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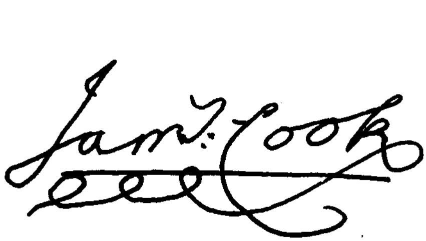

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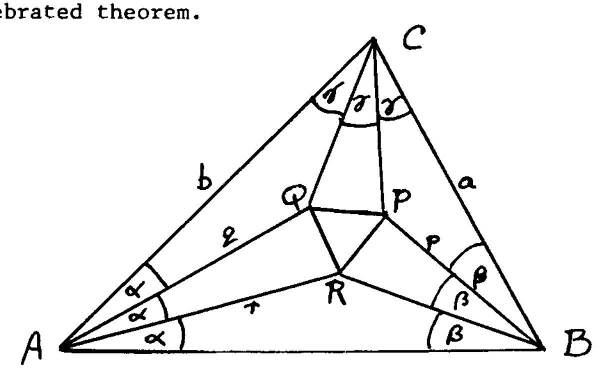
COMPUTER PROOF OF MORLEY'S THEOREM

G. Szekeres

After a recent one-day course on MACSYMA at the University of New South Wales I decided to test my newly-acquired skills by producing a machine proof of Morley's celebrated theorem.



Position the triangle ABC so that A is the origin (0,0) and B is (0,1). Let α, β and γ be the 1/3 angles (so that $\alpha + \beta + \gamma = 60^o$). Let $r = AR$, $q = AQ$, $p = BP$, $a = BC$ and $b = AC$ as shown.

$$r = \frac{\sin\beta}{\sin(\alpha+\beta)}$$

$$a = \frac{\sin 3\alpha}{\sin 3(\alpha+\beta)} \qquad b = \frac{\sin 3\beta}{\sin 3(\alpha+\beta)}$$

$$p = \frac{a\sin\gamma}{\sin(60^o - \alpha)} \qquad q = \frac{b\sin\gamma}{\sin(60^o - \beta)}$$

The vertices of the little triangle are

$$\begin{array}{lll} P & : & (1 - p\cos 2 3, \quad p\sin 2\beta) \\ Q & : & (q\cos 2\alpha, \quad q\sin 2\alpha) \\ R & : & (r\cos\alpha, \quad r\sin\alpha) \end{array}$$

From here the machine can dutifully express the coordinates of P, Q and R as rational functions (with integer coefficients) of the five symbols $s = \sin\alpha$, $t = \sin\beta$, $\sqrt{1 - s^2}$, $\sqrt{1 - t^2}$ and $\sqrt{3}$. Thereupon I asked the machine to calculate the horrendous expression for $||Q - R||^2/||P - R||^2$, "rationalizing" the denominators at each step. In a few minutes the machine produced 1 for the value of the last expression, so proving the theorem. I imagine a great deal of Euclidean plane geometry can be converted to a "trivial" machine proof of this kind.

Happy tenth birthday, JCMN.

PROBLEMS

P. Erdos

(a) Let G be a graph of n vertices and $[n^2/4] + 1$ edges. Prove that it contains a triangle with each vertex of degree $\geq n/3$. Also that the factor 1/3 is best possible.

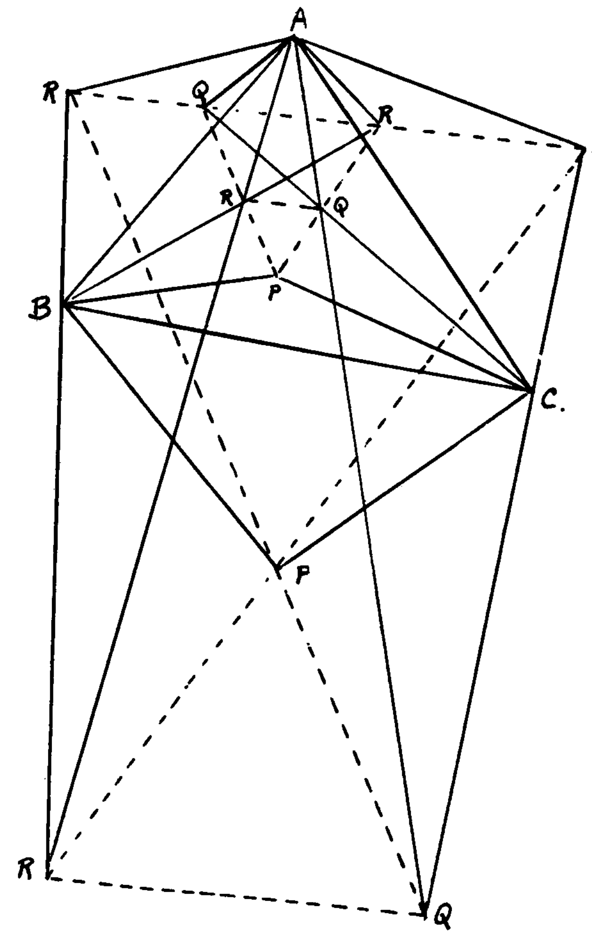
(b) Is it true that given 23 out of 30 consecutive integers one can always find 4 of them pairwise relatively prime? (For 22 instead of 23 it becomes untrue, as may be shown by the case of all the 22 multiples of 2, 3 or 5 among the 30.)

THE EIGHTEEN MORLEY TRIANGLES

In trying to interpret the computer proof (p.4140 above) by G. Szekeres of Morley's theorem, it helps to think about the philosopy of plane trigonometry. The simple view, as taught in my youth, is that lengths are simply distances and cannot be negative, and that the angles of a triangle are all positive and have sum = 180 degrees. The alternative view is that lengths are directed segments along directed lines, and that angles are rotations. In this algebraic formulation a triangle with (in the ordinary sense) sides a, b and c, and angles A, B and C could equally well be described as having sides a, -b and c, and angles A + 180°, B and C + 180°. The sine and cosine rules for the triangle remain true, but, for instance, the inscribed circle becomes an escribed circle, for the algebraic formulation does not distinguish between the inside and the outside of the triangle, or between the internal and external bisectors of an angle. In the algebraic form of trigonometry there are two ways of bisecting, and three ways of trisecting, any angle.

The computer is instructed to take a strictly algebraic view of trigonometry, and so anything that it proves about trisectors of angles will be equally valid for all three ways of trisection.

Therefore we should look to see exactly what the computer has proved. Taking any triangle ABC (which we may assume described in the simple way with all sides and angles positive) it has constructed the equilateral triangle PQR by using angles α, β and γ. What has it assumed about them? Only that $3\alpha = A$, $3\beta = B$, $3\gamma = C$ and $\alpha + \beta + \gamma = \pm 60^{\circ}$. Note the $\pm$ sign; what the computer assumes about the sum of the three angles is that its cosine is 1/2 and its sine is $\surd 3/2$ where $\surd 3$ is an algebraic indeterminate with square equal to 3, and so when we interpret this algebraic indeterminate as a real number it can be positive or negative. There are 18 possible



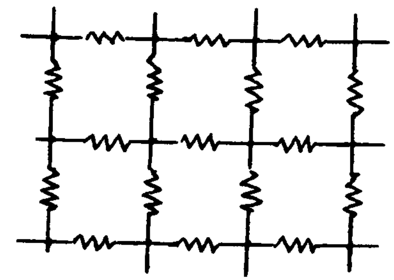
geometrical interpretations of the three angles α, β and γ to which the proof applies. They are as follows:

$A/3$,	$B/3$	$C/3$	(the original case)
$A/3 + 120^\circ$,	$B/3 + 120^\circ$,	$C/3 + 120^\circ$	
$A/3 - 120^\circ$,	$B/3 - 120^\circ$,	$C/3 - 120^\circ$	
$A/3$,	$B/3 + 120^\circ$,	$C/3 + 120^\circ$	(and two more like this)
$A/3 - 120^\circ$,	$B/3$,	$C/3$	(and two more like this)
$A/3 + 120^\circ$,	$B/3 - 120^\circ$,	$C/3 - 120^\circ$	(and two more like this)
$A/3 + 120^\circ$,	$B/3 - 120^\circ$,	$C/3$	(and five more like this)

The eighteen equilateral triangles have combinatorial properties awaiting investigation. Each vertex of one triangle is clearly also a vertex of one of the others. A corollary to Bricard's proof of Morley's theorem is that the angle between AB and QP is $\alpha - \beta$ in the notation above, and it follows that all the 18 Morley triangles are parallel to one another. Each side of a Morley triangle is also a side of some of the others, but of how many of the others?

ELECTRICAL NETWORKS

(JCMN 18. p.13 and 20, p.61)



Denote by $R(m, n)$ the resistance between nodes $(0, 0)$ and (m, n) in a two-dimensional square lattice of one-ohm resistors. It has long been known that $R(m, n)$ is the integral mean of the doubly periodic function

$$(1 - \cos(mx+ny))/(2 - \cos x - \cos y).$$

To establish this, interpret $R(m, n)$ as the voltage of the node (m, n) below that of the origin when a current of two amperes is fed in to the origin. Then it may be seen that there is a recurrence relation

$$R(m, n+1) + R(m, n-1) + R(m+1, n) + R(m-1, n) - 4R(m, n) = 2\delta_{mn}$$

and inequalities $0 \leqslant R(m, n) \leqslant |m| + |n|$ and the obvious symmetry conditions $R(m, n) = R(-n, m) = R(n, m)$.
We shall see eventually that these give existence and uniqueness.

Putting $u = \exp imx$ and $v = \exp iny$, we set up the generating function $f(u, v) = \sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} R(m, n)\, u^m v^n$.
The recurrence relation may be written

$$(u + 1/u + v + 1/v - 4)\ f(u, v) = 2$$

and the theory of Fourier series shows that the Fourier coefficient $R(m, n)$ is the integral mean of

$$u^{-m} v^{-n} \; f(u, v) = \frac{1 - \cos(mx+ny) + i \sin(mx+ny)}{2 - \cos x - \cos y}$$

where the sine term may clearly be discarded. We seem to have established our result, but the careful analyst will uncover a few subtleties.

The difficulty is that $f(u, v) = 2/(u + 1/u + v + 1/v - 4)$ is not an absolutely integrable function of x and y over one period, the Fourier coefficients do not tend to zero, and classical Fourier theory does not apply. This objection may be dealt with by using the theory of generalized (or "improper") functions. Since $|R(m, n)| \leqslant |m| + |n|$ the trigonometric series $\sum\sum R(m, n) \exp i(mx + ny)$ must converge, and its sum, the generating function $f(u, v)$ must exist. From the recurrence relation we know that

$$(u + 1/u + v + 1/v - 4) \; f(u, v) = 2, \text{ for some A.}$$

Remember that now we are dealing with generalized functions. The solution is (for some A, B, C and D)

$$f(u, v) = 2/(u + 1/u + v + 1/v - 4) + A\,\delta(u - 1)\,\delta(v - 1)$$
$$+ B\,\delta(u - 1)\,\delta'(v - 1) + C\,\delta'(u - 1)\,\delta(v - 1) + D\,\delta'(u - 1)\,\delta'(v-1).$$

Since $R(m, n)$ is an even function of m and of n it follows that f is an even function of x and of y, and so the last three terms must be discarded. The Fourier coefficient $R(m, n)$ is the integral mean (over the (x, y) in one period) of

$$u^{-m} v^{-n} f(u, v) = \frac{-\cos(mx+ny) + i \sin(mx+ny)}{2 - \cos x - \cos y} + A\,\delta(u - 1)\,\delta(v - 1).$$

The integral mean of the second term is some constant, call it B. (It is, of course, a known multiple of A.) Now note that the function $1/(2 - \cos x - \cos y)$ has an integral mean, call it C. This C is finite because it is an integral mean in the sense of generalized functions, it is a Hadamard finite part of the classical integral mean which is, of course, infinite.

The Fourier coefficient $R(m, n)$ is therefore the integral mean of $\dfrac{1 - \cos(mx + ny)}{2 - \cos x - \cos y} + B - C$, where B is an arbitrary constant. But now we can use the fact that $R(0, 0) = 0$, and so determine that $B = C$. Therefore $R(m, n)$ is the integral mean of $(1 - \cos(mx+ny))/(2 - \cos x - \cos y)$. This function is integrable in the classical sense, and the integral mean may therefore be determined by classical analysis.

J.B. Parker (in JCMN 20, p.61) has determined the values on the diagonals, where $m = n$

$$R(m, m) = (2/\pi)(1 + 1/3 + 1/5 + \ldots + 1/(2m - 1)).$$

The importance of this result lies in the fact that the recurrence relation

$$R(m, n+1) + R(m, n-1) + R(m+1, n) + R(m-1, n) - 4R(m, n) = 2\delta_{mn}$$

with the symmetry conditions

$$R(m, n) = R(n, m) = R(-n, m)$$

and the values on the diagonals, together determine the solution uniquely. The proof of this assertion is a double induction, slightly tedious, and so we shall not go into it. One conclusion easily seen is that each $R(m, n)$ is of the form $P(m, n) + (2/\pi)\,Q(m, n)$ where P and Q are rational. Since P and Q each satisfies a simple recurrence relation and their values are known on the diagonals, they can be calculated with finitely many additions and subtractions. For example, the values at points adjacent to the diagonals are (for any positive m)

$$R(2m - 1, 2m) = (4/\pi)(1 + 1/5 + \ldots + 1/(4m - 3)) - 1/2$$

$$R(2m, 2m + 1) = (4/\pi)(1/3 + 1/7 + \ldots + 1/(4m - 1)) + 1/2$$

The condition $0 \leqslant R(m, n) \leqslant |m| + |n|$, which appeared to play only a minor part in the calculation, is in fact essential; without it the solution would not be unique. In fact, we could take any values of $R(1, 1)$, $R(2, 2)$ etc., and find a solution satisfying all the other conditions.

TRIANGULAR SOLITAIRE

Blanche Descartes

Most solitaire boards are cross-shaped. A few are triangular, as in Fig.1, with 15 holes. As in ordinary solitaire, we start by filling every hole with a marble, then removing one. If at any time there are three consecutive holes in a line with the first two filled and the third empty we may move the marble from the first hole to the third, jumping over the second marble, which is then removed. Thus, if the holes are labelled as in Fig.2, and holes h and e are occupied and c is empty, we can move the marble in h into c, removing the marble in e. We try to finish leaving one marble only, in the hole that was empty at the start.

0	a	A
0 0	b c	B C
0 0 0	d e f	C A B
0 0 0 0	g h i j	A B C A
0 0 0 0 0	k l m n o	B C A B C
Fig.1	Fig.2	Fig.3

Suppose that the holes are divided into 3 classes, A, B and C, as in Fig.3. Let n_A = the number of occupied holes of class A, and let n_B and n_C be similarly defined. Then a move does not change the parities of the 3 differences, $n_A - n_B$, $n_B - n_C$ and $n_C - n_A$, so they have the same parities at the end of the game as at the beginning. This makes certain positions at the end of the game impossible. But since, when every hole is full, $n_A \equiv n_B \equiv n_C \equiv 1 \bmod 2$, if only one hole is empty at the start, and only one hole occupied at the finish, the two holes must be of the same class.

Let the notation "hc" mean a move of a marble from hole h to the empty hole c, removing the marble that was in the intermediate hole e. If we start by leaving hole a

vacant, a solution is:

da, fd, af, jc, gb, le, nl, km, mf, cj, of, fd, da.

If the initial vacant hole were b, a solution would be:

gb, md, om, ch, jc, af, ln, fm, bg, nl, km, md, gb.

If the initial vacant hole were d, a solution would be:

md, om, jh, cj, df, kd, le, bg, jc, ch, md, gb, ad.

But if e is left vacant I suspect that there is no sequence of moves giving a final position with only one hole occupied, either at e or anywhere else. Is this true? If so, is there a neat proof of it?

COMBINATORIAL PROBLEM

Suppose that we have finitely many finite sets, with A(2) of them having two elements, A(3) having 3, and so on. The intersection of any two of the sets has no more than one element. Find, in terms of A(2), A(3), ..., a lower bound for the number n of elements in the union of all the sets.

Conjecture:

$$n(n-1) \geq \sum_{2}^{\infty} r(r-1)A(r) = 2A(2) + 6A(3) + 12A(4) + \ldots$$

SUBSETS OF A FINITE SET

Jamie Simpson

Let A be a finite set and $\mathcal{A} = \{A_1, A_2, A_3, \ldots\}$ be a collection of subsets of A, possibly including the empty set $\emptyset$. We say that a subset A_i is <u>elementary</u> in $\mathcal{A}$ if it is not the union of any subcollection of $\mathcal{A} \setminus A_i$.

If A has cardinality n and $\mathcal{A}$ consists entirely of elementary subsets, what is the maximum cardinality of $\mathcal{A}$? Call this function M(n).

M(3), for instance, equals 5 since if A is $\{1,2,3\}$ we can use

$$\mathcal{A} = \{\emptyset, \{1\}, \{1,2\}, \{2,3\}, \{3,1\}\} .$$

For low values of n we have M(0) = 1, M(1) = 2, M(2) = 3, M(3) = 5, M(4) = 8, and M(5) = 13. This sequence will be familiar to readers interested in the dynamics of rabbit populations. Unfortunately M(6) is at least 23, not 21 as we might hope.

PROBLEMS

(JCMN 35, p.4068 and 36, p.4096)

C.C. Rousseau

P. Erdös asked for five points in the plane, no three in a line, no four on a circle, and no one equidistant from three others, such that they determine four distinct distances, one occurring once, one twice, one three times, and one four times. One example was given in JCMN 36, using cubic irrationals.

Here is an example constructible by Euclidean methods.

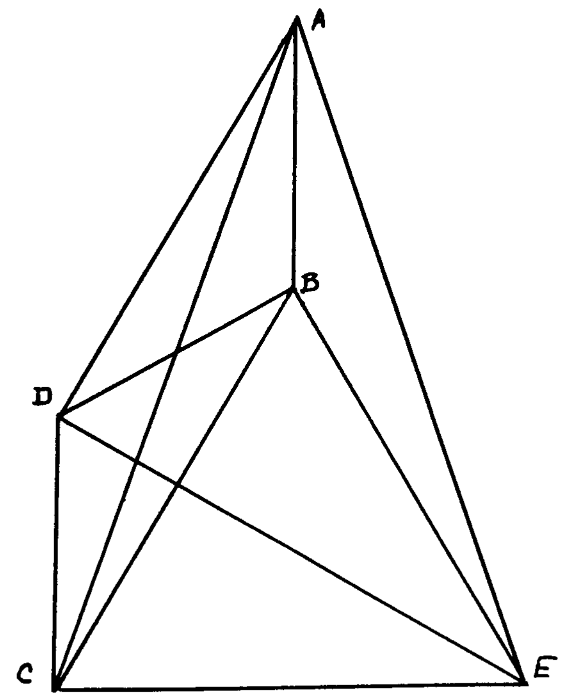
In Cartesian coordinates

A is $(\sqrt{3}/2,\ 5/2)$ B is $(\sqrt{3}/2,\ 3/2)$

C is $(0,\ 0)$ D is $(0,\ 1)$ and

E is $(\sqrt{3},\ 0)$.

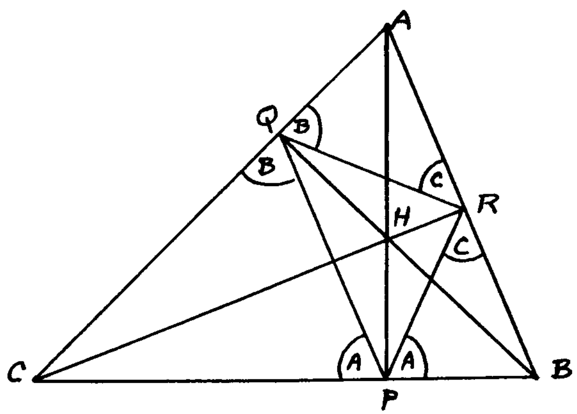
$$AD = BC = BE = CE = \sqrt{3}$$
$$AB = BD = DC = 1$$
$$AE = AC = \sqrt{7}$$
$$DE = 2$$



SYMMEDIAN POINT

(JCMN 33, p.4032 and 35, p.4075)

Consider any triangle ABC and its pedal triangle PQR. Since CQHP is cyclic, angle CPQ = angle CHQ = 90° − angle ACH = A. The angles are therefore as shown



The pedal triangle PQR has sides a cos A, b cos B and c cos C, and has angles $180^\circ - 2A$, $180^\circ - 2B$ and $180^\circ - 2C$. The area of PQR is therefore 2 cos A cos B cos C times the area of ABC. By adding areas of triangles we find that

$$\cos^2 A + \cos^2 B + \cos^2 C + 2\cos A\cos B\cos C = 1.$$

Because $\cos^2 A + \cos^2 B + \cos^2 C = 3$ (Arithmetic mean of $\cos^2 A$, etc.) ≥ 3 (Geometric mean of $\cos^2 A$, etc.) $= 3q^{2/3}$ where $q = \cos A\cos B\cos C$, the identity above tells us that $\quad 3q^{2/3} + 2q \leq 1$

or $\quad (q^{1/3}+1)^2(2q^{1/3}-1) \leq 0$

Therefore $q \leq 1/8$ and

$$8\cos A\cos B\cos C \leq 1,$$

with equality only in the case of an equilateral triangle. This inequality was suggested in JCMN 35 on page 4088.

For an alternative proof note that for obtuse or right-angled triangles the inequality is trivial, and for acute-angled triangles the inequality of arithmetic and geometric means and the concavity of the cosine function of acute angles tells us that

$$q^{1/3} \leq (\cos A + \cos B + \cos C)/3 \leq \cos(A+B+C)/3 = 1/2.$$

Now consider the two questions asked in JCMN 33 and 38. Is the symmedian point on the same side of the Euler axis as the incentre? And is it (like the incentre) always inside the "critical circle" (on GH as diameter)? The answer is YES to both questions.

Using trilinear coordinates we have the following:

Euler axis: $f(x, y, z) = 0$ where
$f(x, y, z) = x\sin A(\sin 2B - \sin 2C) + y\sin B(\sin 2C - \sin 2A)$
$+ z\sin C(\sin 2A - \sin 2B)$.

Symmedian point, K,	(a, b, c)
Incentre, I,	(1, 1, 1)
Centroid, G,	(bc, ca, ab)
Orthocentre, H,	(sec A, sec B, sec C)
Centre of critical circle	(2 cos(B−C) − cos A, 2 cos(C−A) − cos B, 2 cos(A−B) − cos C)
Critical circle,	$g(x, y, z) = 0$ where

$$g = x^2\sin 2A + y^2\sin 2B + z^2\sin 2C - yz\sin A - zx\sin B - xy\sin C.$$

For any point with homogeneous coordinates (x,y,z), provided that $ax + by + cz \geq 0$, the sign of $f(x,y,z)$ tells us on which side of the Euler axis the point is. For the incentre

$$f = \sin A(\sin 2B - \sin 2C) + \sin B(\sin 2C - \sin 2A) + \sin C(\sin 2A - \sin 2B)$$

$$= 2\sin A\sin(B-C)\cos(B+C) + 2\sin B\sin C(\cos C - \cos B) - \sin 2A(\sin B - \sin C)$$

$$= 2\sin\frac{B-C}{2}\left(-2\sin A\cos\frac{B-C}{2}\cos A + 2\sin B\sin C\sin\frac{B+C}{2} - \sin 2A\cos\frac{B+C}{2}\right)$$

$$= 2\sin\frac{B-C}{2}\left(-4\sin A\cos A\cos\frac{B}{2}\cos\frac{C}{2} + 2\sin B\sin C\cos\frac{A}{2}\right)$$
$$= 16\sin\frac{B-C}{2}\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}\left(\cos(B+C)\sin\frac{A}{2} + \sin\frac{B}{2}\sin\frac{C}{2}\right)$$
$$= 8\sin\frac{B-C}{2}\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}\left(\sin\left(\frac{A}{2}+B+C\right) - \sin\left(B+C-\frac{A}{2}\right) + \cos\frac{B-C}{2} - \cos\frac{B+C}{2}\right)$$
$$= 8\sin\frac{B-C}{2}\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}\left(-\cos\left(\frac{B+C}{2}-A\right) + \cos\frac{B-C}{2}\right)$$
$$= 16\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}\sin\frac{A-C}{2}\sin\frac{B-A}{2}\sin\frac{C-B}{2}$$

For the symmedian point $(\sin A, \sin B, \sin C)$,

$$f = \sin^2 A(\sin 2B - \sin 2C) + \sin^2 B(\sin 2C - \sin 2A) + \sin^2 C(\sin 2A - \sin 2B)$$
$$2f = -\cos 2A(\sin 2B - \sin 2C) - \cos 2B(\sin 2C - \sin 2A) - \cos 2C(\sin 2A - \sin 2B)$$
$$= \sin(2A-2B) + \sin(2B-2C) + \sin(2C-2A)$$
$$f = \sin(A-B)\cos(A-B) + \sin(B-A)\cos(B+A-2C)$$
$$= \sin(A-B)(\cos(A-B) - \cos(B+A-2C))$$
$$= 2\sin(C-B)\sin(B-A)\sin(A-C).$$

Comparing this with the expression found previously we see that the two points are on the same side of the Euler axis, and for isosceles triangles both points are on the Euler axis.

To verify that the symmedian point is inside the critical circle, note that the point $(\sin B, -\sin A, 0)$ is on the line at infinity and $g(\sin B, -\sin A, 0) =$

$$\sin^2 B\sin 2A + \sin^2 A\sin 2B + \sin A\sin B\sin C = 3\sin A\sin B\sin C \geq 0$$

and therefore g is positive outside and negative inside the critical circle.

$g(\sin A, \sin B, \sin C) =$

$$\sin^2 A\sin 2A + \sin^2 B\sin 2B + \sin^2 C\sin 2C - 3\sin A\sin B\sin C.$$
$$2g = (1-\cos 2A)\sin 2A + (1-\cos 2B)\sin 2B + (1-\cos 2C)\sin 2C - 6\sin A\sin B\sin C$$
$$= (1-\cos 2A)\sin 2A + 2\cos(B-C)\sin(B+C) - \cos(2B-2C)\sin(2B+2C) - 6\sin A\sin B\sin C$$
$$= \sin A(-2\cos(B+C) + 2\cos(B-C) - 2\cos A(\cos(2B+2C) - \cos(2B-2C)) - 6\sin B\sin C)$$
$$= \sin A(4\sin B\sin C + 4\cos A\sin 2B\sin 2C - 6\sin B\sin C)$$
$$g = \sin A\sin B\sin C(8\cos A\cos B\cos C - 1)$$

which we saw to be negative at the beginning of this note.

For the incentre $(1, 1, 1)$ the function g is

$$\sin 2A + \sin 2B + \sin 2C - \sin A - \sin B - \sin C$$
$$= -4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}\left(1 - 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$$
$$= -(\sin A + \sin B + \sin C)\left(1 - 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$$

which can be seen to be negative by observing that the inequality

$$8\cos A\cos B\cos C \leq 1$$

may be applied to a triangle (such as that formed by the three ex-centres) with angles $(\pi - A)/2$, etc. The symmedian point, like the incentre, is strictly inside the critical circle except in the case of an equilateral triangle for which the critical circle becomes a single point.

REDUCTIO AD ABSURDUM

Blanche Descartes

The question is often asked, can French mathematicians be conscripted into the armed forces? To settle this question, let us suppose that they could be conscripted. Remember also that the French call a mathematician "tête-à-x".

When his military service was over, such a mathematician would be an ex-conscript, or written algebraically,

$$\text{exconscrit} = \text{mathematicien} = \text{tête-à-x} = \theta x.$$

Divide both sides of this equation by ex. We get

$$\text{conscrit} = \frac{\theta}{e} = \text{tête assurée},$$

which is clearly absurd. Hence French mathematicians cannot be conscripted.

FROM CAPTAIN COOK'S JOURNAL

Monday 3rd September 1770

Here we saw Cocoa nutt Trees, Bread Fruit Trees, and Plantain Trees, but we saw no fruit but on the former, and these were small and Green; the other Trees, Shrubs, Plants, etc., were likewise such as is common in the So. Sea Islands and in New Holland.

Upon my return to the Ship we hoisted in the boat and made sail to the Westward, with a design to leave the Coast altogether. This, however, was contrary to the inclination and opinion of some of the Officers, who would have me send a Party of Men ashore to cut down the Cocoa Nutt Trees for the sake of the Nutts; a thing that I think no man living could have justified, for as the Natives had attacked us for meer landing without taking away one thing, certainly they would have made a Vigerous effort to have defended their property; in which case many of them must have been kill'd, and perhaps some of our own people too, and all this for 2 or 300 Green Cocoa Nutts, which when we had got them would have done us little service; besides nothing but the utmost necessity would have obliged me to have taken this method to come at refreshments.

BESSEL FUNCTIONS

What is the simplest way to calculate

$$\int_0^\infty J_0(x)\,dx \quad ?$$

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Your Editor is now also Editor of "The Mathematical Scientist". This journal (TMS for short) was founded by CSIRO in 1976, and from 1985 onwards is being taken over by the Australian Mathematical Society. The theme of TMS is the relevance of mathematics to the world in which we live, and the use of mathematical models in all branches of science. It is primarily a research journal, wanting to publish new work, but it will also print historical notes or surveys or unsolved problems. If you have written anything that seems appropriate to TMS please send it (preferably two copies) to me or any member of the Editorial Board (see below).

EDITORIAL

The JCMN for its first eight years, 1975-1983, was published by the Mathematics Department of the James Cook University of North Queensland, address:

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The issues 1-31 from this period have been reprinted as paperback volumes:

Volume 1 (Issues 1-17)
Volume 2 (Issues 18-24) (out of print)
Volume 3 (Issues 25-31)

We hope to reprint Volume 2 soon. These volumes are available for $10 (Australian) each, including postage, from the Head of the Mathematics Department. I should explain that I am now Head of Department, but will retire at the end of December and leave the University. Since Issue 32 (October 1983) I have edited JCMN and arranged the printing and distribution. In 1986 my wife and I plan to leave Townsville and go to

66 Hallett Road, Burnside,
South Australia, 5066, Australia.

but we expect to produce Issue 39 (February 1986) from Townsville (address as on page 4138).

Basil Rennie