Island, and there to observe the Transit of Venus, they being likewise provided with instruments for that purpose.

Saturday, 3rd.-This day proved as favourable to our purpose as we could wish. Not a Cloud was to be seen the whole day, and the Air was perfectly Clear, so that we had every advantage we could desire in observing the whole of the Passage of the planet Venus over the Sun's Disk. We very distinctly saw an Atmosphere or Dusky shade round the body of the planet, which very much disturbed the times of the Contact, particularly the two internal ones. Dr. Solander observed as well as Mr. Green and myself, and we differ'd from one another in Observing the times of the Contact much more than could be expected. Mr. Green's Telescope and mine were of the same Magnifying power, but that of the Doctor was greater than ours. It was nearly calm the whole day, and the Thermometer Exposed to the Sun about the Middle of the day rose to a degree of heat we have not before met with.

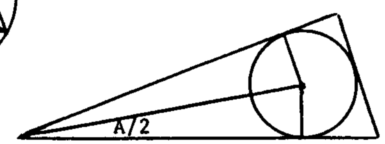
TRIANGLES WITH NEGATIVE ANGLES

Usually in studying plane triangles we take the sides a, b, and c to be positive and the angles A, B, and C to be between 0 and 180 degrees. Let R be the radius of the circumcircle, and r the radius of the inscribed circle, and write s for the semiperimeter. Let r_1, r_2, and r_3 be the radii of the three escribed circles. This gives us twelve parameters describing the triangle; there are nine relations between them, for a triangle has three degrees of freedom. It is easy to find a few of these relations as follows.

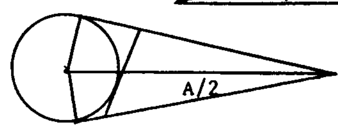
$a = 2R \sin A$



$s-a = r \cot A/2$



$r_1 = s \tan A/2$



$2s = a + b + c$ and $A + B + C = 180^\circ$

Denote this set of equations by (1).

Any equation connecting the twelve parameters may be verified just by manipulation of the equations (1). To see this, observe that we can eliminate the radii r, r_1, r_2 and r_3 of the tritangent circles, and then eliminate s, a, b and c, leaving an equation in R and the three angles. Since R is the only length involved it has to cancel. Then we can eliminate one of the angles and the equation

becomes a relation between two angles, valid in an open set of the product space, and generally it can be verified (though perhaps a logician might insist on the possibility that we may come to an equation that can neither be proved nor disproved)

For example the cosine rule may be obtained from the trigonometrical identity

$$\sin^2(180^\circ-B-C) = \sin^2 B + \sin^2 C - 2\sin B\sin C\cos(180^\circ-B-C).$$

As another example we may calculate

$$\begin{aligned} r &= (s-a)\tan A/2 = R(\sin B+\sin C-\sin A)\tan A/2 \\ &= 2R\cos A/2\ \cos(B/2-C/2)\tan A/2 - 2R\sin^2 A/2 \\ &= 2R\sin A/2(\cos(B/2-C/2)-\cos(B/2+C/2)) \\ &= 4R\ \sin A/2\ \sin B/2\ \sin C/2 \qquad ..(2) \end{aligned}$$

As a third example, $(s-2R-r)/R$

$$\begin{aligned} &= \sin A + \sin B + \sin C - 2 - 4\sin A/2\sin B/2\sin C/2 \\ &= -(1-\sin A)+2\cos A/2\cos(B/2-C/2)-1 \\ &\qquad -2\sin A/2(\cos(B/2-C/2)-\sin A/2) \\ &= -(\cos A/2-\sin A/2)^2+2(\cos A/2-\sin A/2)\cos(B/2-C/2)-\cos A \\ &= (\cos A/2-\sin A/2)(-\cos A/2+2\cos(B/2-C/2)-\cos A/2) \\ &= 2(\cos A/2-\sin A/2)(\cos(B/2-C/2)-\sin(B/2+C/2)) \\ &= 2(\cos A/2-\sin A/2)(\cos B/2-\sin B/2)(\cos C/2-\sin C/2) \qquad ..(3) \end{aligned}$$

This has paved the way for two questions. Is it meaningful to have triangles with negative angles? and is it useful? The answers are "yes" and "possibly".

Let A, B, ... be the twelve variables describing a triangle, define a new family A', B', ... by:-

$A' = 360^\circ - A$	$B' = -B$	$C' = -C$
$a' = a$	$b' = b$	$c' = c$
$s' = s$	$R' = -R$	$r' = -r$
$r_1' = -r_1$	$r_2' = -r_2$	$r_3' = -r_3$ ...(4)

It can be checked that these new parameters satisfy equations like (1), and therefore they will also satisfy any equation obtained from them. For example from (3) we get:-

$$s'-2R'-r' = 2R'(\cos A'/2-\sin A'/2)(\ \dots\ )(\ \dots\ ) \qquad ..(3')$$

Expressing this in terms of the original parameters,

$$s+2R+r = 2R(\cos A/2+\sin A/2)(\cos B/2+\sin B/2)(\ \dots\ ) \qquad ..(5)$$

We have in this way obtained another property of triangles without having to do any real work for it. Multiplying (2) and (5) gives

$$s^2 - (2R+r)^2 = 4R^2\cos A \cos B \cos C$$

which (Murray Klamkin tells me) was in the Educational Times in 1897. It is hard to find anything new about triangles.

For another example take instead of (4) the transformation:-

$$A' = -A \qquad B' = A+C \qquad C' = A+B$$
$$a' = -a \qquad b' = b \qquad c' = c$$
$$s' = s-a \qquad R' = R \qquad r' = -r_1$$
$$r_1' = -r \qquad r_2' = (s-a)\cot B/2 = (s-a)(s-b)/r$$
$$r_3' = (s-a)\cot C/2 = (s-c)\cot A/2 = (s-a)(s-c)/r \qquad ...(6)$$

As before, the new parameters satisfy the equations corresponding to (1), and by the same kind of argument, using (2), we get

$$r_1 = 4R \sin A/2 \cos B/2 \cos C/2 \qquad ...(7)$$

which by (1) implies

$$s = 4R \cos A/2 \cos B/2 \cos C/2$$

and by comparison with (2) we find that the product of the tangents of the half-angles is r/s. However this is not really new, because we already know rather more about the tangents of the half-angles. From the first two equations of (1) it follows that

$$s-4Rt/(1+t^2) = r/t$$

when $t = \tan A/2$, and therefore also when $t = \tan B/2$ and when $t = \tan C/2$, so that we have found the cubic equation

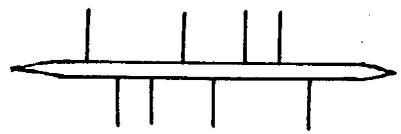
$$st^3 - (4R+r)t^2 + st - r = 0$$

for the tangents of the half-angles. This is essentially Blundon's Theorem (see Andrew Guinand's article, page 4191 above).

It seems that this trick of giving a triangle negative sides or angles is unlikely to find new facts, but it sometimes gives an easy way to calculate old results.

STALE NEWS

The winning boat in the Trial Eights race of the Cambridge University Boat Club at Ely in December 1963 was rigged like this:-



This rig is in accordance with the theory of Thue sequences as explained in the JCMN eighteen years later. See "The Rig of a Rowing Boat" in issue 26, (September 1981) p. 3037, "Of the Earth Murphy" and "The Rig of a Rowing Boat" by H. O. Davies in issue 27, p. 3055 and "Of the Earth Murphy" by J. B. Parker in issue 28, p. 3085.

EDITORIAL

The JCMN for its first eight years, 1975-1983, was published by the Mathematics Department of the James Cook University of North Queensland, address:-

Post Office James Cook,
North Queensland, 4811,
Australia.

The issues 1-31 from this period have been reprinted in three paperback volumes, on sale from the Mathematics Department for 10 Australian dollars each, including postage.

I now have no connection with the James Cook University. Although my wife and I have plans to move to South Australia we have made little progress in that direction, and this issue is being prepared (in June 1986) at our old house, 69, Queen's Road, Hermit Park, NQ 4812. However we expect to make the move in the next few months, and readers are recommended to take our address as being

66, Hallett Road, Burnside,
S. Aust. 5066,
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Basil Rennie.